STELLA MARY'S COLLEGE OF ENGINEERING

(Accredited by NAAC, Approved by AICTE - New Delhi, Affiliated to Anna University Chennai)

Aruthenganvilai, Azhikal Post, Kanyalumari District, Tamilnadu - 629202.

ME8594 DYNAMICS OF MACHINES

(Anna University: R2017)

Prepared By

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Aruthenganvilai, Kallukatti Junction Azhikal Post, Kanyakumari District-629202, Tamil Nadu.

DEPARTMENT OF MECHANICAL ENGINEERING

COURSE MATERIAL

REGULATION	**2017**
YEAR	**III**
SEMESTER	**05**
COURSE NAME	**DYNAMICS OF MACHINES**
COURSE CODE	**ME8594**
NAME OF THE COURSE INSTRUCTOR	**Mr. P. Vijayan**

SYLLABUS:

UNIT I FORCE ANALYSIS 12

Dynamic force analysis – Inertia force and Inertia torque– D Alembert's principle –Dynamic Analysis in reciprocating engines – Gas forces – Inertia effect of connecting rod– Bearing loads – Crank shaft torque – Turning moment diagrams –Fly Wheels – Flywheels of punching presses- Dynamics of Cam- follower mechanism.

UNIT II BALANCING 12

Static and dynamic balancing – Balancing of rotating masses – Balancing a single cylinder engine – Balancing of Multi-cylinder inline, V-engines – Partial balancing in engines – Balancing of linkages – Balancing machines-Field balancing of discs and rotors.

UNIT III FREE VIBRATION 12

Basic features of vibratory systems – Degrees of freedom – single degree of freedom – Free vibration– Equations of motion – Natural frequency – Types of Damping – Damped vibration–Torsional vibration of shaft – Critical speeds of shafts – Torsional vibration – Two and three rotor torsional systems.

UNIT IV FORCED VIBRATION 12

Response of one degree freedom systems to periodic forcing – Harmonic disturbances – Disturbance caused by unbalance – Support motion –transmissibility – Vibration isolation vibration measurement.

UNIT V MECHANISM FOR CONTROL 12

Governors – Types – Centrifugal governors – Gravity controlled and spring controlled centrifugal governors – Characteristics – Effect of friction – Controlling force curves. Gyroscopes –Gyroscopic forces and torques – Gyroscopic stabilization – Gyroscopic effects in Automobiles, ships and airplanes.

TEXT BOOKS:

1. F. B. Sayyad, "Dynamics of Machinery", McMillan Publishers India Ltd., Tech-Max Educational resources, 2011.
2. Rattan, S.S, "Theory of Machines", 4th Edition, Tata McGraw-Hill, 2014. 3. Uicker, J.J., Pennock G.R and Shigley, J.E., "Theory of Machines and Mechanisms", 4th Edition, Oxford University Press, 2014.

REFERENCES:

1. Cleghorn.W. L, "Mechanisms of Machines", Oxford University Press, 2014
2. Ghosh. A and Mallick, A.K., "Theory of Mechanisms and Machines", 3rd Edition Affiliated East-West Pvt. Ltd., New Delhi, 2006.
3. Khurmi, R.S.,"Theory of Machines", 14th Edition, S Chand Publications, 2005.
4. Rao.J.S. and Dukkipati.R.V. "Mechanisms and Machine Theory", Wiley-Eastern Ltd., New
5. Delhi, 1992.
6. Robert L. Norton, "Kinematics and Dynamics of Machinery", Tata McGraw-Hill, 2009.
7. V.Ramamurthi, "Mechanics of Machines", Narosa Publishing House, 2002.

Course Outcome Articulation Matrix

	Program Outcome												***PSO***		
Course Code / CO No	***1***	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***	***10***	***11***	***12***	***1***	***2***	***3***
ME8097 / C426.1	3	2	0	3	0	0	0	0	2	3	0	3	3	3	0
ME8097 / C426.2	3	3	0	3	3	0	0	0	2	3	0	3	3	3	0
ME8097 / C426.3	3	3	0	3	3	0	0	0	2	3	0	3	3	3	0
ME8097 / C426.4	3	3	0	3	3	0	0	0	2	3	0	3	3	3	0
ME8097 / C426.5	3	3	0	3	3	0	0	0	2	3	0	3	3	3	0
Average	3	3	0	3	2	0	0	0	2	3	0	3	3	3	0

In a reciprocating engine mechanism. the crank and the connecting rod are 300mm and 1m long respectively. The crank rotates at a constant speed of 200 rpm Determine (i) the crank angle at which the maximum velocity occurs (ii) Maximum velocity of the piston

Unit-I FORCE Analysis

Given data:

$r = 300\,mm$ $\quad l = 1\,m$
$= 0.3\,m$ $\quad N = 200\,rpm.$

$n = \frac{l}{r} = \frac{1}{0.3} = 3.33.$

$\omega = \frac{2\pi N}{60} = 20.94\ rad/s.$

Solution:

(i) Crank angle at which the maximum velocity occurs

WKT, $V_p = \omega r \left[\sin\theta + \frac{\sin 2\theta}{n}\right]$

For maximum velocity of the piston

$$\frac{dV_p}{d\theta} = \frac{d}{d\theta}\left[\omega r\left(\sin\theta + \frac{\sin 2\theta}{n}\right)\right] = 0$$

$$\omega r\left[\cos\theta + \frac{2\cos 2\theta}{2n}\right] = 0$$

$$n\cos\theta + 2\cos^2\theta - 1 = 0$$

$$\left[\because \cos 2\theta = 2\cos^2\theta - 1\right]$$

$$2\cos^2\theta + 3.33\cos\theta - 1 = 0$$

Given Data:

$L = 0.12m$

$N = 1500$ rpm.

$r = \frac{L}{2} = \frac{0.12}{2}$.

$r = 0.06m$.

$l = 3r$

$l = 0.18$

$n = \frac{l}{r} = 3$

$\theta = \frac{1}{4}$ of stroke from IDC.

$= \frac{1}{4} \times 180°$

$\theta = 45°$

Solution:

$$V_p = r\omega\left[\sin\theta + \frac{\sin 2\theta}{2n}\right].$$

$$V_p = 8.235 \, m/s.$$

$$a_p = r\omega^2\left[\cos\theta + \frac{\cos 2\theta}{n}\right]$$

$$a_p = 1046.83 \, m/s^2.$$

$$\omega_{pc} = \frac{\omega\cos\theta}{n}.$$

$$= 37.02 \, rad/s.$$

$$\alpha_{pc} = \frac{\omega^2 \sin\theta}{n}.$$

$$\alpha_{pc} = 5815.74 \, rad/s$$

$$\alpha_p = \omega^2 r \left[\cos\theta + \frac{\cos 2\theta}{n}\right].$$

$$= 62.83^2 \times 0.1 \left[\cos 45 + \frac{\cos 2\times 45}{4}\right]$$

$$\alpha_p = 279.138\ m/s^2.$$

Note :-

Since $\sin^2\theta$ is small as compared to n^2 $\therefore$ it may be neglected.

(ii) angular velocity

$$\omega_{PC} = \frac{\omega \cos\theta}{n} = \frac{62.83 \cos 45}{4}.$$

$$= 11.107\ rad/s.$$

$$\alpha_{PC} = \frac{\omega^2 \sin\theta}{n} = \frac{62.83 \sin 45}{n}.$$

$$= 697.84\ rad/s^2.$$

A petrol engine has a stroke of 120mm and connecting rod 5 times the crank length. The crank rotates at 1500rpm clockwise Determine (i) velocity and acceleration of the piston. (ii) angular velocity and angular acceleration of the connecting rod when the piston travel one-fourth of the stroke

S [IDC]

In a slider crank meachanism the length of the crank and the connecting rod are 100mm and 400mm respectively. The crank rotates uniformly at 600 rpm clockwise when the crank has turn through 45° from the inner dead centre. Find (i) velocity and acceleration of the slider (ii) angular velocity and angular acceleration of the connecting rod.

Given data:

$r = 0.1 m$ $\qquad$ $N = 600 rpm$

$l = 0.4 m$ $\qquad$ $\theta = 45°$

~~Q - angle made by crank.~~

~~φ - inclination of connecting rod.~~

Solution:

$$\omega = \frac{2\pi N}{60} = 62.83 \text{ rad/s}$$

$$n = \frac{l}{r} = \frac{0.4}{0.1} = 4$$

V_p = velocity of the slider.

$$V_p = r\omega \left[\sin\theta + \frac{\sin 2\theta}{2n}\right]$$

$$= 0.1 \times 62.83 \left[\sin 45 + \frac{\sin 2\times 45}{2\times 4}\right]$$

$$V_p = 5.228 \text{ m/s}$$

Unit - I

Force Analysis

Velocity and acceleration of the reciprocating parts in engine

(i) Displacement of the piston:

r = crank radius.

$$x = r\left[(1-\cos\theta) + \frac{\sin^2\theta}{2n}\right]$$

ϕ - inclination of connecting rod.

(ii) Velocity of the piston

L = length of the connecting rod

$$V_p = r\omega\left[\sin\theta + \frac{\sin 2\theta}{2n}\right]$$

(iii) Acceleration of the piston

$$a_p = \omega^2 r\left[\cos\theta + \frac{\cos 2\theta}{n}\right]$$

θ = angle made by crank

(iv) Angular velocity of connecting rod.

$$\boxed{n = \frac{l}{r}}$$

$$\omega_{pc} = \frac{\omega\cos\theta}{(n^2-\sin^2\theta)^{1/2}} \approx \frac{\omega\cos\theta}{n}$$

(v) Angular acceleration of connecting rod

$$\alpha_{pc} = \frac{-\omega \sin\theta (n^2-1)}{(n^2-\sin^2\theta)^{3/2}} \approx \frac{-\omega^2 \sin\theta}{n}.$$

force transmitted is 1/nth of impressed force assume that the mass of the motor is equally distributed among the five spring. Determine (i) stiffness of each spring (ii) dynamic transmitted to the base at the operating speed. (iii) Natural frequency of the system.

Given Data:

$m = 120\, kg$

$N = 1500\, rpm$

$m_u = 35\, kg$

$e = 0.5\, mm$

$e = 0.5 \times 10^{-3} m$

number of spring = 5

$\xi = 0$

$$\frac{F_T}{F_0} = \frac{1}{11} = 0.091$$

Solution:

(i) Stiffness of each spring

In the absense of damping,

$\xi = 0$

$$\therefore \varepsilon = \pm \frac{1}{1-r^2}$$

$$0.091 = \frac{1}{1-r^2}$$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 1500}{60}$$

$\omega = 157.08\, rad/s$

$$0.09 = \frac{1}{\left(\frac{\omega}{\omega_n}\right)^2 - 1}$$

$$\left(\frac{\omega}{\omega_n}\right)^2 = 12$$

$\omega_n = 45.35\, rad/s$

$$45.35 = \sqrt{\frac{S}{120}}$$

$S = 2.467 \times 10^5 N/m$

$\varepsilon = F_T / F_0$

$F_T = F_0 \times \varepsilon$

$F_0 = m_u \times \omega^2 \times e$

$= 35 \times 157.08^2 \times 0.5 \times 10^{-3}$

$F_0 = 431.79\, N$

$F_T = 431.79 \times 0.091$

$F_T = 39.29\, N$

$$f_n = \frac{\omega_n}{2\pi} = \frac{45.36}{2\pi}$$

$f_n = 7.22\, Hz$

$$\cos 2\theta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-3.33 \pm \sqrt{3.33^2 - 4 \times 2 \times (-1)}}{2 \times (2)}$$

$$\cos\theta = 0.26$$

$$\boxed{\theta = 75^\circ}$$

(ii) Maximum velocity of the piston

sub $\theta = 75^\circ$ in V_p

$$V_{p_{max}} = \omega r \left[\sin\theta + \frac{\sin 2\theta}{2 \times n} \right]$$

$$= 20.94 \times 0.3 \left[\sin 75 + \frac{\sin 150}{2 \times 3.33} \right]$$

$$= 6.54 \text{ m/s.}$$

The crank and connecting rod of a steam engine are 0.35m and 1.55m in length The crank rotates at 180rpm clockwise Determine the velocity and acceleration of the piston when the crank is at 40° from IDC also determine the position of the crank for zero acceleration of the piston.

Given Data:

$$r = 0.35 \text{ m}$$

$$l = 1.55 \text{ m}$$

$$N = 180 \text{ rpm}$$

$$\theta = 40^\circ$$

$$\omega = 18.85 \text{ rad/s}.$$

$$n = l/r = \frac{1.55}{0.35} = 4.12.$$

Solution:

(i) Velocity and acceleration when $\theta = 40^\circ$

$$V_p = \omega r \left[\sin\theta + \frac{\sin 2\theta}{2n}\right].$$

$$V_p = 4.974 \text{ m/s}$$

$$a_p = \omega^2 r \left[\cos\theta + \frac{\cos 2\theta}{n}\right].$$

$$= 100.51 \text{ m/s}^2.$$

(ii) Position of the crank for zero acceleration of the piston

Let θ_1 = position of the crank from IDC. for zero acceleration of the piston

$$a_p = \omega^2 r\left[\cos\theta_1 + \frac{\cos 2\theta_1}{n}\right]$$

$$0 = \omega^2 r n \left[n\cos\theta_1 + \cos 2\theta_1\right]$$

$$n\cos\theta_1 + \cos 2\theta_1 = 0$$

$$4.428\cos\theta_1 + (2\cos^2\theta_1 - 1) = 0$$

$$2\cos^2\theta_1 + 4.428\cos\theta_1 - 1 = 0$$

$$\cos\theta_1 = 0.2066$$

$$\theta_1 = 78.08^\circ$$

Forces on the Reciprocating part of an engine neglecting the weight of the connecting rod.

1) To find inertia force of reciprocating parts (F_P) inertia force, $F_I = m_R a_R$

$$= m_R \omega^2 r \left[\cos\theta + \frac{\cos 2\theta}{n}\right]$$

2) Force acting along connecting rod (F_Q)

$$F_Q = \frac{F_P}{\cos\phi}$$

$$\boxed{F_P = F_L - F}$$

$$\boxed{F_L = (P_1 - P_2) A}$$

3) Thrust on sides of cylinder walls

$$F_N = F_Q \sin\phi = F_P \tan\phi$$

4) Crank pin effort (F_T)

$$F_T = F_Q \sin(\theta + \phi)$$

$$= \frac{F_P}{\cos\phi} \times \sin(\theta + \phi)$$

5) Thrust on crank shaft bearing (F_B)

$$F_B = F_Q \cos$$

$$= \frac{F_P}{\cos\phi} \cdot \cos$$

6) Crank effort on crank shaft (T):

$$T = F_T \times r$$

$$= \left[\frac{F_P}{\cos\theta} \sin(\theta + \phi)\right] r$$

$$= F_P\left[\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}}\right] r$$

7. The length of the crank and connecting rod of the horizontal engine are 200mm and 1m respectively. The crank is rotating at 400rpm. When the crank has turned through 30° from the IDC. The difference of pressure between cover and piston rod is 0.4 N/mm². If the mass of the reciprocating parts is 100kg and cylinder bore is 0.4m Calculate (i) inertia force (ii) Force on piston (iii) piston effort (iv) thrust on the sides of the cylinder. (v) thrust in the connecting rod (vi) Crank effort.

Given Data:

$$r = 0.2\,m$$
$$l = 1\,m$$
$$N = 400\,rpm$$
$$\theta = 30^\circ$$
$$P_1 - P = 0.4\,N/mm^2$$
$$m_R = 100\,kg$$
$$D = 0.4\,m$$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 400}{60} = 41.89\,rad/s$$

$$n = \frac{l}{r} = 5$$

Solution:

(i) inertia force.

$$F_I = m_R \omega^2 r \left[\cos\theta + \frac{\cos 2\theta}{n}\right]$$

$$= 100 \times 41.89^2 \times 0.2 \left[\cos 30 + \frac{\cos 2 \times 30}{5}\right]$$

$$= 33.903\,kN.$$

(ii) Force on piston

~~Design~~.

$$\boxed{F_L = (P_1 - P_2) A}$$

$$= (P_1 - P_2) \times \frac{\pi}{4} \times d^2.$$

$$F_L = 50.265\,kN.$$

Piston effect

$$F_P = F_L - F_i$$

$$= 50.265 - 33.903$$

$$= 16.362 \text{ kN}$$

Thrust on the cylinder

$$F_N = F_P \tan \phi$$

$$\phi = \sin^{-1} \frac{\sin \theta}{n}$$

$$\phi = 5.739^\circ$$

$$F_N = 16.36 \tan 5.739$$

$$= 1.6442 \text{ kN}$$

Thrust in connecting rod

$$F_Q = \frac{F_P}{\cos \phi} = 16.44 \text{ kN}$$

Crank effort

$$F_T = F_Q \sin(\theta + \phi)$$

$$F_T = 9.604 \text{ kN}$$

Turning moment of the crank shaft

$$T = F_T \times r$$

$$= 9.604 \times 0.2$$

$$= 1.92 \text{ kNm}$$

A horizontal steam engine running 240 rpm has bore of 300mm and stroke 600mm. The connecting rod is 1.25m long and the mass of the reciprocating part is 60kg when the crank is 60° past its IDC. The steam pressure on the cover side of the piston is 1.125 N/mm². while that of the crank side is 0.125 N/mm² neglecting the area of the piston rod. Determine (i) Force on the piston rod and turning moment on the crank shaft.

Given Data

$N = 240 rpm$

$D = 0.3 m$, $L = 0.6 m$

$l = 1.25 m$, $m_R = 60 kg$

$\theta = 60°$, $P_1 = 1.125 N/mm^2$

$P_1 = 1.125 \times 10^6 N/m^2$

$P_2 = 0.125 \times 10^6 N/m^2$

Solution :

$\omega = \frac{2\pi N}{60} = 25.13 rad/s$

$n = \frac{l}{r} = \frac{l}{L/2} = 4.1667$

Net load on piston

$F_L = (P_1 - P_2) \times A$

$= (P_1 - P_2) \times \frac{\pi}{4} D^2$

$= 70685.83 N$

Inertia force

$F_I = (m_R) \omega^2 r \left(\cos\theta + \frac{\cos 2\theta}{n}\right)$

$= 60 \times 25.13^2 \times 0.3 \left(\cos 60 + \frac{\cos 2 \times 60}{4.1667}\right)$

$F_I = 4319.68 N$

$F_P = F_L - F_I$

$= 66.36 \times 10^3 N$

$T = F_P r \left(\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}}\right)$

$= 66.36 \times 10^3 \times 0.3 \left(\sin 60 + \frac{\sin 120}{2\sqrt{(4.1667^2)^2 - \sin^2 60}}\right)$

$= 19.357 kNm.$

A horizontal steam engine running at 210 rpm has a bore of 190mm (D) and stroke of 350mm. The piston rod is 20mm (d) in diameter and connecting rod length is 950 mm. The mass of the reciprocating part is 8kg and the frictional resistance is equivalent to a force of 350N. Determine (i) when the crank is at 115° from IDC the mean pressure be 4500 N/m² on the cover side and 100 N/m² on the crank side (i) thrust on the connecting rod (ii) thrust on the cylinder wall. (iii) load on the bearing (iv) turning moment on the crank shaft

Given data:

$N = 210\ rpm$

$D = 190mm = 0.19m$

$L = 0.35m$

$d = 0.02m$

$l = 0.95m$, $\boxed{\theta = 115°}$

$m_R = 8kg$, $R_F = 350N$

$P_1 = 4500 N/m^2$

$P_2 = 100 N/m^2$

$$W = \frac{2\pi N}{60} = 21.99\ rad/s$$

$$n = \frac{l}{r} = 5.43$$

Solution:

First of all we find piston efford

$$F_P = F_L - F_I - R_F$$

$$F_L = P_1A_1 - P_2A_2$$

A_1 - area of piston on cover side

$$A_1 = \frac{\pi D^2}{4} = \frac{\pi}{4} \times 0.19^2$$

$$= 0.02835 m^2.$$

Area of piston on crank side

$$A_2 = A_1 - a$$

$$= 0.02835 - \frac{\pi}{4} \times d^2$$

$$= 0.02835 - \frac{\pi}{4} \times 0.02^2$$

$$A_2 = 0.028\ m^2$$

$$F_L = (4500 \times 0.02835) - (100 \times 0.028)$$

$$= 124.775 N$$

$$F_I = mR\omega^2 r \left[\cos\theta + \frac{\cos 2\theta}{n}\right]$$

$$= 8 \times 21.99^2 \times 0.175 \left[\cos 115 + \frac{\cos 230}{5.43}\right]$$

$$F_I = -366.24N$$

$$F_P = 124.77 + 366.24 - 350$$

$$F_P = 141.01N$$

(i) Thrust acting on the connecting rod.

$$F_Q = \frac{F_P}{\cos\phi}$$

$$\sin\phi = \frac{\sin\theta}{n} = \frac{\sin 115}{5.43}$$

$$\phi = 9.6081$$

$$F_Q = \frac{141.01}{\cos 9.6081}$$

$$F_Q = 143.02N$$

(ii) Thrust on cylinder wall

$$F_N = F_P \tan\phi$$
$$= 141.01 \tan 9.60$$
$$= 23.87\,N$$

(iii) load on the bearing

$$F_B = F_{Cr} \cos(\theta + \phi)$$
$$= 143.02 \cos(115 + 9.6081)$$

~~Fly wheel~~ $= -81.23\,N$

(iv) Turning moment on the crank shaft.

$$T = F_T \times r.$$
$$T = \left[\frac{F_P}{\cos\phi}\sin(\theta+\phi)\right] r.$$
$$= \left[\frac{141.042}{\cos 9.6081}\sin(115+9.6081)\right] 0.115$$
$$= 20.6\,Nm.$$

Dynamically Equivalent System:

$$m_1 + m_2 = m$$
$$m_1 l_1 = m_2 l_2$$
$$m_1 l_1^2 = m_2 l_2^2 = mk^2.$$
$$m_1 = \frac{l_2 m}{(l_1 + l_2)} \quad \text{and} \quad m_2 = \frac{l_1 m}{l_1 + l_2}.$$
$$k^2 = l_1 l_2$$

Frequency of oscillation, $n = \dfrac{1}{t_P}$

$$= \frac{1}{2\pi}\sqrt{\frac{g}{l}}.$$

FLY WHEEL:

1) Coefficient of fluctuation of speed.

$$C_S = \frac{N_1 - N_2}{N} = \frac{\omega_1 - \omega_2}{\omega} = \frac{V_1 - V_2}{V}$$

2) Coefficient of steadyness, $\dfrac{1}{C_S} = m.$

$$m = \frac{1}{C_S}.$$

3. Energy stored in Flywheel

$$E = \frac{1}{2} I \omega^2.$$

4. Maximum fluctuation of energy

$$\Delta E = \frac{1}{2} I (\omega_1^2 - \omega_2^2)$$

$$\Delta E = I \omega^2 C_s.$$

Operation of Flywheel

i) Maximum fluctuation of flywheel

$$\Delta E = E_1 - E_2.$$

$$= E_1 \left[1 - \frac{(\theta_2 - \theta_1)}{2\pi} \right]$$

where ; $\frac{\theta_2 - \theta_1}{2\pi} = \frac{t}{4r} = \frac{t}{2\pi}$ $(S = 2r)$

Note :

No. of rivets closed per hers

$$= \frac{\text{Energy supplied by motor in hour}}{\text{Energy required for riveting}}$$

Shear Area $= \pi d t$

τ_1 = shear area × Energy

Let m be the mass of flywheel.

k is radius of gyration

$$N = \frac{N_1 + N_2}{2}.$$

Mean speed during the cycle N, $C_s = \frac{N_1 - N_2}{N}.$

C_s - flutuation of speed.

1. An engine flywheel has a mass of 6.5 ton and the radius of gyration is 2m. If the maximum and minimum speed are 120 rpm and 118 rpm respectively. Find (i) mean speed of flywheel (ii) coefficient of fluctuation of speed (iii) ~~Maxim~~ Max flutuation of energy

Given Data:

$$m = 6500\,kg \;,\; k = 2\,m$$

$$N_1 = 120\,rpm \;,\; N_2 = 118\,rpm.$$

Solution:

(i). Mean speed

$$N = \frac{N_1 + N_2}{2} = \frac{120+118}{2}$$

$$= 119\,rpm$$

(ii) Coefficient of fluctuation of Speed:

$$C_s = \frac{N_1 - N_2}{N} = \frac{120-118}{119}$$

$$C_s = 0.0168$$

(iii) Maximum fluctuation of energy

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 119}{60} = 12.46\,rad/s.$$

Max fluctuation energy, $\Delta E = I\omega^2 C_s$

$$= mk^2 \times \omega^2 \times C_s$$

$$= 6500 \times 2^2 \times 12.46^2 \times 0.0168$$

$$= 67.831\,kNm.$$

The vertical double acting steering develops 75 kN at 250 rpm the max. flutuation of energy in 30 % of workdone per stroke the max + min speed are not to vary more than 1 % on either side of the mean speed. Find the mass of flywheel if radius of gyration is 0.6 m

Given Data:

$P = 75\,kW$

$N = 250\,rpm$

$\Delta E = 30\%$ of workdone per stroke

$C_s = \pm 1\%$

$k = 0.6\,m$

Solution:

$\omega = \frac{2\pi N}{60} = 26.179\ \frac{rad}{s}$

W.K.T,

$\text{Workdone/cycle} = \frac{P \times 60}{N}$

$= \frac{75 \times 10^3 \times 60}{250}$

$= 18000\,Nm$

$\therefore \Delta E = 30\%$ of work done per stroke.

$= \frac{30}{100} \times 18000$

$\Delta E = 5400\ Nm.$

$\Delta E = I\omega^2 C_s$

$5400 = mk^2\omega^2 C_s$

$5400 = m \times 0.6^2 \times 26.17^2 \times 0.02$

$m = 1094.26\,kg$

The radius of gyration of a flywheel is 1m and the fluctuation of speed is not to exceed 1% of the mean speed of the flywheel. If the mass of the flywheel is 3340 kg and the steam engine develops 150 kW at 135 rpm. Find (i) Maximum fluctuation of energy (ii) Coefficient of fluctuation of energy

Given data:

$k = 1m$

$C_S = \pm 1\%$

$m = 3340 kg$

$P = 150 kW$

$N = 135 rpm$

$\omega = \frac{2\pi N}{60} = 14.4 \frac{rad}{s}$

Solution:

1. $\Delta E = I \omega^2 C_S$

$= mk^2 \times \omega^2 \times C_S$

$= 3340 \times 1^2 \times 14.14^2 \times 0.01$

$\Delta E = 6675.3 Nm$

$C_E = \frac{\Delta E}{Workdone/cycle}$

$Workdone/cycle = \frac{P \times 60}{N}$

$= \frac{150 \times 10^3 \times 60}{135}$

$Workdone/cycle = 66666.7 Nm$

$C_E = \frac{6675.3}{66,666.7}$

$C_E = 0.1$

1.4 Turning Moment Diagram

1, Turning moment diagram of a petrol engine drawn to vertical scale 1mm to 6 Nm and horizontal scale of 1mm to 1°. The turning moment repeats its self after every half revolution of the engine. The area above & below the mean torque line are 305, 710, 50, 350, 980 and 275 mm^2. Mass of the rotating part is 40 kg at a radius of gyration of 140mm. Calculate the coefficient of fluctuation of speed if the mean speed 1500 rpm.

Solution:

Turning moment 1mm = 6 Nm

crank angle 1mm = 1°

$m = 40kg$

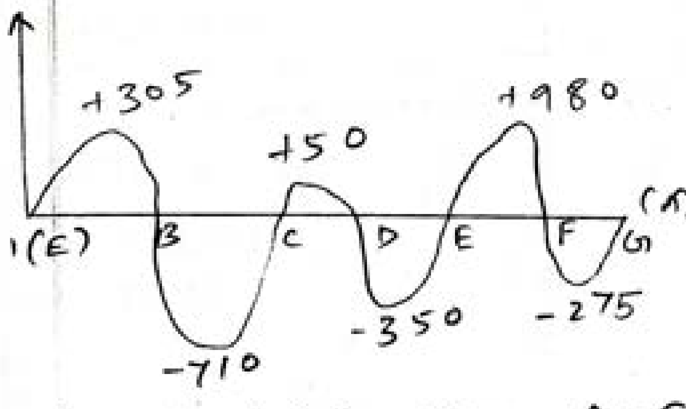
$k = 140mm$

$N = 1500 rpm$

$\omega = \frac{2\pi N}{60} = 157.08 \text{ rad/s}$

$\left.\begin{matrix}1mm^2 \text{ on} \\ \text{Turning} \\ \text{moment}\end{matrix}\right\} = 6 \times \left(1 \times \frac{\pi}{180}\right)$

$= 0.105 \text{ Nm}$



Let the total energy, A = E

point	Energy (mm^2)
A	E
B	E + 305 (max)
C	E+305−710 = E − 405
D	E − 405+50 = E − 355
E	E−355−350 = E − 705 (min)
F	E−705+980 = E + 275
G	E+275 − 275 = E

ΔE = maximum energy − Minimum energy

$= E+305 - E + 705$

$\Delta E = 1010 \text{ N}mm^2$.

$\Delta E = 1010 \times 0.105$

$= 106.05 \text{ Nm}$

$\Delta E = I\omega^2 C_s$.

$\Delta E = mk^2 \omega^2 C_s$

$106.05 = 40 \times (0.14)^2 \times 157.08^2 \times C_s$

$C_s = 0.00548$

The turning moment diagram from a multi-cylinder engine has been drawn to a scale of 1mm is equal to 4500Nm vertically and 1mm is equal 2.4° Horizontally. The intercepted area between the output torque curve and mean resistance line, taken in order from one end are 342, 230, 245, 303, 115, 232, 227 and 164 mm^2. When the engine is running at 150rpm. If the mass of the flywheel is 1000kg and the total fluctuation of speed does not exceed 3% of mean speed. Find the minimum value of radius of gyration

Given Data.

Turning moment 1mm = 4500Nm

Crank angle 1mm = 2.4°

$$\left.\begin{array}{l}\therefore 1mm^2 \text{ on} \\ \text{turning} \\ \text{moment} \\ \text{diagram}\end{array}\right\} = 4500 \times \left(2.4 \times \frac{\pi}{180}\right)$$

$= 188.49\,Nm$

$N = 1500 rpm$

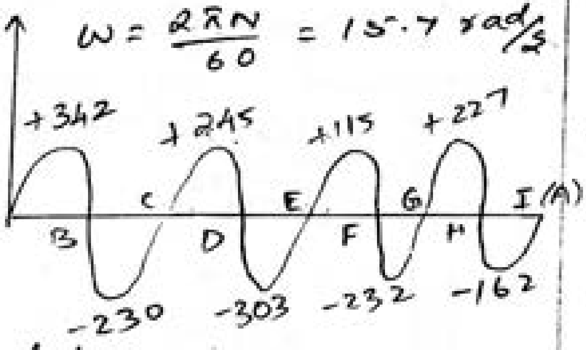
$m = 1000 kg$

$C_s = \frac{N_1 - N_2}{N} = 3\%$

$C_s = 0.03$

Solution:

$\omega = \frac{2\pi N}{60} = 15.7\ rad/s$



Let the total energy,

A = E

Point	Energy
A	E
B	E+342.
C	E+342-230 = E+112
D	E+112+245 = E+357 (max)
E	E+357-303 = E+54
F	E+54+115 = E+169
G	E+169-232 = E-63
H	E-63+227 = E+164
I	E+164-164 = E.

$$\Delta E = \left.\begin{array}{c}\text{Maximum} \\ \text{energy}\end{array}\right\} - \left\{\begin{array}{c}\text{Minimum} \\ \text{energy.}\end{array}\right.$$

$= E + 357 - E + 63$

$\Delta E = 420\,mm^2$

$\Delta E = 420 \times 188.49$

$\Delta E = 79.166\,KNm$

$\Delta E = I\omega^2 C_s$.

$79.166 \times 10^3 = mk^2 \times (15.7)^2 \times 0.03$.

$k = 3.27\,m$.

x ———— x

1.5 Torque Problem

1. Workdone per cycle $= \int_0^{\pi} T d\theta$

2. Angular acceleration of fly wheel $\}$; $I\alpha = (T - T_{mean})$.

3. Work done $= T\theta$

$$\frac{\text{Work done}}{\text{revolution}} = T \times 2\pi$$

1. The torque delivered by a two stroke engine is represented by $T = (1000 + 300\sin 2\theta - 500\cos 2\theta)$ Nm where θ is the angle turned by the crank from the inner dead center. The engine speed is 250 rpm the mass of the flywheel is 400 kg and radius of gyration 400 mm. Determine the power developed by the engine (ii) Total percentage fluctuation of speed (iii) The angular acceleration of fly wheel when the crank has rotated through an angle of 60° from the IDC (iv) maximum angular acceleration and retardation of the flywheel.

Given Data

$T = (1000 + 300\sin 2\theta - 500\cos 2\theta)$

$N = 250$ rpm

$m = 400$ kg

$k = 400$ mm.

$\omega = \frac{2\pi N}{60} = 26.18$ rad/s.

Solution:

i) power developed by the engine.

$$\text{Workdone cycle} = \int_0^{\pi} T d\theta.$$

$$= \int_0^{\pi} (1000 + 300\sin 2\theta - 500\cos 2\theta) d\theta$$

$$= \left[1000(\theta) + 300\left(\frac{-\cos 2\theta}{2}\right) - 500\left(\frac{\sin 2\theta}{2}\right)\right]_0^{\pi}$$

$= 1000\pi$ Nm

$= 3141.59$ Nm

$$T_{mean} = \frac{\text{Workdone per cycle}}{\text{Crank angle}}$$

$$= \frac{1000\pi}{\pi}$$

$T_{mean} = 1000$ Nm

Power developed $\} = T_{mean} \times \omega$

$= 1000 \times 26.18$

$P = 26.18 \text{ kW}.$

x———————x

(ii) Total percentage fluctuation of speed:

Here, the cycle of operation is repeated every π revolution.

$T = T_{mean}$

$1000 + 300\sin 2\theta - 500\cos 2\theta = 1000$

$300 \sin 2\theta = 500 \cos 2\theta$

$\tan 2\theta = 5/3$

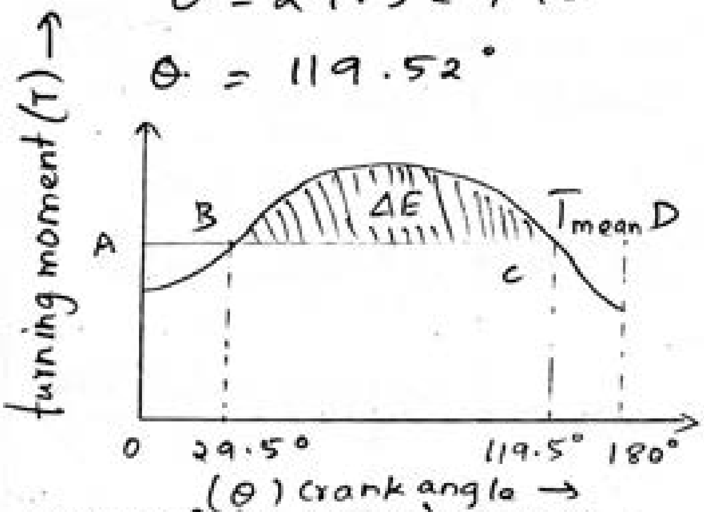
$\theta = 29.52°$

$\cancel{\theta = 29.52 \times \frac{\pi}{180}}$

$\cancel{\theta = 0.5152 \text{ rad/s}}$

$\theta = 29.52 + 90$

$\theta = 119.52°$



Therefore maximum fluctuation energy, ΔE

$$\Delta E = \int_{29.5}^{119.5} (T - T_{mean})\, d\theta$$

$$\Delta E = \int_{29.52}^{119.5} [1000 + 300\sin 2\theta - 500\cos 2\theta - 1000]\, d\theta$$

$$\Delta E = \int_{29.5}^{119.5} (300\sin 2\theta - 500\cos 2\theta)\, d\theta$$

$$= \left[300\left[\frac{-\cos 2\theta}{2}\right] - 500\left[\frac{\sin 2\theta}{2}\right]\right]_{29.5}^{119.5}$$

$$= \left[300\left(\frac{-\cos(2\times 119.5)}{2}\right) - \frac{500\sin 2\times 119.5}{2}\right] - \left[300\left(\frac{-\cos 2\times 29.5}{2}\right) - \frac{500\sin 2\times 29.5}{2}\right]$$

$= [77.26 + 214.29] - [-77.26 - 214.29]$

$= 583.1 \text{ Nm}.$

$\Delta E = I\omega^2 C_s$

$583.03 = mk^2 \omega^2 C_s.$

$583.08 = 400 \times 0.4^2 \times 26.28^2 \times C_s$

$C_s = 0.01329$

$C_s = 1.329\%$

(iii) Angular velocity of flywheel when, $\theta = 60°$

$$I \cdot \alpha = \frac{(T - T_{mean})}{\cancel{2}}$$

$$\alpha = \frac{T - T_{mean}}{I}$$

$$\alpha = \frac{T - T_{mean}}{mk^2}.$$

$$\alpha = \frac{(1000 + 300\sin 2\theta - 500\cos 2\theta - 1000)}{400 \times 0.4^2}$$

$= 7.96 \text{ rad/s}^2.$

(iv) Maximum acceleration and retardation of fly wheel

$$\frac{d}{d\theta}(T - T_{mean}) = 0$$

$$\frac{d}{d\theta}(1000 + 300\sin 2\theta - 500\cos 2\theta - 1000) = 0.$$

$[300\cos 2\theta \times 2 + 500\sin 2\theta \times 2] = 0$

$300\cos 2\theta = -500\sin 2\theta$

$\tan 2\theta = -300/500$

$\theta = -15.48°$

Angular Acceleration of Flywheel.

~~when θ = 60°~~

∴ ~~The formula is~~ .

when $2\theta = -30.96 + 180$

$= 149.04$

$2\theta = -30.96 + 360$

$= 329.04$

∴ when $T_1 = (T - T_{mean})$

$2\theta = 149.04$

$= 300 \sin 2\theta - 500 \cos 2\theta$

$= 583.09\ Nm$

$2\theta = 329.04$

when $T_1 = (T - T_{mean})$

$= -583.09\ Nm$

The value of $T - T_{mean}$ at maximum and minimum torque T arae same.

The maximum acceleration is equal to maximum retardation ∴ $T_1 - T_{mean} = I\alpha$

$583.09 = 400 \times 0.4^2 \times \alpha$

$\alpha = 9.11\ rad/s^2$.

The torque exerted on the crank shaft of a two stroke engine is given by the equation $T = (14500 + 2300 \sin 2\theta - 1900 \cos 2\theta)$ where θ is the crank angle displacement from the IDC assuming the resisting torque to be constant determine (i) the power of the engine when the speed is 150 rpm (ii) The moment of inertia of flywheel if the speed variation is not to exceed ±0.5% of the mean speed (iii) the angular acceleration of the flywheel when the crank turned through 30° from the IDC

Given Data :

$T = 14500 + 2300 \sin 2\theta - 1900 \cos 2\theta$

$N = 150\ rpm$;

$C_s = \pm 0.5\% = 1\%$ (only for 2 stroke).

$\omega = \frac{2\pi N}{60} = 15.71\ rad/s$.

Solution :

(i) power developed by the engine

workdone/cycle $= \int_0^{\pi} T\, d\theta$

$= \int_0^{\pi} (14500 + 2300 \sin 2\theta - 1900 \cos 2\theta)\, d\theta$

$= \left[14500(\theta) + 2300\left(\frac{-\cos 2\theta}{2}\right) - 1900\left(\frac{\sin 2\theta}{2}\right) \right]_0^{\pi}$

$= 14500\pi\ Nm$.

$T_{mean} = \frac{\text{Workdone/cycle}}{\text{Crank angle}}$

$= \frac{14500\pi}{\pi}$

$T_{mean} = 14500\ Nm$

power developed by engine $\} = T_{mean} \times \omega$

$= 14500 \times 15.71$

$= 227.79$ kW

(ii) Moment of inertia of fly wheel, if the speed variation is not to exceed $\pm 0.5\%$ of the mean speed.

$T = T_{mean}$

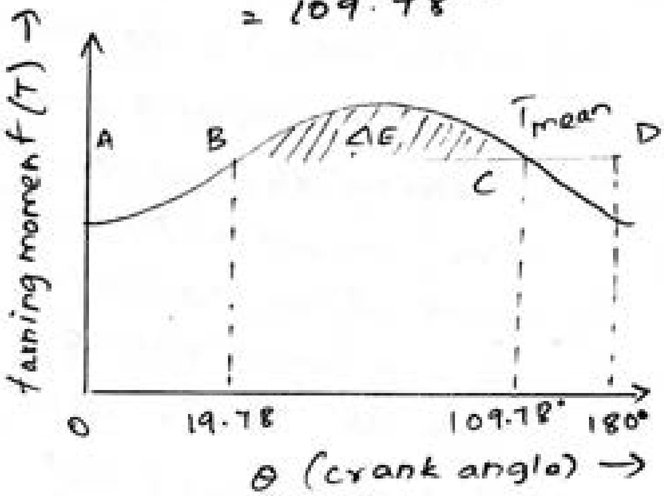
$14500 + 2300\sin 2\theta - 1900\cos 2\theta = 14500$

$2300\sin 2\theta = 1900\cos 2\theta$

$\theta_1 = 19.78°$

$\theta_2 = \theta_1 + 90$

$= 109.78°$



$\therefore$ Max flutuation energy

$$\Delta E = \int_{19.78}^{109.78} (T - T_{mean})\, d\theta$$

$$\Delta E = \int_{19.78}^{109.78} (14500 + 2300\sin 2\theta - 1900\cos 2\theta - 14500)\, d\theta$$

$$\Delta E = \int_{19.78}^{109.78} (2300\sin 2\theta - 1900\cos 2\theta)\, d\theta$$

$$= \left[2300\left(\frac{-\cos 2\theta}{2}\right) - 1900\left(\frac{\sin 2\theta}{2}\right)\right]_{19.78}^{109.78}$$

$$= \left\{\left[2300\left(\frac{-\cos 2\times 109.78}{2}\right) - 1900\left(\frac{\sin 2\times 109.78}{2}\right)\right] - \left[2300\left(\frac{-\cos 2\times 19.78}{2}\right) - 1900\left(\frac{\sin 2\times 19.78}{2}\right)\right]\right\}$$

$= \{[$

$\Delta E = 2983.28$ Nm

$\Delta E = I\omega^2 C_s$

$2983.28 = mk^2\omega^2 C_s$

$2983.28 = I (15.71)^2 \times 0.01$

$I = 1208.765$ kg m^2

(iii) when $\theta = 30$

$I\alpha = T - T_{mean}$

$$\alpha = \frac{T - T_{mean}}{I}$$

$$= \frac{14500 + 2300\sin 2\theta - 1900\cos 2\theta - 14500}{1208.765}$$

$= 1.903\sin 60 - 1.572\cos 60$

$\alpha = 0.862$ rad/s^2

×

Unit - II

Balancing of Rotating Mass.

Balancing is the process of designing (or) modifying machinery so that the unbalanced is reduced to an acceptable level and if possible is eliminated.

1. Static balance

A system of rotating masses ifito static balance (if the ~~---~~ combined masses of the system lies on the axis of rotation

2. Dynamic balance

When the system of rotating masses exists centrifugal force aswell as resultant couple is called dynamic balance.

Analytical Method.

=> First of all find out the centrifugal force. (product of mass and radius of rotation)

=> Resolve the centrifugal force horizontal and vertical and find the sum ~~of~~ $.

The torque developed by two stroke engine. $T = (1000 + 300\sin 2\theta - 500\cos\theta)$ Nm where θ is the angle made by the crank from IDC the engine speed is 250rpm. The mass of flywheel is 400kg. radius of gyration is 400mm. Determine (i) total percentage fluctuation of speed (ii) angular accelaration of flywheel when the crank has rotated through an angle of 60° from IDC (iii) Maximum angular retardation of flywheel.

Given Data:

$T = (1000 + 300\sin 2\theta - 500\cos\theta)$ Nm

$N = 250$ rpm, $m = 400$ kg, $k = 0.4$ m

$\theta = 60$

$$\omega = \frac{2\pi N}{60} = 26.18 \text{ rad/s}.$$

$$I = mk^2 = 400 \times 0.4^2.$$

$$I = 64 \text{ kg m}^2.$$

Solution:

$$\left.\begin{matrix}\text{Workdone} \\ \text{per cycle}\end{matrix}\right\} = \int_0^{2\pi} T\,d\theta$$

$$= \int_0^{2\pi} (1000 + 300\sin 2\theta - 500\cos\theta)\,d\theta$$

$$= \left[1000\theta - 150\cos 2\theta - 500\sin\theta\right]_0^{2\pi}$$

$$= [1000 \times 2\pi - 146.4 - 54.72] - [150]$$

$$= 6232.07 \text{ Nm}.$$

$$\text{Workdone/cycle} = 6232.07 \text{ Nm}.$$

$T_{mean} =$

Sum of horizontal component

$$\Sigma H = m_1 r_1 \cos\theta_1 + m_2 r_2 \cos\theta_2 + \cdots$$

Sum of Vertical component

$$\Sigma V = m_1 r_1 \sin\theta_1 + m_2 r_2 \sin\theta_2 + \cdots$$

⇒ Magnitude of resultant centrifugal force

$$F_c = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

⇒ If θ is the angle which the resultant force make with the horizontal, then

$$\tan\theta = \frac{\Sigma V}{\Sigma H}$$

⇒ The balancing force is then equal to the resultant force but in opposite direction.

⇒ Find out the magnitude of the balancing mass.

$$F_c = m \cdot r$$

where,

m - balacing mass

r - radius of rotation

1. Four mass $m_1, m_2, \boxed{m_3, m_4}$ are 200kg, 300kg, 240kg, and 260kg respectively The corresponding radii of rotation 0.2m, 0.15m, 0.25m and 0.3m and the angle between successive masses 45°, 75° and 135° Find the position and magnitude of balance mass required if its radius of rotation is 0.2m.

Given data:

$m_1 = 200kg \qquad r_1 = 0.2m$

$m_2 = 300kg \qquad r_2 = 0.15m$

$m_3 = 240kg \qquad r_3 = 0.25m$

$m_4 = 260kg \qquad r_4 = 0.3m.$

$\theta_1 = 0 \Rightarrow \theta_2 = 0+45 = 45°$

$\theta_3 = 45+75 = 120° \Rightarrow \theta_4 = 120+135 = 255°$

$r = 0.2m.$

Solution:

$m_1 r_1 = 200 \times 0.2 = 40\ kgm$

$m_2 r_2 = 300 \times 0.15 = 45\ kgm.$

$m_3 r_3 = 240 \times 0.25 = 60\ kgm$

$m_4 r_4 = 260 \times 0.3 = 78\ kgm.$

Resolving horisontaly.

$\Sigma H = m_1 r_1 \cos\theta_1 + m_2 r_2 \cos\theta_2 + m_3 r_3 \cos\theta_3$
$+ m_4 r_4 \cos\theta_4.$

$= 40\cos 0 + 45\cos 45 + 60\cos 120 + 78\cos 255$

$\Sigma H = 21.63$ kgm.

$\Sigma V = m_1 r_1 \sin\theta_1 + m_2 r_2 \sin\theta_2 + m_3 r_3 \sin\theta_3 + m_4 r_4 \sin\theta_4$

$= 40\sin 0 + 45\sin 45 + 60\sin 120 + 78\sin 255$

$\Sigma V = 8.43$ kgm.

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

$$= \sqrt{(21.63)^2 + (8.43)^2}$$

$R = 23.21$ kgm

$$\theta = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right) = \tan^{-1}\left(\frac{8.43}{21.63}\right).$$

$\theta = 21.29°$

$$\therefore m = \frac{R}{r} = \frac{23.21}{0.2} = 116 \text{ kg}$$

Graphical Method.

=> First of all draw the graphical ~~method~~. space diagram with the position of several masses.

=> Find out the centrifugal force exerted by each mass on the rotating shaft now draw the vector diagram. with the obtained centrifugal force $(m \times r)$

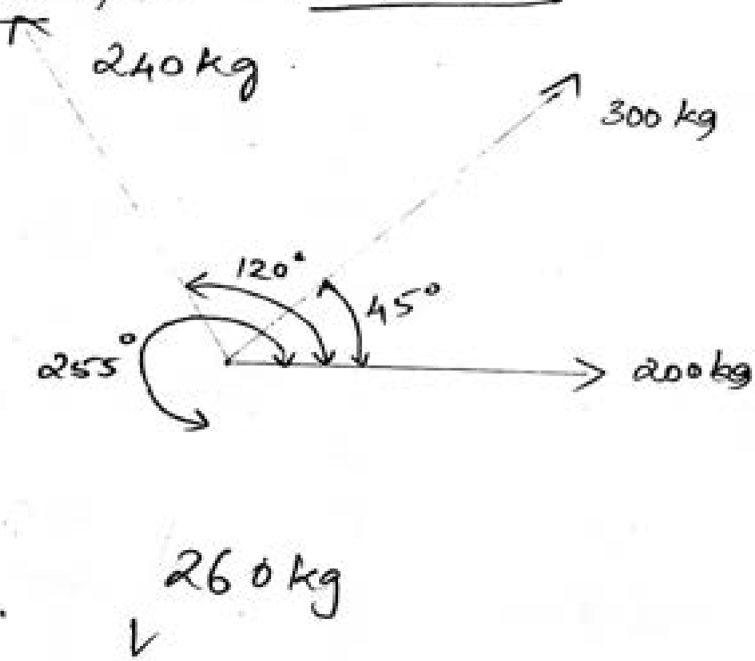
i.e., ab represents the centrifugal force exerted by the mass. and radii (or) AB $m_1 r_1$

AB - magnitude of direction
choose som suitable scale similarly.
draw DC, CD and DD, to represent
the centrifugal force. of other masses

* Now as per polygon law of forces
the closing side 'AE' represents the
resultant force in magnitude and
direction.

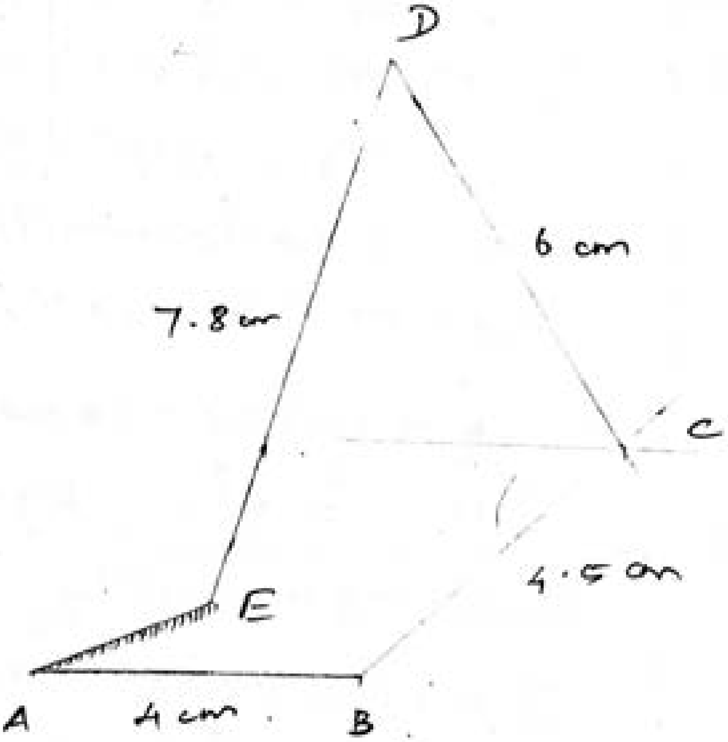
* The balancing force is then equal
to the resultant force.

Values taken from previous
problem.



scale.

10 kgm = 1 cm.



$AE = 2.3$ cm

$= 2.3 \times 10$

$= 230$ kg cm

2\. A rigid motor has all ends as unbalanced in one plane and can be considered to consist of three masses 5 kg, 3 kg and 8 kg and an angle of 165° and 85° counter clockwise diration from m_1 and m_3. The radii are $r_1 = 20$ cm, $r_2 = 8$ cm, $r_3 = 14$ cm. Determine the balanceng mass requred at the radius of 10 cm. Specify the location of mass with respect to m

Give data:

$m_1 = 5$ kg $\quad r_1 = 20$ cm $= 0.2$ m

$m_2 = 3$ kg $\quad r_2 = 8$ cm $= 0.08$ m.

$m_3 = 8$ kg $\quad r_3 = 14$ cm $= 0.14$ m.

$r = 0.1$ m

$$\theta_1 = 0 \qquad m_1 r_2 = 1\,kgm$$
$$\theta_2 = 165 \qquad m_2 r_2 = 0.24\,kgm$$
$$\theta_3 = 275^\circ \qquad m_3 r_3 = 1.12\,kgm.$$

Resolving horizontally

$$\Sigma H = m_1 r_2 \cos\theta_1 + m_2 r_2 \cos\theta_2 + m_3 r_3 \cos\theta_3$$

$$= 1\cos 0 + 0.24\cos 165 + 1.12\cos 275$$

$$\Sigma H = 0.8651\,kgm.$$

Resolving vertically.

$$\Sigma V = m_1 r_1 \sin\theta_1 + m_2 r_2 \sin\theta_2 + m_3 r_3 \sin\theta_3.$$

$$= 1\sin 0 + 0.24\sin 165 + 1.12\sin 275$$

$$\Sigma V = -1.053.$$

$$R = \sqrt{\Sigma V^2 + \Sigma H^2}$$

$$= \sqrt{0.8657^2 + (-1.053)^2}$$

$$R = 1.3636$$

$$\theta = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right) = \tan^{-1}\left(\frac{-1.053}{0.8657}\right)$$

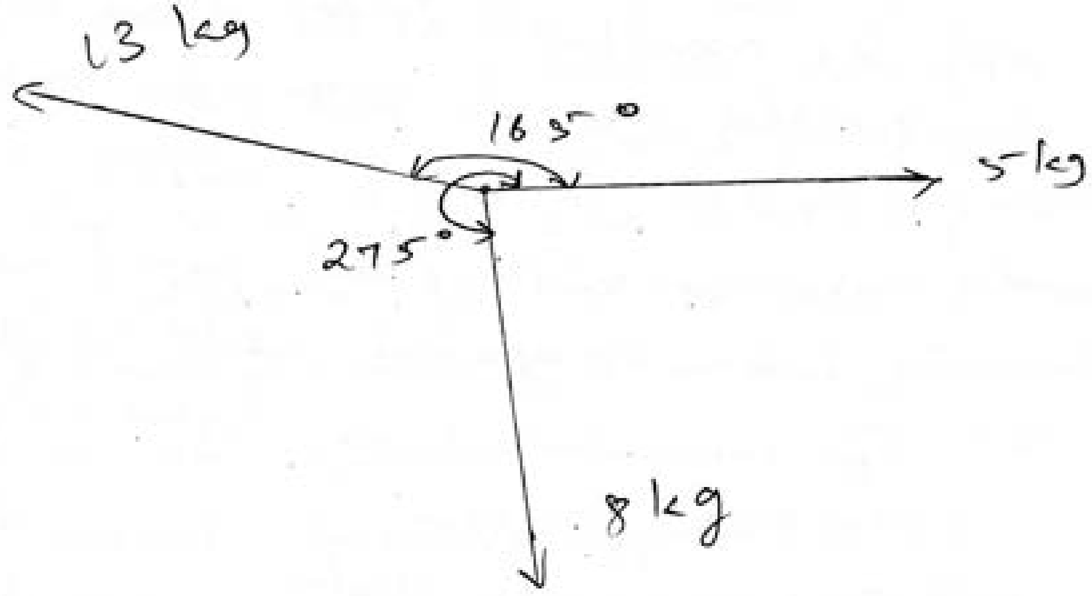
$$\theta = -50.57^\circ$$

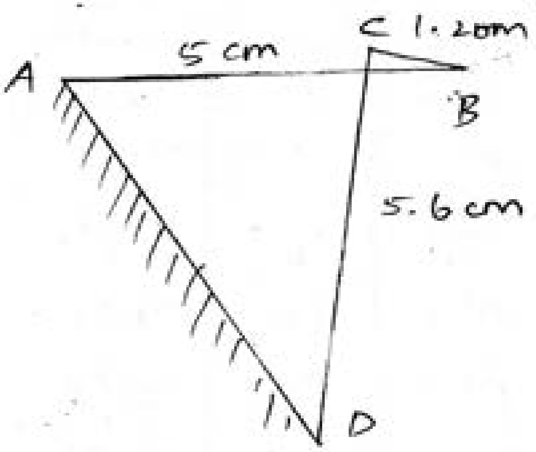
$$m = \frac{R}{r} = \frac{1.3636}{0.1} = 0.1363$$

Graphical Method

Scale
1kgm = 5cm

$\theta_1 = 0$ $m_1 r_1 = 5$ cm
$\theta_2 = 165$ $m_2 r_2 = 1.2$ cm
$\theta_3 = 275.0$ $m_3 r_3 = 5.6$ cm





Balancing of several masses rotating in different plane.

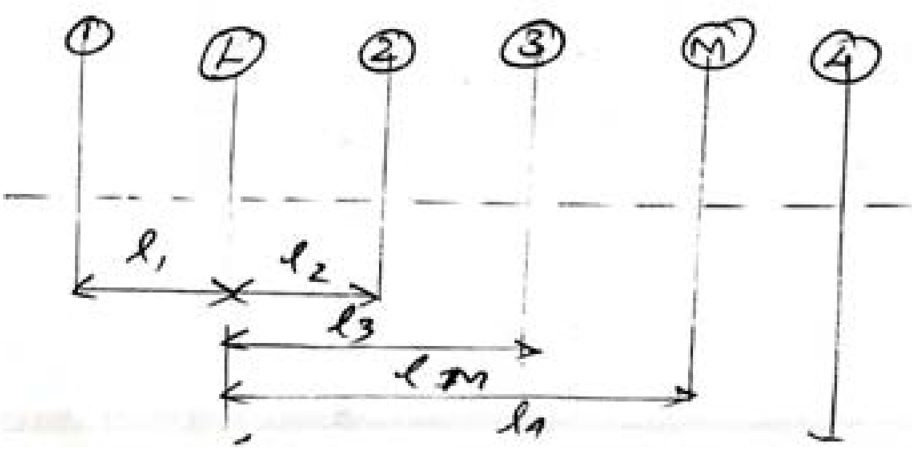
⇒ Let us consider four masses revolving in plane 1, 2, 3, 4 respectively

⇒ The magnitude of the balancing masses masses are M_L and m_M in planes L and M in plane L and M.

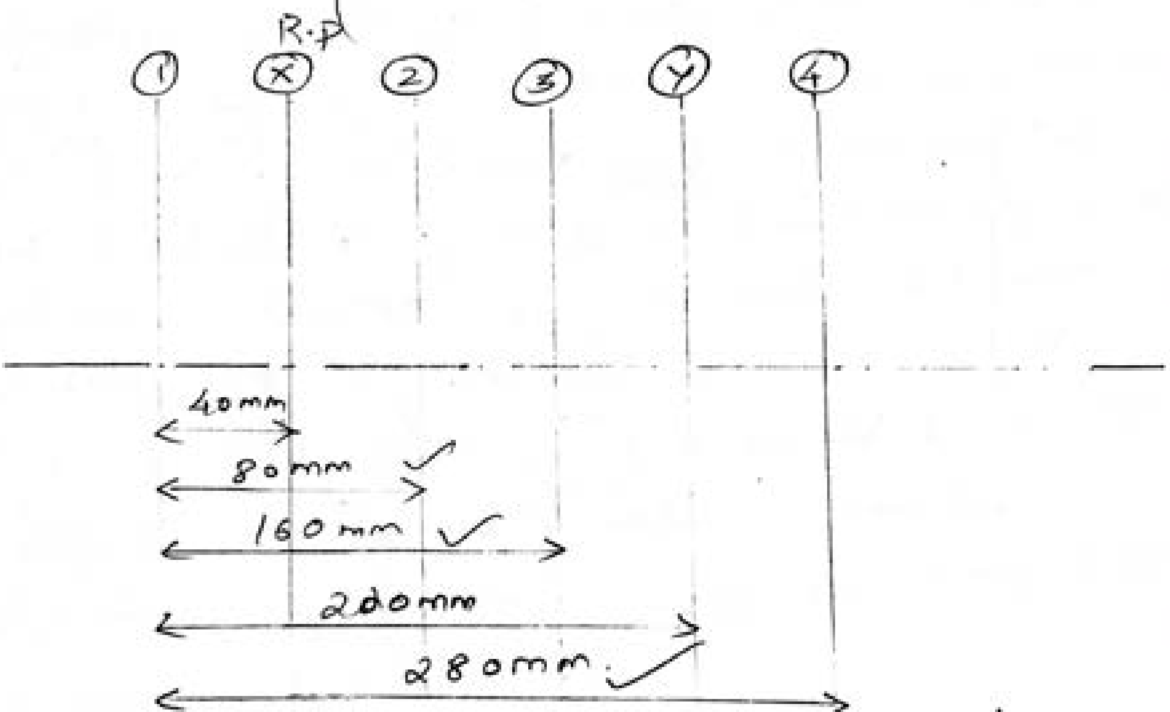
→ Take one of the plane say L as reference plane (rp). Left of the rp considered as negative and right is positive

⇒ Tabulate the data.

Plane	Mass M	Radii r	Centrifugal m·r	Distance from Plane (L)	Couple (mrl)
1	m_1	r_1	$m_1 r_1$	$-l_0$	$-m_1 r_1 l_1$
L(R.P)	m_L	r_L	$m_L r_L$	0	0
2	m_2	r_2	$m_2 r_2$	l_2	$m_2 r_2 l_2$.
3	m_3	r_3	$m_3 r_3$	l_3	$m_3 r_3 l_3$
M	m_M	r_M	$m_M r_M$	l_M	$m_m r_m l_m$
4	m_4	r_4	$m_4 r_4$	l_4	$m_4 r_4 l_1$.



5.13
1. A rotating shaft carries four unbalanced masses 18 kg, 14 kg, 16 kg and 12 kg at radii 5 cm, 6 cm, 7 cm and 6 cm respectively.
The 2nd, 3rd and 4th masses revolve in planes 8 cm, 16 cm and 28 cm respectively measured from the plane of the first mass and are angularly located at 60°, 135° and 270° respectively measured clockwise from first mass looking from this mass end of the shaft.
The shaft is dynamically balanced by two masses both located at 5 cm radii and revolving in planes midway between those of 1st and 2nd masses and midway between those of 3rd and 4th masses. Determine graphically or otherwise, the magnitudes of the masses and their respective angular positions.

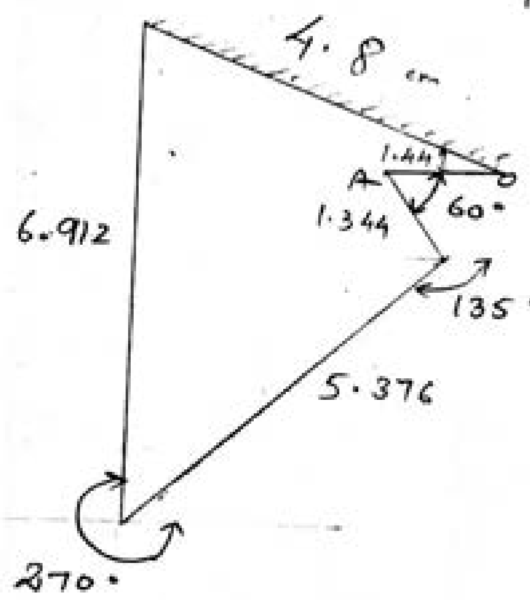


$$\begin{array}{r} 280 \\ 160 \\ \hline 120 \end{array} \qquad \begin{array}{r} 60 \\ 160 \\ \hline 220 \end{array}$$

$M_1 = 18\,kg$. $r_1 = 0.05\,m$ Angle
$m_2 = 14\,kg$ $r_2 = 0.06\,m$ 60
$M_3 = 16\,kg$ $r_3 = 0.07\,m$ 135
$m_4 = 12\,kg$ $r_4 = 0.06\,m$. 270

Plane	mass m	Radii r	Centrifugal (m.r) Force polygon	Distance from plane (l)	couple (m r l) (couple polygon)
1	18	0.05	0.9 4.5	−0.04	−0.036 1.44
X (R.P)	mx	0.05	0.05 mx	0	0
2	14	0.06	0.84 4.2	0.04	0.0336 1.344
3	16	0.07	1.12 5.6	0.12	0.1344 5.376
Y	my	0.05	0.05 my 3.33	0.18	0.009 my 13.
4	12	0.06	0.72 3.6	0.24	0.1728 6.912

Couple polygon:



Scale

$1\,cm = 0.025\,kg/m^2$.

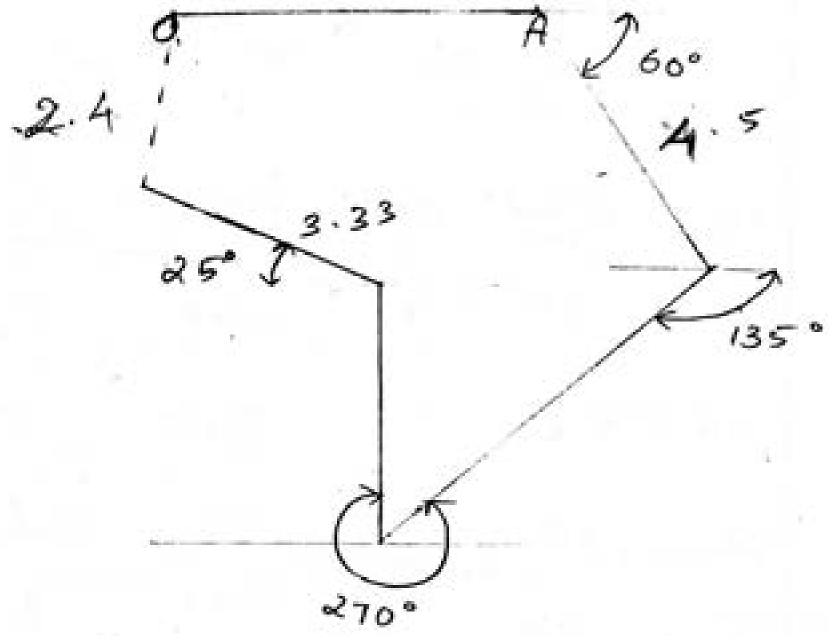
$0.009\,my = 4.8 \times 0.025$

$My = 13.33\,kg$

Force polygon:

Scale

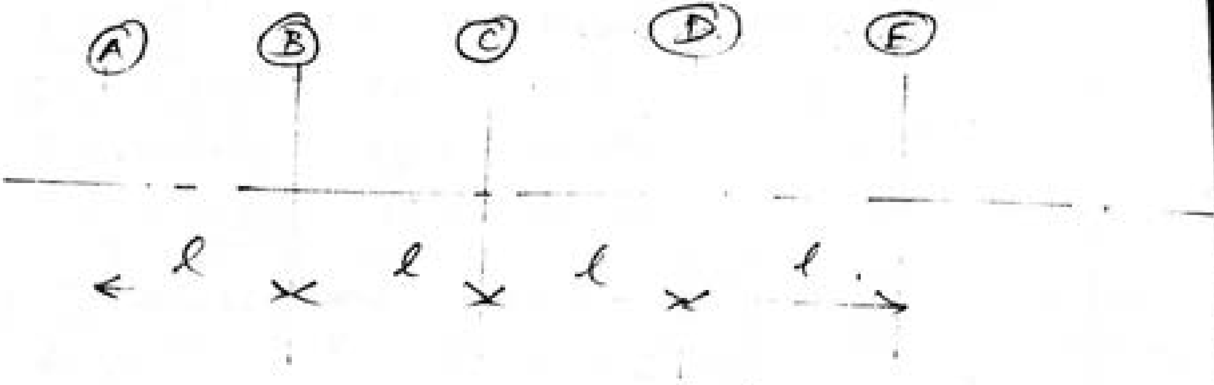
1 cm = 0.2 kg m



$0.05\, m_x = 2.4 \times 0.2$

$m_x = 9.6\ kg$

2. A shaft carried five masses A, B, C D and E which revolve at the same radius is planes which are equidistant from one another. The magnitude of A, ~~B~~, C, D are 50 kg, 40 kg and 80 kg respectively. The angle between A and C is 90° and that between C and D is 135°. Determine the magnitude of the masses in planes B and E and their positions to put the shaft in complete rotating balance.



Plane	Mass m	Radii r	Centrifugal m.r kg m (Forced polygon)	Distance from plane	Couple (m.r.l)
A	50	r	50r	$-l$	50
B(R.P)	M_B	r	$M_B r$	0	0
C	40	r	40r	l	40
D	80	r	80r	$2l$	160
E	M_E	r	$M_E r$	$3l$	$3M_E$

0.8
3.2

5.10
5.9
5.8
5.7
5.6

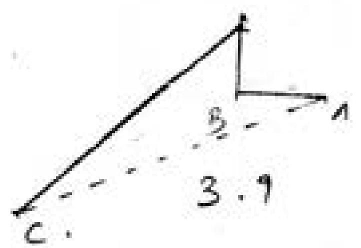
Given Data:

$r_A = r_B = r_C = r_D = r_E = r$

$m_A = 50kg$, $M_C = 40kg$, $M_D = 80kg$.

$\angle AOC = 90$ & $\angle COD = 135°$.

Couple polygon



$3M_E = 3.9 \times 50$

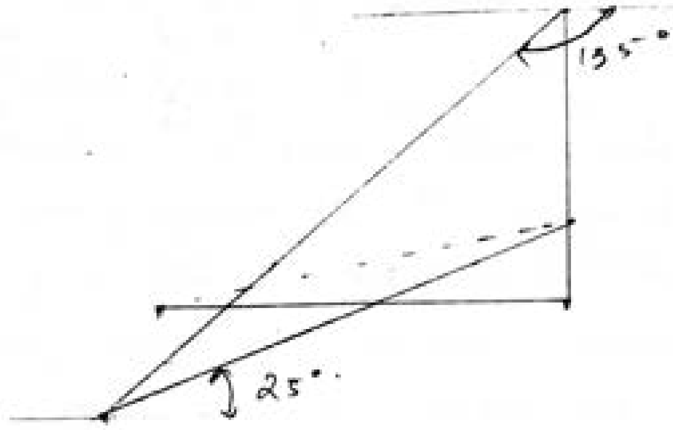
$M_E = \frac{3.9 \times 50}{3}$

$M_E = 65$

Forced polygon:

Scale.

$10r = 1cm$



3\. A rotating shaft carries four masses. A, B, C, D which are radially attached to it. The mass centres are 30mm, 38mm, 40mm and 35mm respectively from the axis of rotation. The mass A, C & D are 7.5kg, 5kg, 4kg respectively. The axial distance between the plane of rotation of A & B is 400mm and between B & C 500mm. The masses A & C are at right angle ~~to~~ to each other. find for a complete balance.

Solution:

$M_a = 7.5kg$, $M_c = 5kg$, $M_d = 4kg$.

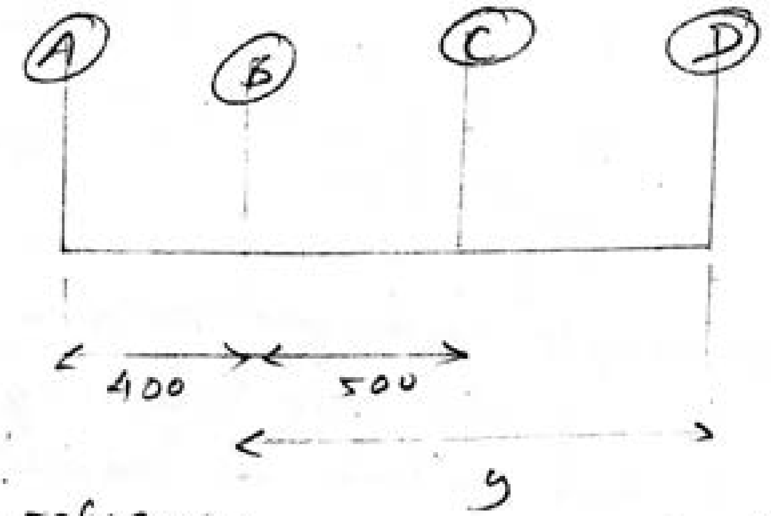
$r_a = 30\ mm$

$r_b = 38mm$ $\qquad \angle AOC = 90°$.

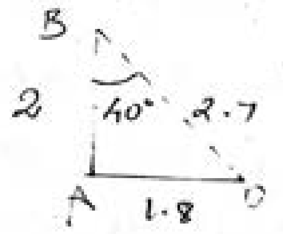
$r_c = 40mm$.

$r_d = 35mm$.

Plane	Mass M	Radii r	Centrifugal. mr	Distance from R.P	Couple mr.l
A	7.5	0.03	0.225	-0.4	-0.09
B (R.P)	M_B	0.038	$0.038 M_B$	0	0
C	5	0.04	0.2	0.5	0.1
D	4	0.035	0.14	y	0.14 y.



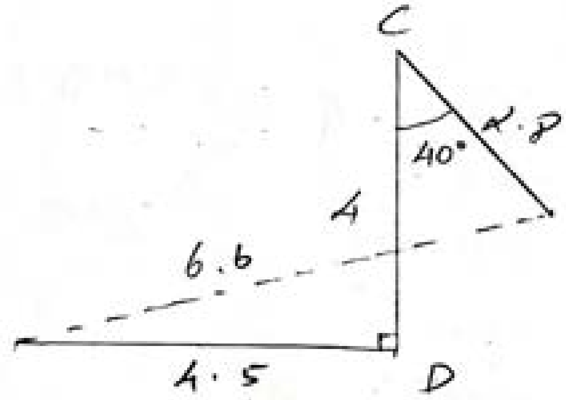
Couple polygon:



scale
1cm = 0.05
0.14 y = 2.7 × 0.05
y = 0.96

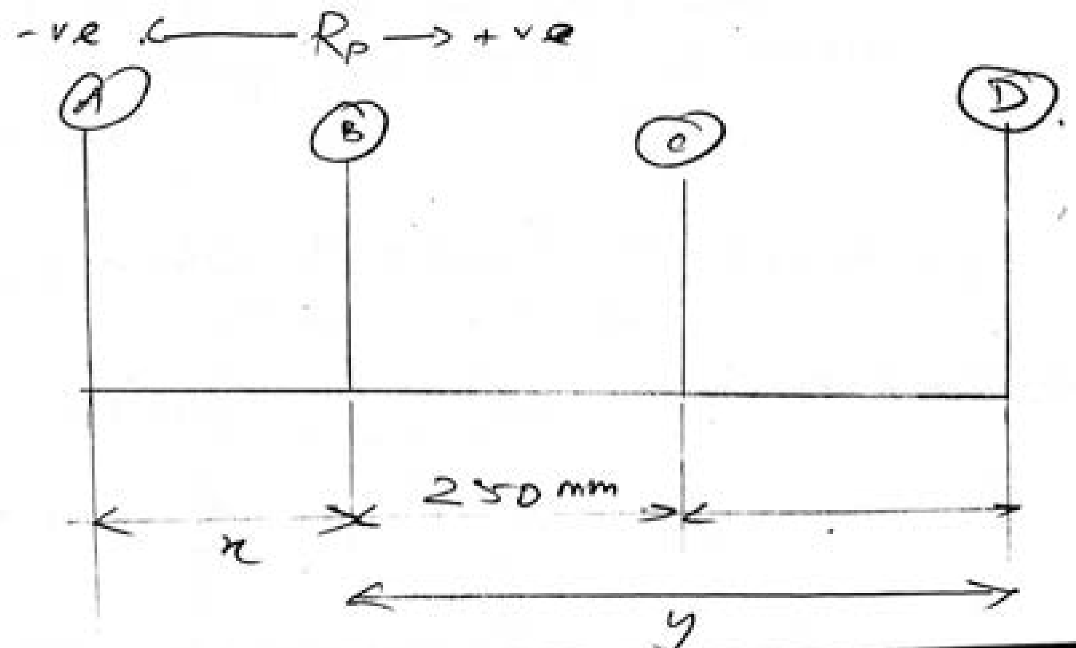
Force polygon:

1 cm = 0.05
$0.038 M_B = 6.6 \times 0.05$
$M_B = 8.68$



4. A shaft carries four rotating masses A, B, C and D which are completely balanced. The masses B, C and D are 50 kg, 80 kg and 70 kg respectively. The masses C and D makes angles of 90° and 195° respectively with mass B in the same sense. The masses A, B, C and D are concentrated at radius 75 mm, 100 mm, 50 mm and 90 mm respectively. The plane of rotation of masses B and C are 250 mm apart. Determine (i) the magnitude of mass A and its angular position and (ii) the position of planes A and D

Plane	Mass m	Radii r	Centrifugal (m.r)	Distance from R.P (l)	Couple m r l.
A	m_A	0.075	$0.075 m_A$	$-x$	$-0.075 m_A x$
B (R.P)	50	0.1	5	0	0
C	80	0.05	4	0.25	1
D.	70	0.09	6.3	y	6.3y.



5. A, B, C and D are four masses carried by a rotating shaft. at radii 100mm, 150mm, 150mm and 200mm respectively. The planes in which the masses rotates are space at 500mm apart and the magnitude of masses B, C, and D are 9kg, 5kg and 4 kg respectively. Find the required mass A and the relative angular settings of the four masses so that the shaft mast be in complete balance.

Unit - III

Free Vibration

Any motion which repeat itself after an interval of time is called vibration. There are three type of vibration

1. longitudinal ($\parallel^{el}$) to force.
2. transverse ($\perp r$) to force
3. Torsional (G) to force.

1. Longitudinal Vibration:

When the particle of the shaft moves parallel to the axis of the shaft then the vibration are known as longitudinal. Vibration

2. Transverse Vibration

When the particle of the shaft moves perpendicular to the axis of the shaft, then the vibration are called as transverse vibration

3. Torsional Vibration

When the particle of the shaf moves in a circle about the axis of the shaft then the vibration are known as torsional vibration.

Longitudianal Vibration

Undamped Free longitudinal Vibration :

$$\omega_n = \sqrt{\frac{S}{n}}$$

$$f_n = \frac{1}{t_p} = \frac{1}{2\pi}\sqrt{\frac{s}{m}}$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{9.81}{\delta}} = \frac{0.4985}{\delta}$$

$$t_p = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{\delta}{m}}$$

$$f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{s}{m}} = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}} = \frac{0.495}{\sqrt{\delta}}$$

Let, m - mass of the body suspended from the spring

δ - Static deflection of the spring

s - Stiffness of the spring

ω_n - circular frequency

t_p - Time period.

$$\delta = \frac{Wl}{A \cdot E}$$

$$W = m \cdot g$$

A = Area of cross section

E = Young's modulus

l - length.

1. A shaft of 100 mm diameter and 1m long is fixed at one end and the other end carries a flywheel of mass 1 ton take Young's modulus for the shaft material as 200 GN/m^2. Find the natural frequency of longitudinal vibration.

Given Data:

$$d = 100\,mm = 0.1\,m$$
$$l = 1\,m$$
$$m = 1\,ton = 1000\,kg$$
$$E = 200 \times 10^9\,N/m^2.$$

Solution:

$$f_n = \frac{1}{2\pi}\sqrt{\frac{9.81}{\delta}} = \frac{0.493}{\delta}$$

$$\delta = \frac{Wl}{AE} = \frac{mgl}{AE}$$

$$= \frac{1000 \times 9.81 \times 1}{\frac{\pi}{4} \times d^2 \times 200 \times 10^9}$$

$$= \frac{9810}{\frac{\pi}{4} \times 0.1^2 \times 2 \times 10^{11}}$$

$$\delta = 6.245 \times 10^{-6}\,m$$

$$f_n = \frac{0.493}{6.245 \times 10^{-6}}$$

$$\boxed{f_n = 197.28.}$$

2. A spring mass system has spring stiffness S N/m and a mass of m kg. It has the natural frequency of vibration as 12 Hz and extra 2 kg mass is coupled to m and the natural frequency is reduced by 2 Hz. Find the values of S and m.

★. $f_{n_1} = 12\ Hz$, $m_1 = m$

$f_{n_2} = 12 - 2$ $\quad m_2 = m+2$.

$= 10\ Hz$

WKT, $f_n = \frac{1}{2\pi}\sqrt{\frac{S}{m}}$

For mass m,

$$f_{n_1} = \frac{1}{2\pi}\sqrt{\frac{S}{m}} = 12$$

$$12 = \frac{1}{2\pi}\sqrt{\frac{S}{m}}$$

$$75.3982 = \sqrt{\frac{S}{m}}$$

$$S = 5684.89\,m$$

(ii) for m+2

$$10 = \frac{1}{2\pi}\sqrt{\frac{S}{m+2}}.$$

$$3947.84 = \frac{S}{m+2}.$$

$$3947.84\,m + 7895.68 = S.$$

$$3947.84\,m + \underline{7895.68} = 5684.89\,m$$

$$7895.68 = 1737.05\,m$$

$$m = 4.5454\ kg$$

$$S = 5684.89 \times 4.5454$$

$$S = 25840.4\ N/m$$

3. A steel wire ($E = 1.96 \times 10^{11} N/m^2$) is of 2 mm diameter and is 30m long. It is fixed at the upper end and carries a mass mkg at its lower end. find m so that the frequency of longitudinal vibration is four cycle/second (Hz).

Given data:

$E = 1.96 \times 10^{11} N/m^2$.

$d = 2mm = 0.002m$

$l = 30m$

$f_n = 4Hz$.

Solution

WKT,

$$f_n = \frac{1}{2\pi}\sqrt{\frac{9.81}{\delta}}$$

$$4 = \frac{1}{2\pi}\sqrt{\frac{9.81}{\delta}}$$

$$\boxed{\delta = 0.01553 \text{ m}}$$

$$\delta = \frac{W \times l}{A \cdot E}$$

$$\delta = \frac{m \times g \times l}{\frac{\pi}{4} \times d^2 \times E}$$

$$0.01553 = \frac{m \times 9.81 \times 30}{\frac{\pi}{4} \times 0.002^2 \times 1.96 \times 10^{11}}$$

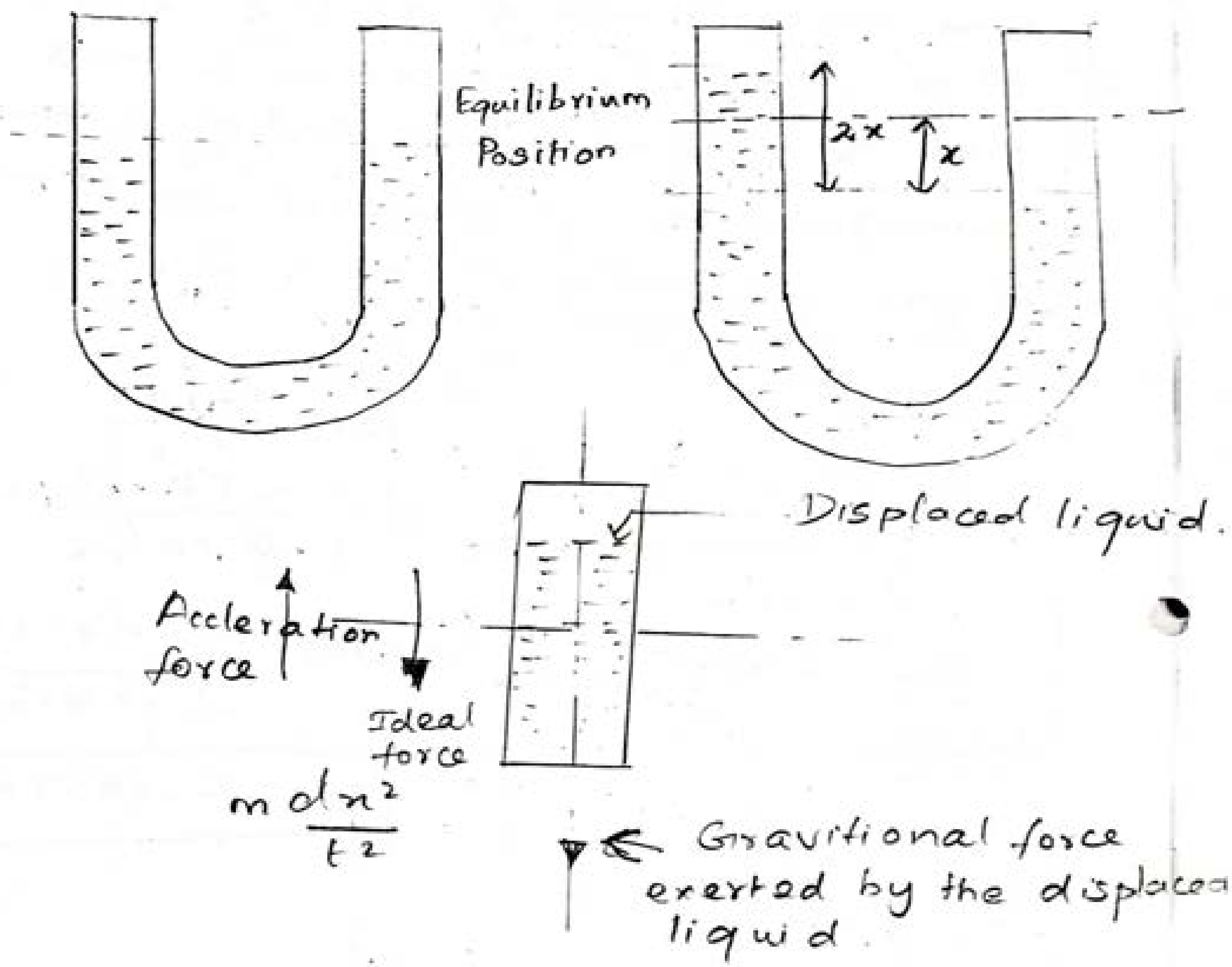
$$\boxed{m = 32.4957 \text{ kg}}$$

4. Natural frequency of oscillation of compound pendullum. Determine the equation of motion when a liquid column. vibration in a U-tube

(i) Newtown's method.

(ii) Energy method.

Find the Natural frequency.



The given u-tube is in equilibrium position.

Let, a - area of cross section of U-tube
ρ - mass density of water
l - total length of liquid column.
x - Distance through which liquid is displace from the equilibrium position.

(i) Newton method:

Total mass of the liquid, m =
= density x volume
= $\rho \times a \times l$.

Mass of the liquid displaced, $m_1 = (\rho a 2x)$

Gravitational force exerted by the displaced liquid $\left.\right\}$ $= m_1 \times g$ $= \rho a 2 x g$

The various forces acting on displaced liquid are

$$\text{accelerating force} = m\frac{d^2x}{dt^2} \; (\uparrow)$$

$$\text{Inertia force} = m\frac{d^2x}{dt^2} \; (\downarrow)$$

$$\text{gravitational or external force} = (\rho a 2x) g \; (\downarrow)$$

According to D'Alembert's principle:

$$\text{inertia} + \sum \text{external force} = 0$$

$$m\frac{d^2x}{dt^2} + (\rho a 2x) g = 0$$

$$\text{(or)} \quad (\rho a l)\frac{d^2x}{dt^2} + (\rho a 2x) g = 0$$

$[\because m = \rho a l]$

$$\text{(or)} \quad \frac{d^2x}{dt^2} + \left(\frac{2g}{l}\right) x = 0$$

This is required differential equation of motion. Comparing above equation with fundamental equation of SHM, we get

$$\omega_n^2 = \frac{2g}{l} \quad \text{(or)} \quad \omega_n = \sqrt{\frac{2g}{l}}$$

$\therefore$ Natural frequency of vibrations, $f_n = \dfrac{\omega_n}{2\pi} = \dfrac{1}{2\pi}\sqrt{\dfrac{2g}{l}}$

(ii) Energy method:

According to energy method

$$\frac{d}{dt}(K.E + P.E) = 0$$

Kinetic energy of liquid $= \frac{1}{2} mv^2$

$$P.E = mgh$$

$$PE = (\rho a x) g x = \rho a g x^2$$

$$\frac{d}{dt}\left[\frac{1}{2}\rho a l\left(\frac{dx}{dt}\right)^2 + \rho a g x^2\right] = 0$$

$$\frac{1}{2}\rho a l \, 2\left(\frac{dx}{dt}\right)\left(\frac{d^2x}{dt^2}\right) + \rho a g (2x)\left(\frac{dx}{dt}\right) = 0$$

$$\rho a l \frac{dx^2}{dt^2} - \rho a g \, 2x = 0$$

$$\frac{d^2x}{dt^2} + \left(\frac{2g}{l}\right)x = 0$$

Formula:

Effect of inertia of constrain in longitudinal vibration, $f_n = \frac{1}{2\pi}\sqrt{\frac{s}{m + \frac{m_c}{3}}}$

Springs in series

$$\delta = \delta_1 + \delta_2$$

$$\frac{W}{s} = \frac{W}{s_1} + \frac{W}{s_2}$$

$$\frac{1}{s_{eq}} = \frac{1}{s_1} + \frac{1}{s_2}.$$

Springs not in series.

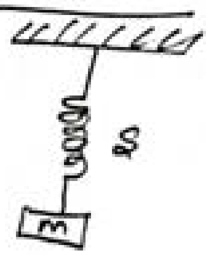
$$s_{eq} = s_1 + s_2.$$

1. Determine the equivalent spring stiffness and natural frequency of the vibrating system (i) The mass is suspended to a spring (ii) The mass is suspended to the bottom of two springs in series (iii) The mass is fixed between two springs (iv) The mass is fixed to the midpoint of a spring (v) The mass is suspended at the bottom of two spring in 11el

Take $S_1 = 1500 Nm$, $S_2 = 900 Nm$, $m = 12 kg$

Solution:

①



$$f_n = \frac{1}{2\pi}\sqrt{\frac{S_{eq}}{m}}$$

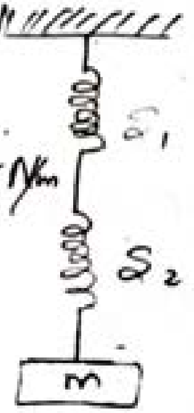
$$S_{eq} = S_1$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{1500}{12}}$$

$$= 1.78 Hz.$$

②

$$\frac{1}{S_{eq}} = \frac{1}{S_1} + \frac{1}{S_2}$$



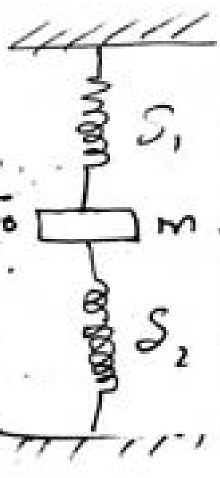
$$S_{eq} = 562.5 N/m$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{S_{eq}}{m}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{562.5}{12}}$$

$$f_n = 1.0896 Hz.$$

③ $S_{eq} = S_1 + S_2$.

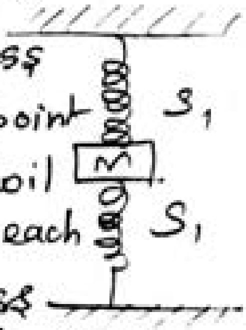


$S_{equ} = 2400 Nm$.

$$f_n = \frac{1}{2\pi}\sqrt{\frac{2400}{12}}$$

$$f_n = 2.25 Hz$$

④ As the mass is fixed at midpoint the number of coil becomes half on each side ∴ Stiffness of spring on each side



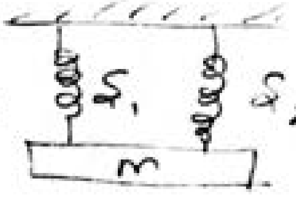
$$= \frac{S_1}{1/2} = 2S_1$$

$$\therefore S_{eq} = 2S_1 + 2S_1 = 4S_1$$

$$S_{eq} = 6000 Nm.$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{6000}{12}} = 3.55$$

⑤



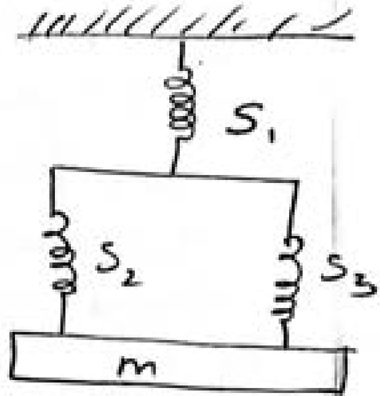
$$S_{eq} = S_1 + S_2$$

$$= 2400$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{2400}{12}}$$

$$= 2.25 Hz.$$

9 Find the equivalent system of vibration of the system shown in fig. If $S_1 = 5000$ N/m $S_2 = S_3 = 8000$ N/m and $m = 25$ kg. Find the natural frequency of vibration of the system



Given Data:

$S_1 = 5000$ N/m

$S_2 = S_3 = 800$ N/m

$m = 25$ kg.

Solution:

Effective stiffness of spring of the bottom 2 springs $= S_{e_1}$

$$S_{e_1} = S_2 + S_3$$

$$S_{e_1} = 16000 \text{ N/m}.$$

$$\frac{1}{S_{eq}} = \frac{1}{S_1} + \frac{1}{S_{e_1}}$$

$$S_{eq} = 3809.52 \text{ N/m}$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{S_{eq}}{m}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{3809.52}{12}}$$

$$f_n = 1.96 \text{ Hz}.$$

Critical damping constant and damping ratio

Damping.

The resistance offered by a body to motion of a vibrating system.

Damper:

The device used for resisting vibration is called damper.

Damping constant = c

now $\left(\frac{c}{2m}\right)^2 - \left(\frac{k}{m}\right) = 0$

Take $c = C_c$

$$\left(\frac{C_c}{2m}\right)^2 = \frac{k}{m}$$

$$\frac{C_c}{2m} = 2m\sqrt{\frac{k}{m}}$$

$$C_c = 2m\,\omega_n$$

Damping Ratio

Damping Ratio $\Sigma = \frac{c}{C_c}$

Three condition for spring Marsdash. port system.

i) Over damped System

$\Sigma > 1$

$$x = A_1 e^{(-\Sigma + \sqrt{\Sigma^2 - 1})\omega_n t} + A_2 e^{(-\Sigma - \sqrt{\Sigma^2 - 1})\omega_n t}$$

ii) Critical damped system, $\Sigma = 1$

$$x = (A_1 + A_2) e^{-\omega_n t}$$

iii) Under Damped system $\Sigma < 1$

$$x = \cancel{X} e^{-\Sigma \omega_n t} \sin(\omega_d t + \phi)$$

$\therefore \omega_d$ - damped natural frequency.

Formulas:-

1) Natural frequency

$$\omega_n = \sqrt{\frac{k}{m}} \text{ (or) } \sqrt{\frac{s}{m}} \text{ (or) } \sqrt{\frac{g}{\delta}}$$

2) Critical damping co-efficient

$$C_c = 2m\omega_n$$

3) Damping Ratio

$$\Sigma = \frac{c}{C_c}$$

4) Lograthmin decreament.

$$\delta = \frac{2\pi\Sigma}{\sqrt{1-\Sigma^2}} \text{ (or) } \ln\frac{x_n}{x_{n+1}}$$

5) Damped Natural frequency

$$\omega_d = \sqrt{1-\Sigma^2} \times \omega_n.$$

i) If 'n' is no of cycle is given then ratio of two consecutive amplitude $(\delta) = \frac{1}{n}\ln\frac{x_1}{x_n}$

b) 1. Find the equation of motion for the spring mass dashpot as shown in fig. for the cases (i) $\Sigma = 2$. (ii) $\Sigma = 1$, $\Sigma = 0$. The mass is displaced by a distance of 30 mm.

Note:

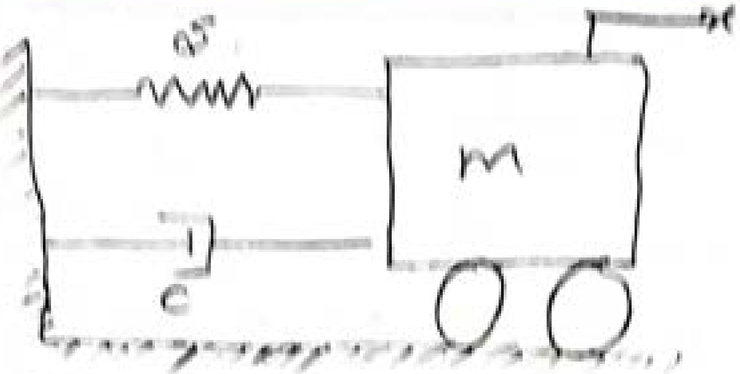
There are three types of damping

(i) Over damped

$$\varepsilon > 1$$

(ii) critically damped $\varepsilon = 1$.

(iii) Under damped $\varepsilon < 1$



Given Data:

$$x = 30\text{ mm} = 0.03\text{ m}$$

(i) $\xi = 2$

$$x = C_1 e^{(-\xi + \sqrt{\xi^2 - 1})\omega_n t} + C_2 e^{(-\xi - \sqrt{\xi^2 - 1})\omega_n t} \longrightarrow (1)$$

$$= C_1 e^{-0.268\omega_n t} + C_2 e^{-3.732\omega_n t}$$

(1) At $t = 0$, $x = 0.03\text{ m}$

$$0.03 = C_1 + C_2 \longrightarrow ①$$

(2) At $t = 0$; $v = \frac{dx}{dt} = 0$

$$\frac{dx}{dt} = -0.268\,\omega_n C_1 e^{-0.268\omega_n t} - 3.732\,\omega_n C_2 e^{-3.732\omega_n t}$$

$$\frac{dx}{dt} = -0.268\omega_n C_1 e^{-0.268\omega_n t} - 3.732\omega_n \cdot C_2 e^{-3.732\omega_n t}$$

$$0 = -0.268\,\omega_n C_1 - 3.732\,\omega_n C_2.$$

$$[0.268C_1 + 3.732C_2]\omega_n = 0$$

$$0.268C_1 + 3.732C_2 = 0 \quad ②$$

Solving equ ① and ② :

$$c_1 = 0.0323 \ \& \ C_2 = -2.32 \times 10^{-3}$$

Sub the value of c_1 and c_2 in Equ(i)

$$x = 0.0323 e^{-0.268\omega_n t} - (2.32 \times 10^{-3}) e^{-3.732\omega}$$

(ii) $\xi = 1$

$$x = [C_1 + C_2 t] e^{-\omega_n t}$$

1. $t = 0$, $x = C_1 e^{-\omega_n t} + C_2 t e^{-\omega_n t}$

$$C_1 = 0.03$$

2. $t = 0$, $v = \frac{dx}{dt} = 0$

$$V = \frac{dx}{dt} = -\omega_n C_1 e^{-\omega_n t} - \omega_n C_2 t e^{-\omega_n t} + C_2 e^{-\omega_n t}$$

$$0 = -\omega_n (0.03) - 0 + C_2$$

$$0 = -\omega_n (0.03) + C_2$$

$$C_2 = \omega_n (0.03)$$

$$C_1 = 0.03 , \ C_2 = \omega_n (0.03)$$

$$x = (0.03 + 0.03\omega_n) e^{-\omega_n t}$$

Equation of motion $\varepsilon = 0.3$

$$\varepsilon = 0.3 \ (\text{ie}; \ \varepsilon < 1)$$

$$x = X \cdot e^{-\varepsilon \omega_n t} \sin\left(\sqrt{1-\varepsilon^2}\,\omega_n t + \phi\right)$$

wher X and ϕ are arbituary constant, sub $\varepsilon = 0.3$, we get

$$x = X e^{-0.3\omega_n t} \sin(0.954\,\omega_n t + \phi)$$

Apply the initial condition to the above equation to find X and ϕ values.

(i) At, $t = 0$, $x = 0.3\,m$

$$X \cdot \sin\phi = 0.03.$$

(ii) At, $t = 0$, $V = \frac{dx}{dt} = 0.$

$$V = \frac{dx}{dt} = X e^{-0.3\omega_n t} \left[\cos(0.954\omega_n t + 0.954\omega_n\right].$$

$$0 = -\omega_n C_1 + C_2[0+1]$$

$$C_2 - \omega_n C_1 = 0$$

$$C_2 = 0.03\,\omega_n$$

$$x = (0.03 + 0.03\omega_n)e^{-\omega_n t}.$$

(iii) $\varepsilon = 0.3$

$$x = X e^{-\varepsilon\omega_n t} \sin(\omega_d t + \phi)$$

$$x = X e^{-\varepsilon\omega_n t} \sin\left(\sqrt{1-\varepsilon^2}\,\omega_n t + \phi\right)$$

$\Rightarrow t = 0$, $x = 0.03$

$$x = X \sin\phi$$

$$0.03 = X\sin\phi$$

At $t = 0$, $v = \frac{dx}{dt} = 0$

$$\frac{dx}{dt} = Xe^{-0.3\omega_n t}\left[\cos(0.95\omega_n t+\phi) \times 0.95\omega_n\right]$$

$$\times \sin(0.95\omega_n t+\phi)\left[(1-0.3\omega_n)e^{-0.3\omega_n t}\right]$$

b) Apply initial condition.

$$0 = 0.95\omega_n X\cos\phi - 0.3\omega_n X\sin\phi$$

$$\tan\phi = \frac{0.95}{0.3} = 3.18$$

$$\phi = \tan^{-1}(3.18) = 72.54°$$

Sub ϕ value in eqn (2).

$$X = \frac{0.03}{\sin(72.5)} = 0.0314$$

Sub the value of 'X' and ϕ in

$$x = 0.034 + e^{-0.3\omega_n t}\sin(0.95\omega_n t + 72.54)\,m$$

2. A vibrating system consist of mass 8 kg, spring of stiffness 5.6 N/mm and a dashpot of damping coefficient of 40 N/m/s. Find (i) critical damping coefficient (ii) damping factor (iii) natural frequency of damped vibration. (iv) Logorithmithic decreasment (v) Ratio of two consecutive amplitude (vi) Number of cycle after which the amplitude is reduced to 20%.

Given Data:

$m = 8\,kg$, $S = 5.6 \times 10^3\,N/m$.

$c = 40\,N/m/s$.

Solution : Critical Damping coefficient

① $C_c = 2 m \omega_n$

$$\omega_n = \sqrt{S/m} = 26.4575.$$

$$C_c = 2 \times 8 \times 26.4575$$

$$C_c = 422.32 \ N/m/s.$$

Damping factor :

② $\varepsilon = \frac{c}{C_c} = 0.0945$

③ Natural frequency of damping vibration

$$f_d = \frac{\omega_d}{2\pi}$$

$$\omega_d = \sqrt{1-\varepsilon^2} \times \omega_n$$

$$= \sqrt{1-(0.0944)^2} \times 26.45$$

$$\omega_d = 26.33 \ rad/s.$$

$$f_d = \frac{26.33}{2\pi}$$

$$f_d = 4.19 \ Hz.$$

4. Logrithmic decreament

$$\delta = \frac{2\pi\varepsilon}{\sqrt{1-\varepsilon^2}}$$

$$= \frac{2\pi \times 0.0945}{\sqrt{1-0.0945^2}}$$

$$= 0.596$$

cut

5. Consecutive amplitude.

$$\delta = \ln\left[\frac{x_n}{x_{n+1}}\right]$$

(or)

$$\frac{x_n}{x_{n+1}} = e^{\delta}$$

$$0.5957 = \ln\left[\frac{x_n}{x_{n+1}}\right]$$

$$\frac{x_n}{x_{n+1}} = 1.81.$$

6. Number of cycle after which the amplitude is reduced to 20%.

X_0 = amplitude of starting position

X_n = amplitude after n cycle

$$X_n = 20\%$$

$$X_n = 0.2 \times 0.$$

$$\delta = \frac{1}{n}\ln\left[\frac{x_0}{x_n}\right]$$

$$\delta = \frac{1}{n}\ln\left(\frac{x_0}{0.2x_0}\right].$$

$$0.596 = \frac{1}{n}\ln\left[\frac{1}{0.2}\right].$$

$$0.595 = \frac{1}{n}[1.609).$$

$$n = 2.70 \text{ cycle}.$$

1. A single damped vibrating system a suspended mass of 8 kg makes 30 oscillation in 18 seconds. The amplitude decreases to 0.25 of the initial value after 5 oscillation determine striffness of spring, lograthemic decrement, damping factor, damping coefficient.

Given Data:

$$m = 8\,kg$$

$$f_n = \frac{30}{18}$$

$$f_n = 1.67\,Hz$$

$$x_5 = 0.25 x_0$$

To find:

$$S = ?$$

Solution:

$$\omega_n = \sqrt{\frac{S}{m}}$$

$$\omega_n = 2\pi f_n$$

$$= 2 \times \pi \times 1.66$$

$$\omega_n = 10.49\, r/s$$

$$\delta = \frac{1}{n} \ln\left[\frac{x_0}{x_n}\right]$$

$$\delta = \frac{1}{5} \ln \frac{x_0}{0.25 \times x_0}$$

$$\delta = 0.277$$

$$\varepsilon = \frac{c}{c_c}$$

$$c_c = 2m\omega_n$$

$$c_c = 167.84\, N/m/s$$

$$\delta = \frac{2\pi\varepsilon}{\sqrt{1-\varepsilon^2}}$$

$$0.277 = \frac{2\pi\varepsilon}{\sqrt{1-\varepsilon^2}}$$

$$0.044 = \frac{\varepsilon}{\sqrt{1-\varepsilon^2}}$$

$$\sqrt{1-\varepsilon^2} = \frac{\varepsilon}{0.044}$$

$$1-\varepsilon^2 = \frac{\varepsilon^2}{1.936 \times 10^{-3}}$$

$$1-\varepsilon^2 = 516\varepsilon^2$$

$$517\varepsilon^2 = 1$$

$$\varepsilon = 0.04$$

$$\varepsilon = \frac{c}{c_c}$$

$$c = 0.04 \times 167.84$$

$$c = 6.7136\, N/m/s$$

$$\omega_n = \sqrt{\frac{S}{m}}$$

$$10.49^2 = \frac{S}{m}$$

$$S = 880.32\, N/m$$

1. The machine mounded on spring and fitted with a mass of 60kg and there are three springs each of stiffness 12 N/mm. The amplitude of vibration reduces from 45mm to 8mm in two complete oscillation assuming the damping force varies as it velocity changes. Determine ratio of frequency of damping and undamped vibration

Given data:

$$m = 60 kg$$

$$S = 12000 \; N/m.$$

$$\frac{x_0}{x_2} = \frac{45}{8}.$$

$$\frac{x_0}{x_2} = 5.62 mm = 5.62 \times 10^{-3} m$$

Solution:

stiffness of spring in three spring

$$S = 3 \times 12000$$

$$S = 36000 \; N/m.$$

$$\xi = \frac{c}{c_c}.$$

$$c_c = 2m\omega_n.$$

$$\omega_n = \sqrt{\frac{36 \times 10^3}{60}} = 24.49 \; r/s.$$

$$C_c = 2m\omega_n = 2 \times 60 \times 24.49.$$

$$C_c = 2938.8 \; N/m/s.$$

$$\delta = \frac{1}{n} \ln\left(\frac{x_0}{x_n}\right) = \frac{1}{2} \ln\left(\frac{x_0}{x_2}\right)$$

$$= \frac{1}{2} \ln(0.00562).$$

$$\delta = \frac{1}{2}\ln(5.62)$$

$$\delta = 0.863$$

$$\delta = \frac{2\pi\Sigma}{\sqrt{1-\Sigma^2}}$$

$$0.863 = \frac{2\pi\Sigma}{\sqrt{1-\Sigma^2}}$$

$$\sqrt{1-\Sigma^2} = 7.28\Sigma.$$

$$1-\Sigma^2 = 52.99\Sigma^2.$$

$$\Sigma = 0.136.$$

$$\Sigma = \frac{c}{c_c}$$

$$c = 399.67 \; N/m/s$$

$$\text{Time period} = \frac{2\pi}{\omega_d}.$$

$$\omega_d = \sqrt{1-\Sigma^2} \times \omega_n.$$

$$= \sqrt{1-0.136^2} \times 24.49.$$

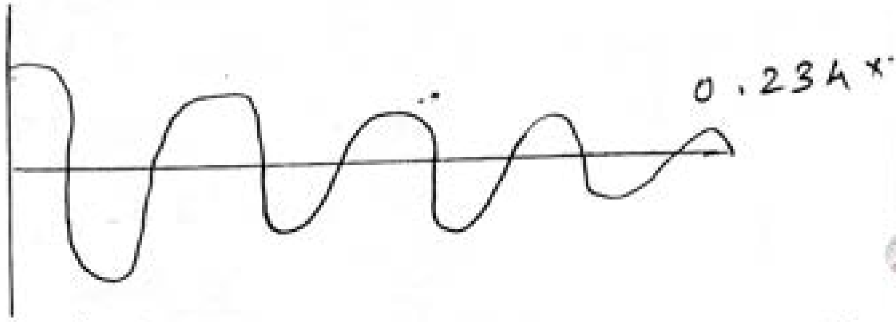
$$\omega_d = 24.26 \; r/s$$

$$\text{Time period} = \frac{2\pi}{24.26} = 0.2595$$

$$\text{Undamped vibration} = \frac{\omega_d}{\omega_n} = 0.99.$$

Grandamped problem:

1. The machine has a mass of 200 kg. It is placed on 2 differed isolator and the corresponding. Free vibration is recorded as shocon in figure (i) damping factor, f_n, undamped vibration, δ, C_c, C.



Given data:

$$m = 200\,kg \quad , n = 4.$$

$$\delta = \frac{1}{n} \ln\left[\frac{x_0}{x_n}\right] = \frac{1}{4}\left[\frac{1.52}{0.234}\right]$$

$$\delta = \frac{1}{4}\left[\frac{1.52}{0.234}\right]$$

$$\delta = 0.472.$$

$$\delta = \frac{2\pi \Sigma}{\sqrt{1-\Sigma^2}}$$

$$0.472 = \frac{2\pi \Sigma}{\sqrt{1-\Sigma^2}}$$

$$\sqrt{1-\Sigma^2} = \frac{2\pi\Sigma}{0.472}$$

$$1-\Sigma^2 = 177.2\Sigma^2$$

$$\Sigma = 0.074 < 1$$

∴ it is underdamping natural.

frequency of damped vibration.

$$f_d = \frac{\text{number of cycle}}{\text{length}} = \frac{4}{0.38}$$

$f_d = 10.52\ Hz$

Circular frequency of damped vibration

$\omega_d = f_d \times 2\pi$

$\omega_d = 2\pi \times 10.52$

$\omega_d = 66.1\ r/s$

$\omega_d = \sqrt{1-\varepsilon^2} \times \omega_n$

$$\frac{66.1}{\sqrt{1-0.074^2}} = \omega_n$$

$\omega_n = 66.35\ rad/s$

$$f_n = \frac{\omega_n}{2\pi} = \frac{66.35}{2\pi}$$

$f_n = 10.56\ Hz$

$$\omega_n = \sqrt{\frac{S}{m}} \Rightarrow 66.35^2 = \frac{S}{200}$$

$S = 879138\ N/m$

$C_C = 2\omega_n m$

$= 2 \times 200 \times 6.3$

$C_C = 2650\ N/s/m$

$$\varepsilon = \frac{C}{C_C}$$

$$0.074 = \frac{C}{2652}$$

$C = 1964.48$

2. A gun barell weights 300 kg has a recoil spring of a stiffness 250 N/mm. The barell recoil 0.8m on firing. Determine (i) initial recoil velocity of gun (ii) critical damping coefficient of dashport engaged at the end of recoil stock.

Given Data:

$$m = 400\ kg$$

$$v = 20\ m/s$$

$$x = 1m.$$

Solution:

Kinetic energy of the barell = work done in spring.

$$\frac{1}{2} m v^2 = \frac{1}{2} \times S \times x^2$$

$$\frac{1}{2} \times 400 \times 20^2 = \frac{1}{2} \times S \times 1^3$$

$$S = 16 \times 10^4\ N/m.$$

(ii) $C_c = 2 m \omega_n$

$$\omega_n = \sqrt{\frac{S}{m}} = \sqrt{\frac{16 \times 10^4}{400}}$$

$$= 20\ rad/s$$

$$CC = 2 \times 400 \times 20$$

$$= 16 \times 10^3\ N/s/m$$

3 A barrel of the large gun recoil a spring on firing, at the end of recoiling the dashport is engaged that allows the barrel to return to its initial position. in a minimum time with out oscillation. The gun barrel has a mass of 400kg and a initial recoil velocity of the gun barrel at the instant of firing is 20 m/s. The barrel recoil 1m on firing. Find (i) Critical damping coefficient of the dashport

(ii) Spring stiffness

Given data:

$m = 400 kg.$

$v = 2 m/s$

$x = 1 m.$

Solution

(ii) Spring stiffness.

kinetic energy of the barrel } = Workdone in spring

$$\frac{1}{2} m v^2 = \frac{1}{2} \times S \times x^2.$$

$$\frac{1}{2} \times 400 \times 20^2 = \frac{1}{2} \times S \times 1^2.$$

$$S = 160000 \ N/m.$$

(i) $C_c = 2m\omega_n$.

$$\omega_n = \sqrt{\frac{s}{m}} = \sqrt{\frac{160000}{40}} = 20 \text{ rad/s}$$

$$C_c = 2 \times 400 \times 20$$

$$C_c = 16000 \text{ N/s/m}.$$

4. Find the equation of motion for the spring mass dashpot system as shown for the cases when (i) $\xi = 2$. (ii) $\xi = 1$ (iii) $\xi = 0.25$ and $x = 2$ mm

Given data:

$$x = 0.025 \text{ m}$$

Condition (1) $\xi = 2$

$$x = c_1 e^{(-\xi + \sqrt{\xi^2 - 1})\omega_n t} + C_2 e^{(-\xi - \sqrt{\xi^2 - 1})\omega_n t}$$

$$x = c_1 e^{(-2+\sqrt{3})\omega_n t} + C_2 e^{(-2-\sqrt{3})\omega_n t}$$

$$x = c_1 e^{-0.268\omega_n t} + C_2 e^{-3.732\omega_n t}$$

Put $t = 0$, $x = 0$

$$c_1 + c_2 = 0.025 \quad \text{—(1)}.$$

At $t = 0$, $v = dx/dt$

$$\frac{dx}{dt} = -0.268\omega_n c_1 e^{-0.268\omega_n t} - 3.732\omega_n C_2 e^{-3.732\omega_n t}$$

Put $t = 0$, $\frac{dx}{dt} = 0$

$$-0.268\omega_n c_1 - 3.732\omega_n C_2 = 0$$

$$0.268 C_1 + 3.732 C_2 = 0$$

Put $c_1 = 0.025 - C_2$

$$0.268(0.025-C_2)+3.732C_2=0$$

$$0.0067-0.268C_2+3.732C_2=0$$

$$C_2=-1.92\times10^{-3}.$$

Sub value in eqn ①.

$$C_1-1.92\times10^{-3}=0.025$$

$$C_1=0.02692.$$

$$x_1=0.02692e^{(-0.268)\omega_n t}-(-1.92\times10^{-3})e^{(-1.92\times10^{-3})\omega_n t}$$

Condition (ii).

$$\Sigma=1.$$

$$x=(c_1+c_2t)e^{-\omega_n t}$$

$$x_1=c_1e^{-\omega_n t}+C_2te^{-\omega_n t}$$

Apply $t=0$, $x=0.025$

$$C_1=0.025$$

Apply $t=0$, $v=\frac{dx}{dt}=0$

$$\frac{dx}{dt}=-\omega_n c_1e^{-\omega_n t}-\omega_n t_2 t^{-\omega_n t}+C_2e^{-\omega_n t}$$

$$0=-\omega_n C_1+C_2$$

$$C_2=\omega_n C_1$$

$$C_2=0.025\omega_n$$

$$x=0.025e^{-\omega_n t}+0.025\omega_n te^{-\omega_n t}$$

$$x=(0.025+0.025\omega_n t)e^{-\omega_n t}$$

$$x=(0.025+0.025\omega_n t)e^{-\omega_n t}$$

Case (iii) $\Sigma=0.25$

$$x=Xe^{-\Sigma\omega_n t}$$

$$x=Xe^{-\Sigma\omega_n t}\sin(\omega_d t+\phi).$$

$$x = X e^{-0.25\omega_n t} \sin(\sqrt{1-\xi^2} \times \omega_n t + \phi).$$

$$x = X e^{-0.25\omega_n t} \sin(0.968\omega_n t + \phi) \longrightarrow (1)$$

when $t = 0$, $x = 0.025$

$$0.025 = X \sin\phi$$

At $t = 0$, $V = \frac{dx}{dt} = 0$

$$\frac{dx}{dt} = X e^{-0.25\omega_n t} [\cos(0.96\omega_n t + \phi) \times 0.096$$

$$+ X \sin(0.954\omega_n t + \phi) [(-0.3\omega_n) e^{-0.3\omega_n t}$$

Apply initial condition

$$0 = 0.96\omega_n X \cos\phi - 0.3\omega_n X \sin\phi$$

$$\tan\phi = \frac{0.96}{0.25}$$

$$\phi = 75.40°.$$

$$\alpha = \frac{0.25}{\sin 75.4} = 0.258.$$

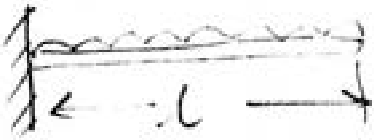
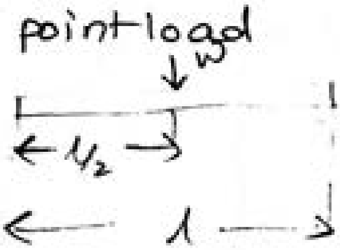
$$x = X e^{-0.3\omega_n t} \sin(0.96\omega_n t + 75.4).$$

$$x = 0.258 e^{-0.3\omega_n t} \sin(0.96\omega_n t + 75.4)$$

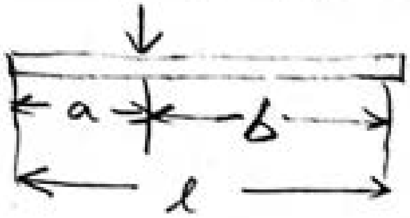
Transverse Vibration.

When the particle of the shaft (or) disc moves ~~pa~~ perpendicular to the axis of vibration then the vibration is known as transverse vibration.

Formula:

Types of beam of load condition	Deflection	Natural frequency
1. Cantilever		
(i) With point load at free end. 	$\delta = \frac{Wl^3}{3EI}$ (at free end)	$f_n = \frac{1}{2\pi}\sqrt{\frac{s}{m}}$ $= \frac{0.4985}{\sqrt{\delta}}$
(ii) Uniformly distributed load of w per unit length. 	$\delta = \frac{wl^4}{8EI}$ (at free end)	$f_n = \frac{1}{2\pi}\sqrt{\frac{s}{\frac{33}{140}m_c}}$ $= \frac{0.621}{\sqrt{\delta}}$ (where $m_c = \frac{wl}{g}$)
2. Simply supported beam		
(i) With central point load 	$\delta = \frac{Wl^3}{48EI}$ (at centre)	$f_n = \frac{1}{2\pi}\sqrt{s/m}$ $= \frac{0.4985}{\sqrt{\delta}}$

ii) with eccentric point load w



$$\delta = \frac{Wa^2b^2}{3EIl}$$

(at point load)

$$f_n = \frac{1}{2\pi}\sqrt{\frac{s}{m}}$$

$$= \frac{0.4985}{\sqrt{\delta}}$$

With uniformly distributed load of w per unit length.

$$\delta_s = \frac{5}{384} \times \frac{wl^4}{EI}$$

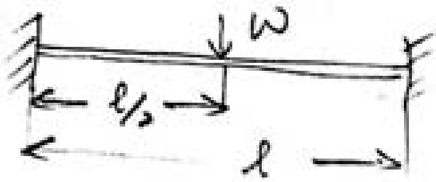
$$f_n = \frac{1}{2\pi}\sqrt{\frac{s}{\frac{17}{35}m}}$$

$$= \frac{0.5615}{\sqrt{\delta_s}}$$

$$m_c = \frac{wl}{g}$$

3. Fixed beam.

i) With central point load w



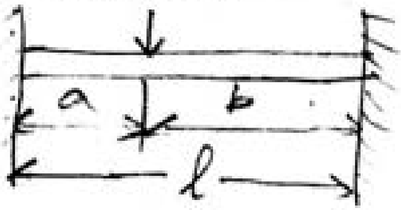
$$\delta = \frac{wl^3}{192EI}$$

(at centre)

$$f_n = \frac{1}{2\pi}\sqrt{s/m}$$

$$= \frac{0.4985}{\sqrt{\delta}}$$

ii) With eccentric point load w



$$\delta = \frac{Wa^3b^3}{3EIl^3}$$

(at point load)

$$f_n = \frac{1}{2\pi}\sqrt{s/m}$$

$$= \frac{0.4985}{\sqrt{\delta}}$$

(iii) With uniformly distributed load w per unit length

$$\delta_s = \frac{Wl^4}{384EI}$$

(at centre)

$$f_n = \frac{1}{2\pi}\sqrt{\frac{s}{\frac{13}{35}m_c}}$$

$$= \frac{0.571}{\sqrt{\delta_s}}$$

$$m_c = \frac{wl}{g}.$$

A shaft of beam length 0.75 m supported freely at end, it carrying a body of mass 90 kg at 0.25 m from one end. Find the natural frequency of transverse vibration assume. $E = 200\ GN/m^2$. and shaft diameter 50 mm.

Given data:

$$l = 0.75\ m$$
$$m = 90\ kg$$
$$a = 0.25\ m$$
$$E = 200 \times 10^9\ N/m^2.$$
$$d = 50\ mm = 0.05\ m.$$

Solution:

For simply supported beam.

$$\delta = \frac{W a^2 b}{3EIl}.$$

$$I = \frac{\pi}{64} \times d^4 = 3.067 \times 10^{-7}\ m.$$

$$W = m \times g = 90 \times 9.81$$
$$= 882.9\ N$$

$$b = l - a = 0.75 - 0.25$$
$$b = 0.5\ m$$

$$\delta = \frac{882.9 \times (0.25)^2 \times (0.5)^2.}{3 \times 200 \times 10^9 \times 3.067 \times 10^{-7} \times 0.}$$

$$\delta = 9.99 \times 10^{-5}\ m.$$

$$f_n = \frac{1}{\sqrt{2\pi}}\sqrt{\frac{s}{m}} = \frac{0.495}{\sqrt{\delta}}$$

$$f_n = \frac{0.495}{\sqrt{9.99 \times 10^{-5}}}$$

$$f_n = 49.86\ Hz$$

A shaft supported freely at the ends has a mass of 120kg and bare 250mm from one end. and the shaft diameter is 40mm. Determine the frequency of transverse vibration. If the length of the shaft 700mm. $E = 200 \times 10^9\ N/m^2$

Given Data:

$$m = 120kg$$

$$a = 250mm = 0.25m.$$

$$d = 40mm = 0.04\ m$$

$$l = 700mm = 0.7m.$$

$$E = 200 \times 10^9\ N/m^2.$$

To find:

δ, f_n

Solution :

$$\delta = \frac{W a^2 b^2}{3 \times E I l}.$$

$$I = \frac{\pi}{64} \times d^4 = \frac{\pi}{64} \times 0.04^4$$

$$I = 1.2566 \times 10^{-7} m.$$

$$b = l - a = 0.7 - 0.25 = 0.45 m$$

$$W = m \times g = 120 \times 9.81$$

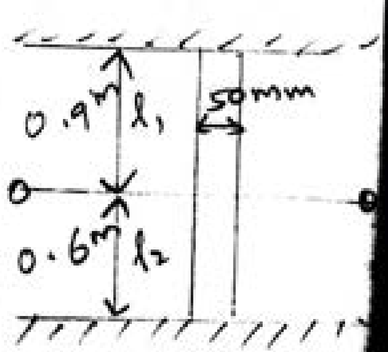
$$\delta = \frac{120 \times 9.81 \times 0.25^2 \times 0.45^2}{0.7 \times 3 \times 1.2566 \times 10^{-7} \times 200 \times 10^9}$$

$$\delta = 2.8229 \times 10^4.$$

$$f_n = \frac{0.495}{\sqrt{\delta}} = \frac{0.495}{\sqrt{2.8229 \times 10^4}}$$

$$f_n = 29.4616 \text{ Hz}$$

A fly wheel is mounted on a vertical shaft as shown in figure. Both end of the shaft are fixed and its diameter is 50 mm. The fly wheel has a mass of 500kg. Find the natural frequency of longitudinal and transverse vibration. Take $E = 200 \times 10^9 N/m^2$.



Given Data :

Considering a fixed beam with a eccentric load for transverse vibration.

$$\delta = \frac{W a^3 b^3}{3 E I l^3}.$$

$$I = \frac{\pi}{64} \times d^4 = \frac{\pi}{64} \times 0.05^2.$$

$$I = 3.0679 \times 10^{-7} m^4.$$

$$\delta = \frac{500 \times 9.81 \times 0.9^3 \times 0.6^3}{3 \times 200 \times 10^9 \times 3.0679 \times 10^{-7} \times 1.5^3}$$

$$\delta = 1.24 \times 10^{-4} \text{ m}$$

$$f_n = \frac{0.495}{\sqrt{\delta}} = \frac{0.495}{\sqrt{1.24 \times 10^{-4}}}$$

$$f_n = 14.15 \text{ Hz}.$$

Longitudinal Vibration:

Let

$m_1 \longrightarrow$ Mass of fly wheel carried by length (l_1)

$m_2 \longrightarrow$ ($m - m_1$) mass of the flywheel carried by length (l_2)

$\therefore$ Extension of l_1

$$\delta = \frac{w_1 l_1}{AE}$$

$\therefore$ Compression of l_2.

$$\delta = \frac{w_2 l_2}{AE}$$

$$\frac{m_1 g_1 l_1}{AE} = \frac{(mg - m_1 g) \times l_2}{AE}$$

$$m_1 l_1 = (m - m_1) l_2.$$

$$m_1 l_1 = (500 - m_1) 0.6.$$

$$m_1(0.9) = (500 - m_1)0.6$$

$$m_1(0.9) = 300 - 0.6m_1$$

$$300 = 0.9m_1 + 0.6m_1$$

$$300 = 1.5m_1$$

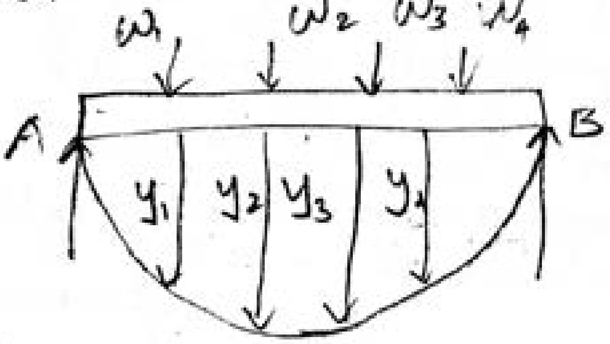
$$m_1 = 200 kg$$

$$M_2 = m - M_2$$

$$= 500 - 200$$

$$m_2 = 300 kg$$

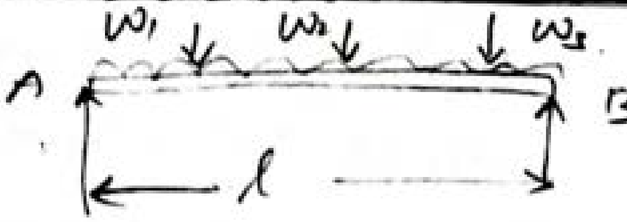
Natural frequency of free transverse vibration for the shaft subjected to point load.



y_1, y_2, y_3, y_4 - deflection under w_1, w_2, w_3, w_4

$$\omega = \sqrt{\frac{g \Sigma my}{\Sigma my^2}} \text{ rad/s}$$

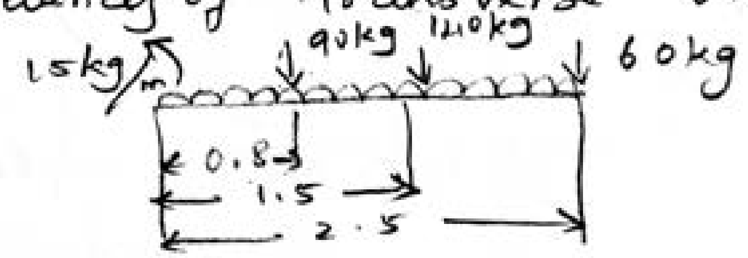
2. Natural frequency of free transverse vibration for a shaft subjected to UDL.



$$f_n = \frac{0.4985}{\sqrt{\delta_1 + \delta_2 + \delta_3 + \dots + \frac{\delta_s}{1.27}}}$$

$$F_n = \left[\frac{0.4985}{\sqrt{\delta_1 + \delta_2 + \delta_3}}\right]$$ if UDL is absent

A shaft of 40 mm diameter and 2.5 m long has a mass of 15 kg/m it is simply supported at the shaft and carrying 3 masses 90 kg, 140 kg and 60 kg at 0.8 m, 1.5 m and 2 m respectively from the left support. Take E = 200 GN/m². Find the frequency of Transverse vibration.



Given Data:

$d = 40\,mm = 0.04\,m$

$l = 2.5\,m$

$m_1 = 90\,kg$

$m_2 = 140\,kg$

$m_3 = 60\,kg$

$E = 20 \times 10^9\,N/m^2$

Solution:

$$\delta_1 = \frac{W_1 a^2 b^2}{3EIl}.$$

$$I = \frac{\pi}{64} \times d^4 = 1.2566 \times 10^{-7} \, m^4.$$

$$a = 0.8$$

$$b = (2.5 - 0.8) = 1.7m$$

$$\delta_1 = \frac{m_1 \times g \times 0.8^2 \times 1.7^2}{3 \times 200 \times 10^9 \times 1.2566 \times 10^{-7} \times 2.5}$$

$$= \frac{1271.376}{188495.5592}$$

$$\delta_1 = 6.7449 \times 10^{-3} \, m.$$

$$\delta_2 = \frac{W_2 a^2 b^2}{3EIl}. \quad \left[\begin{array}{l} a = 1.5 \\ b = 2.5 - 1.5 = 1 \end{array}\right]$$

$$= \frac{140 \times 9.81 \times 1^2 \times 1.5^2}{3 \times 200 \times 10^9 \times 1.2566 \times 10^{-7} \times 2.5}$$

$$= \frac{3090.15}{188.490}$$

$$\delta_3 = 16.39 \times 10^{-3} \, m$$

$$a = 2.$$

$$b = (2.5 - 2)$$

$$b = 0.5$$

$$\delta = \frac{588.6 \times 2^2 \times 0.5^2}{3 \times 200 \times 10^9 \times 1.2566 \times 10^{-7} \times 2.5}$$

$$= \frac{588.6}{188490},$$

$$\delta_3 = \cancel{3.1227 \times 10^{-3}\ m.}$$

$$\delta_s = \frac{5 \times w l^4}{324 \times IE}$$

$$= \frac{5 \times 15 \times 2.5^4}{324 \times 1.2566 \times 10^{-7} \times 200 \times 10^9}$$

$$\delta_s = 3.5996 \times 10^{4}\ m.$$

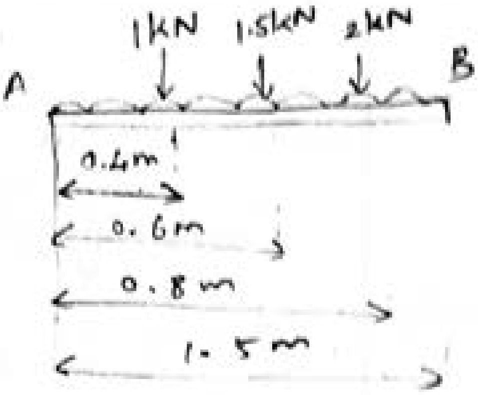
$$\delta_n = \frac{0.4985}{\sqrt{\delta_1 + \delta_2 + \delta_3 + \frac{\delta_s}{1.27}}}$$

$$= \frac{0.485}{\sqrt{8.6677 \times 10^{-3} + 16.40 \times 10^{-3} + 3.13 \times 10^{-3} + \frac{3.6 \times 10^{-4}}{1.27}}}$$

$$= 2.96\ Hz.$$

1. A shaf 30mm diameter and 1.5m long as a mass of 16 kg/m if in simply supported at the end and carrys three load 1 kN, 1.5 kN & 2kN at 0.4 m, 0.6 m & 0.8 m respectively from the left support. Find the frequeny of transverse vibration (i) mass of the shaft (ii) Consider mass of the shaft.

Given Data:



$d = 30\,mm = 0.03\,m$

$UDL = 16\,kg/m$.

$l = 1.5\,m$.

$W_1 = 1 \times 10^3\,N$

$W_2 = 1.5 \times 10^3\,N$

$W_3 = 2 \times 10^3\,N$.

To find $\delta_1, \delta_2, \delta_2$

$$\delta_1 = \frac{W_1 a^2 b^2}{3EIL}$$

$$I = \frac{\pi}{64} \times d^4 = \frac{\pi}{64} \times 0.03^4.$$

$$I = 3.97 \times 10^8$$

$$a = 0.4\,m$$

$$b = 1.5 - 0.4$$

$$b = 1.1\,m$$

$W_1 = m_1$

$= 1 \times 10^3$.

$$\delta_1 = \frac{1 \times 10^3 \times 0.4^2 \times 1.1^2}{3 \times 200 \times 10^9 \times 3.97 \times 10^{-8} \times 1.5}$$

$$= 5.41 \times 10^{-3}\,m$$

$$\delta_2 = \frac{1.5 \times 10^3 \times 0.6^2 \times 0.9^2}{3 \times 200 \times 10^9 \times 3.97 \times 10^{-8} \times 1.5}$$

$$= 12.22 \times 10^{-3}\,m$$

$$\delta_3 = \frac{2 \times 10^3 \times 0.8^2 \times 0.7^2}{3 \times 200 \times 10^9 \times 3.97 \times 10^{-8}}$$

$$= 17.527 \times 10^{-3}.$$

$$f_n = \frac{0.4985}{\sqrt{\delta_1 + \delta_2 + \delta_3}}.$$

$$= \frac{0.4985}{\sqrt{5.41\times10^{-3} + 12.22\times10^{-3} + 17.52\times10^{-3}}}$$

$$f_n = 2.65\,Hz.$$

$$\delta_s = \frac{5\times w l^4}{324 \times I E}$$

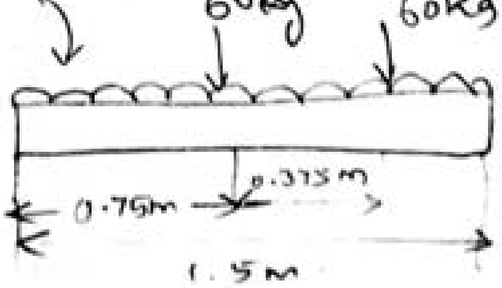
$$= 1.326\times10^{-4}\,m.$$

$$f_n = \frac{0.4985}{\sqrt{\delta_1 + \delta_2 + \delta_3 + \dots + \frac{\delta_s}{1.27}}}$$

$$= 2.62\,Hz.$$

A shaft 1.5m long supported in flexible bearing at the ends. carrying two wheels each of 60kg mass situated at the centre of the shaft and other at the distance of 375mm from the centre. The shaft is hollow external diameter 75 mm and inner diameter 40 mm. The density of the shaft material 7700kg/m³. Find the frequency of transverse vibration. Take $E = 200\times10^9 N/m^2$.

Given Data:



$l = 1.5m.$

$m_1 = 60kg$

$m_2 = 60kg$

$\rho = 7700\,kg/m^3.$

$D = 0.75\,m$ $d = 0.04\,m.$

Solution:

Since the density of the shaft material is given 7700 kg/m^3

The mass of the shaft, $m_s = \frac{\text{mass of the shaft}}{\text{length}}$

$= \text{Area} \times \text{length} \times \text{density}$

$= \frac{\pi}{4}(D^2 - d^2) \times l \times \rho$

$= \frac{\pi}{4}(0.075^2 - 0.04^2) \times 1.5 \times 7700$

$m_s = 36.51 \text{ kg/m}$

$I = \frac{\pi}{4} \times (D^4 - d^4)$

$= \frac{\pi}{64} \times (0.075^4 - 0.04^4)$

$I = 1.42 \times 10^{-6} \text{ m}^4$

$\delta_1 = \frac{W_1 a_1^2 b_1^2}{3EIl}$

$= \frac{60 \times 9.81 \times 0.75^2 \times 0.75^2}{3 \times 200 \times 10^9 \times 1.42 \times 10^{-6} \times 1.5}$

$= 1.456 \times 10^{-4} \text{ m}$

$\delta_2 = \frac{W_2 a_2^2 b_2^2}{3EIl}$

$= \frac{60 \times 9.81 \times 1.125^2 \times 0.375^2}{3 \times 200 \times 10^9 \times 1.42 \times 10^{-6} \times 1.5}$

$= 8.197 \times 10^{-5} \text{ m}$

$$\delta_s = \frac{5 \times wl^4}{345 \times IE}$$

$$= \frac{5 \times 36.5 \times (1.5)^4}{345 \times 1.42 \times 10^{-6} \times 200 \times 10^9}.$$

$$\delta_s = 9.43 \times 10^{-6} m.$$

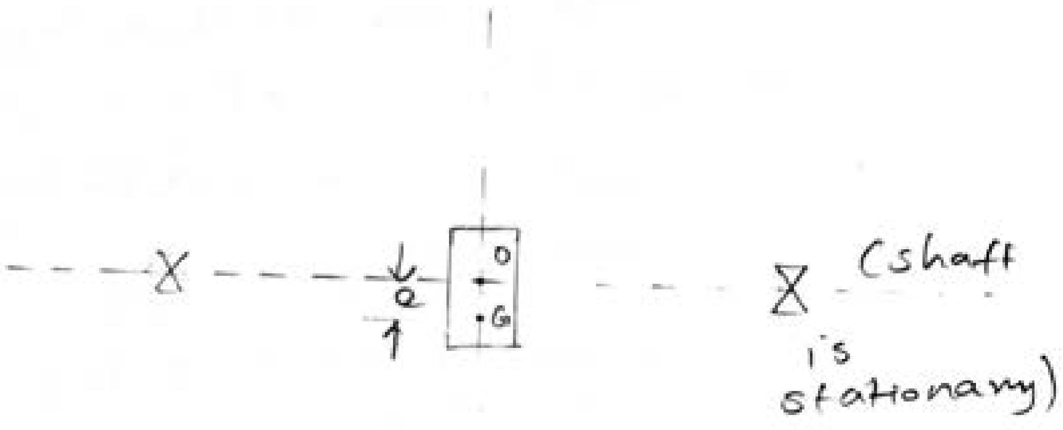
$$f_n = \frac{0.495}{\sqrt{1.45 \times 10^{-4} + 8.195 \times 10^{-5} m + \ldots + \frac{9.43 \times 10^{-6}}{1.27}}}$$

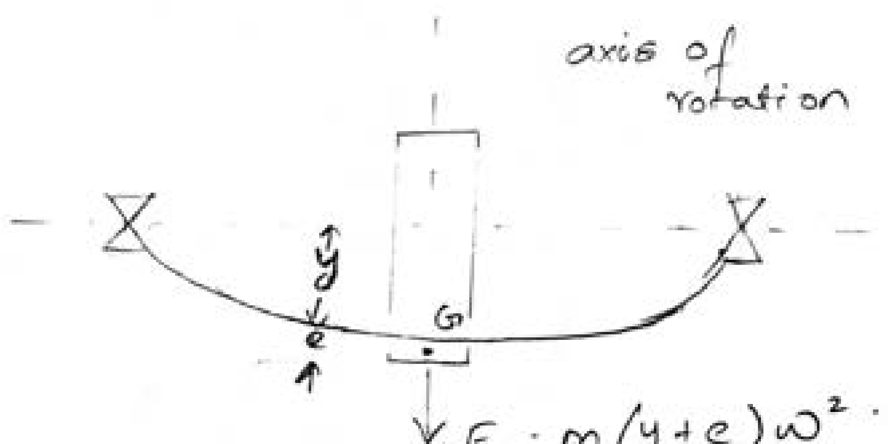
$$f_n = 32.56 Hz$$

Whirling of Shaft

Critical (or) whirling (or) whiping speed is the speed at which a shaft tends to vibrate violently in transverse vibration.

Derivation of whirling shaft.





To find y:

m = mass of rotor

S = stiffness of shaft

e = eccentricity of rotor

y - additional deflection of rotor due to centrifugal force.

ω - angular velocity of shaft.

Centrifugal force $= m(y+e)\omega^2$

restoring force $= Sy$

at equilibrium.

Centriforce force = restoring force.

$$m(y+e)\omega^2 = Sy.$$

$$my\omega^2 + me\omega^2 = Sy.$$

$$me\omega^2 = Sy - my\omega^2.$$

$$y = \frac{me\omega^2}{S - m\omega^2}.$$

$$y = \frac{\omega^2 e}{\frac{1}{m}(S - m\omega^2)}$$

$$y = \frac{\omega^2 e}{\frac{S}{m} - \omega^2}.$$

$$y = \frac{\omega^2 e}{\omega_n^2 - \omega^2}$$

$$y = \frac{e}{\left(\frac{\omega_n^2}{\omega^2} - 1\right)}$$

when $\omega = \omega_n$ the deflection y is infinitely large & resonance occurs

$$\omega_{cr} = \omega = \sqrt{S/m}$$

$$2\pi N_{cr} = \sqrt{S/m} \quad (\because \omega = 2\pi N)$$

1. The rotor has a mass of 12kg and its mounted midway on a 24mm dia horizontal shaft supported at the ends by two bearings. The bearing are 1m apart. shaft rotates at 2400 rpm. If the centre of mass of the rotor is 0.11mm away from the geometric centre of the rotor due to the manufacturing defect. Find the amplitude of the steady state vibration and dynamic force transmitted to the bearing.
Take $E = 200 \times 10^9 \ N/m^2$.

Given Data:

$$m = 12 kg$$
$$d = 0.024 m.$$
$$N = 2400 rpm$$
$$e = 0.11 mm$$
$$E = 200 \times 10^9 \ N/m^2.$$

Solution:

$$I = \frac{\pi}{64} \times d^4$$
$$I = 1.629 \times 10^{-8} \ m^4.$$

Supported at the end.

$$\delta = \frac{Wl^3}{48EI}$$
$$= \frac{12 \times 9.81 \times 1^3}{48 \times 200 \times 10^9 \times 1.629 \times 10^{-8}}$$
$$\delta = 7.528 \times 10^{-4} \ m$$

$$y = \frac{e}{\left(\frac{\omega_n^2}{\omega^2} - 1\right)}$$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 2400}{60} = 251.33\,r/s$$

$$\omega_n = \sqrt{\frac{g}{\delta}} = \sqrt{\frac{9.81}{7.528 \times 10^{-4}}}$$

$$\omega_n = 114.155\ rad/s$$

$$y = \frac{0.11 \times 10^{-3}}{\left(\frac{114.155^2}{251.33^2} - 1\right)}$$

$$y = -1.386 \times 10^{-4}\ m$$

$$\delta y = m \times \omega_n^2 \times y$$

$$= 12 \times 114.155 \times -1.386 \times 10^{-4}$$

$$\delta y = -21.6\ N$$

Load on each bearing.

$$= \frac{Mg}{2} + \frac{\delta y}{2}$$

$$= \frac{12 \times 9.81}{2} - \frac{21.6}{2}$$

$$= 48.06\ N.$$

2. In following data relatively of the shaft fixed held in long bearing the length of the shaft is 12m & the dia is 14 mm. The mass of rotor at mid position is 16kg. The eccentricity of centre of rotor is 0.4mm. Modulus of elasticity of shaft material E = 200 GPa. The permissible stress in shaft material is $70 \times 10^6 \, N/m^2$. Determine (i) the critical speed of the shaft. (ii) Range of speed over which it is unsafe to run the shaft. Assume shaft to the mass.

Given Data:

$$l = 1.2 \, m$$
$$d = 14 \, mm = 0.014 \, m$$
$$M = 16 \, kg$$
$$E = 200 \times 10^9 \, N/m^2$$
$$\sigma = 70 \times 10^6 \, N/m^2$$
$$e = 0.4 \, mm = 0.0004 \, m$$

Solution:

Shaft is held in bearing

$$I = \frac{\pi}{64} \times d^4 = \frac{\pi}{64} \times 0.014^4$$
$$= 1.88 \times 10^{-9} m$$

Now for fixed beam.

$$\delta = \frac{W l^3}{192 \times E \times I}$$
$$= \frac{16 \times 9.81 \times 1.2^3}{192 \times 200 \times 10^9 \times 1.88 \times 10^{-9}}$$
$$= 3.75 \times 10^{-3} \, m$$

$$f_n = \frac{1}{2\pi}\sqrt{g/\delta} = \frac{1}{2\pi}\sqrt{\frac{9.81}{3.75\times10^{-3}}}$$

$$f_n = 8.14\ Hz$$

$$N_{cr} = f_n \times 60 = 488.4$$

$Hz \longrightarrow rpm$

$$= 488.4\ rpm.$$

Range of Speed.

$$\frac{M}{I} = \frac{\sigma}{y}$$ where, M = bending moment of fixed beam.

due to the centre load, M = Nm

I = Moment of inertia

σ = permissible stress.

$y = d/2$ (distance of extreme layer from neutral axis)

M = load × distance.

$$M = \frac{W_1 l}{8}$$

$$\frac{W_1 l/8}{1.88\times10^{-9}} = \frac{70\times10^{9}}{0.014/2}$$

$$\frac{\frac{W_1\times1.2}{8}}{1.88\times10^{-9}} = \frac{70\times10^{6}}{\frac{0.014}{2}}$$

$$\frac{W\times1.2}{8} = 18.8$$

$$W_1 = 125.373\ Nm$$

Additional deflection due to the load.

$$y = \frac{W_1}{W} \times \delta = \frac{125.333}{m \times g} \times 3.75 \times 10^{-3}$$

$$y = 2.99 \times 10^{-3} m$$

Here, 'y' in the deflection of the shaft from the shaft axis which can be either positive or negative.

$$y = \frac{\pm e}{\left(\frac{\omega_{cr}}{\omega}\right) - 1} = \frac{\pm e}{\left(\frac{N_{cr}}{N}\right) - 1}$$

$$2.99 \times 10^{-3} = \frac{\pm 0.0004}{\left(\frac{488.4}{N}\right) - 1}$$

$$2.99 \times 10^{-3} = \frac{\pm 0.0004}{\frac{488.4 - N}{N}}$$

$$2.99 \times 10^{-3} \times N = \frac{\pm 0.0004}{488.4 - N}$$

$$2.99 \times 10^{-3} \times N (488.4 - N) = \pm 0.0004$$

$$1.45912 - 2.99 \times 10^{-3} N = -0.0004.$$

$$N = 480.77 \text{ rpm}$$

$$N = 566.73 \text{ rpm}.$$

Torsional Vibration:

$$\omega = \sqrt{q/I}$$

Time period $t_p = 2\pi/\omega = 2\pi\sqrt{I/q}.$

Natural frequency $f_n = \frac{1}{t_p} = \frac{1}{2\pi}\sqrt{q/R}.$

Torsional stiffness $q = \frac{CR}{l}.$

$C \rightarrow$ modulus of rigidity & $J \rightarrow \frac{\pi}{32} d^4$

$$\Rightarrow \delta_n = \frac{1}{2\pi}\sqrt{\frac{CJ}{Il}}$$

1. Determine the frequency of torsional vibration of disc as shown in fig if both the ends of the shaf are fixed and dia of the shaft 40mm and disc has a mass of 96 kg and radius of gyration of 0.4mm. Take modulus of rigidity $C = 84\ GN/m^2$.

Given data:

$$l_1 = 1m, \ l_2 = 0.8m, \ m = 96 kg$$

$$d = 40mm = 0.04 m, \ C = 84 \times 10^9 N/m^2$$

$$x = 0.0004 m.$$

Solution:

$$I = mk^2 \Rightarrow 96 \times 0.0004^2.$$

$$= 1.536 \times 10^{-5} kg.m^2$$

$$q = CJ/l$$

For two

$$q = \frac{CJ}{l_1} + \frac{CJ}{l_2}$$

$$J = \frac{\pi}{32} \times d^4 = \frac{\pi}{32} \times 0.04^4 = 2.68 \times 10^{-1}$$

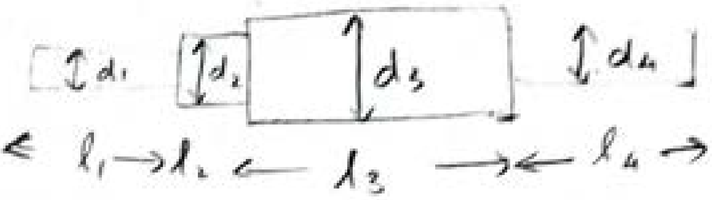
$$q = \frac{84 \times 10^9 \times 2.68 \times 10^{-6}}{1} + \frac{84 \times 10^6 \times 2.68 \times 10^{-1}}{0.8}$$

$$q = 47.79 \times 10^3 Nm$$

$$f_n = \frac{1}{2\pi}\sqrt{g/D}$$

$$= \frac{1}{2\pi}\sqrt{\frac{7.49\times10^{3}}{1.53\times10^{-5}}} = 8.85\,Hz$$

Torsional equivalent shaft:



let $d_1, d_2, d_3, d_4 \Rightarrow$ diameter of length l_1, l_2, l_3, l_4

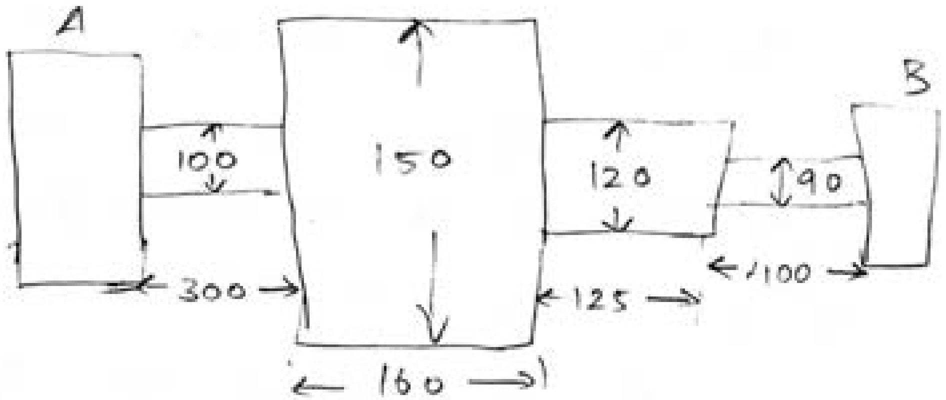
$\theta_1, \theta_2, \theta_3, \theta_4 \rightarrow$ Angle of twist for length. l_1, l_2, l_3, l_4

$J_1, J_2, J_3, J_4 \Rightarrow$ Polar moment of inertia of the shaft of diameter d_1, d_2, d_3, d_4

$$\theta_c = \theta = \theta_1 + \theta_2 + \theta_3 + \theta_4.$$

$$\frac{Tl}{CJ} = \frac{T l_1}{C}$$

A shaft shown in figure. carries two masses. The mass A is 300kg with radius of gyration 0.75m and mass B is 500kg with radius of gyration 0.9m. Determine the frequency of torsional vibration.
It is decide to have load at midpoin This section of the shaft of 120mm diameter by changing the dia of section having the 90mm dia. what will be the new diameter.



Take $G = 84 \times 10^9 \, N/m^2$ it is reduced to the equivalent torsiona shaft at dia of 100 m

Given data:

$d_1 = 100mm$ $d_2 = 150mm$ $d_3 = 120mm$
$l_1 = 300mm$, $l_2 = 160mm$, $l_3 = 125m$
$d_4 = 90mm$, $M_A = 500 \, kg$,
$M_B = 500 \, kg$, $l_4 = 400mm$
$G = 84 \times 10^9 \, N/m^2$, $k_2 = 0.9m$, $K_1 = 0.75m$

Solution:

$$l_e = l_1 + l_2\left(\frac{d_1}{d_2}\right)^4 + l_3\left(\frac{d_1}{d_3}\right)^4 + l_4\left(\frac{d_1}{d_4}\right)^4$$

$$= 300 + 160\left(\frac{100}{150}\right)^4 + 125\left(\frac{100}{120}\right)^4 + 400\left(\frac{100}{90}\right)^4$$

$$= 300 + \cancel{31.6049 + 60.2816}\ 31.6049 + 60.2816 + 609.6632$$

$$l_e = 1001.5497$$

WKT, For two rotor system

$$\boxed{I_A l_A = I_B l_B}$$

$$I_A = M_A k_A^2.$$

$$= 300 \times 0.75$$

$$I_A = 168.75\,\cancel{k}m^2.$$

$$I_B = M_B k_B^2$$

$$= 500 \times 0.9^2$$

$$I_B = 405\,m$$

$$168.75\, l_A = 405\, l_B$$

$$l_B = l_e - l_A$$

$$168.75\, l_A = 405\,(l_e - l_A)$$

$$168.75\, l_A = 405\,(1001.5 - l_A)$$

$$l_A = 0.7069\ m$$

$$l_B = 0.295\ m$$

Frequency at A

$$f_{n_A} = \frac{1}{2\pi}\sqrt{\frac{GJ}{I_A l_A}}$$

$$J = \frac{\pi}{32} \times d^4$$

$$= \frac{\pi}{32} \times 0.09^4.$$

$$J = 6.441 \times 10^{-6}$$

$$f_{n_A} = \frac{1}{2\pi}\sqrt{\frac{84\times10^9 \times 6.441\times10^{-6}}{168.75 \times 0.7067}}$$

$$f_{n_A} = 10.72 \text{ Hz}$$

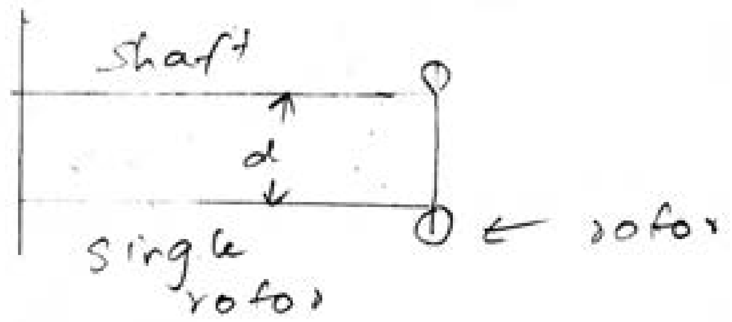
Frequency at B

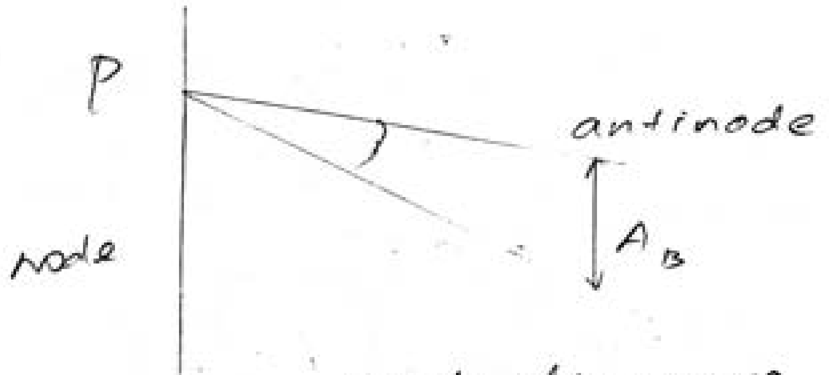
$$f_{n_B} = \frac{1}{2\pi}\sqrt{\frac{84\times10^9 \times 6.441\times10^{-6}}{405 \times 0.295}}$$

$$f_{n_B} = 10.71 \text{ Hz}.$$

Free torsional vibration of a single rotor system

A shaft fixed at one end and carrying a rotor on the free end is known as single rotor system.





node diagram

1. A shaft of 100 mm dia and 1m long is fixed at one end and the other end carries a fly wheel of mass 1 ton. The radius of gyration of fly wheel is 0.5m. Find the frequency of torsional vibration if the modulus of rigidity for the shaft material is 80 GN/m².

Given data:

$$d = 0.1\,m$$
$$l = 1\,m$$
$$m = 1000\,kg$$
$$k = 0.5\,m$$
$$C = 80 \times 10^9\ N/m^2$$

Solution:

$$f_n = \frac{1}{2\pi}\sqrt{\frac{CJ}{Il}}$$

$$J = \frac{\pi}{32} \times d^4$$

$$= \frac{\pi}{32} \times 0.1^4$$

$$= 9.817 \times 10^{-6}\ m^4$$

$$I = mk^2 = 1000 \times 0.5^2$$

$$= 250\ m$$

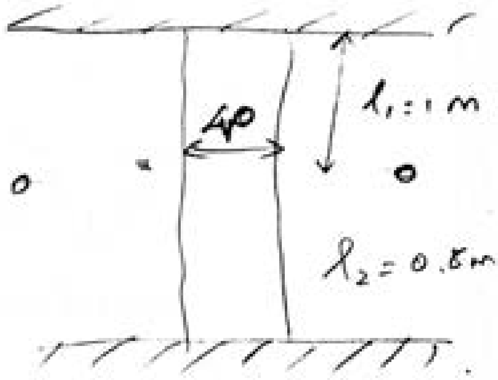
$$f_n = \frac{1}{2\pi}\sqrt{\frac{CJ}{Il}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{80 \times 10^9 \times 9.817 \times 10^{-6}}{250 \times 1}}$$

$$f_n = 8.92\ Hz$$

Determine the torsional frequency of disc of mass 96 kg which are in vertical rod of dia 40mm. The rod is 1.8m in length and the disc is fixed a 1m from the top both the ends of the rod are fixed. The radius of gyration of disc is 0.4 m. Take $G = 85$ GPa

Given data:



$d = 40$ mm
$= 0.04$ m
$m = 96$ kg
$l_1 = 1$ m
$l_2 = 0.8$ m
$K = 0.4$ m
C or $G = 85 \times 10^9$ N/m²

Solution:

$$I = mk^2$$
$$= 96 \times 0.4^2$$
$$= 15.36 \text{ kg/m}^2$$

$$J = \frac{\pi}{32} \times d^4 = 2.51 \times 10^{-7} \text{ m}^4$$

$$q_1 = \frac{CJ}{l} = \frac{85 \times 10^7 \times 2.51 \times 10^{-7}}{1}$$

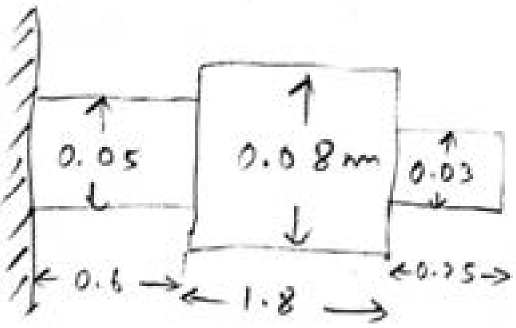
$$q_1 = 21335 \text{ Nm}$$

$$q_2 = \frac{CJ}{l_2} = \frac{85 \times 10^9 \times 2.51 \times 10^{-7}}{0.8}$$

$$q_2 = 26668.75 \text{ Nm}$$

$$q = q_1 + q_2 = 48003.75 \text{ Nm}$$

A stepped shaft is 0.05 m in diameter for the first 0.6 m length, 0.08 m dia for next 1.8 m and 0.03 m dia. for the remaining 0.25 m length while the 0.05 m dia end is fixed. The 0.03 m dia end of the shaft carries a rotor of mass moment of inertia of 14.7 kg m². If the modulus of rigidity of the shaft material is 0.83×10^{11} N/m² find the natural frequency of torsional vibration neglecting the inertia effect of the shaft.



Given data:

$$d_1 = 0.05\,m, \quad d_2 = 0.08\,m, \quad d_3 = 0.003\,m$$

$$l_1 = 0.6\,m, \quad l_2 = 1.8\,m, \quad l_3 = 0.25\,m.$$

$$I = 14.7\ kg\,m^2, \quad C = 0.83 \times 10^{11}\ N/m^2$$

The torsional equivalent shaft.

Assume its diameter 0.05 m.

$$l_e = l_1 + l_2\left(\frac{d_1}{d_2}\right)^4 + l_3\left(\frac{d_1}{d_3}\right)^4.$$

$= 0.6 +$

$l_e = 2.8036\ m.$

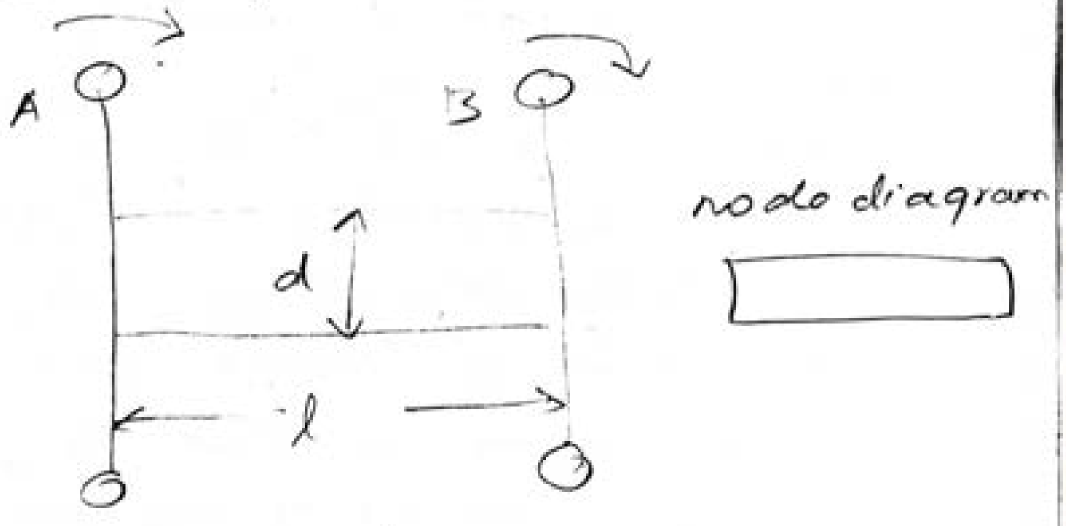
$J = \frac{\pi}{32} \times d^4 = \frac{\pi}{32} \times 0.05^4$

$= 6.1359 \times 10^{-7}\ m^4$

$f_n = \frac{1}{2\pi} \sqrt{\frac{CJ}{I\, l_e}}$

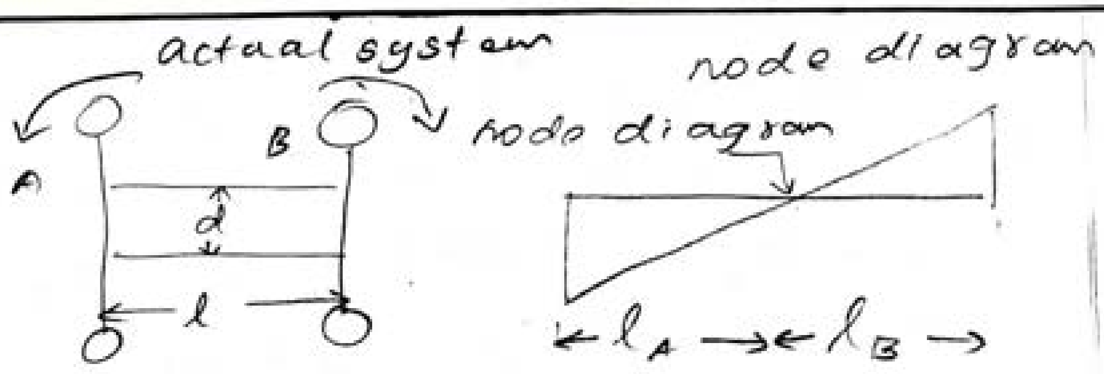
$f_n = 8.6\ Hz$

Free torsional vibration of a.
Two rotor system.

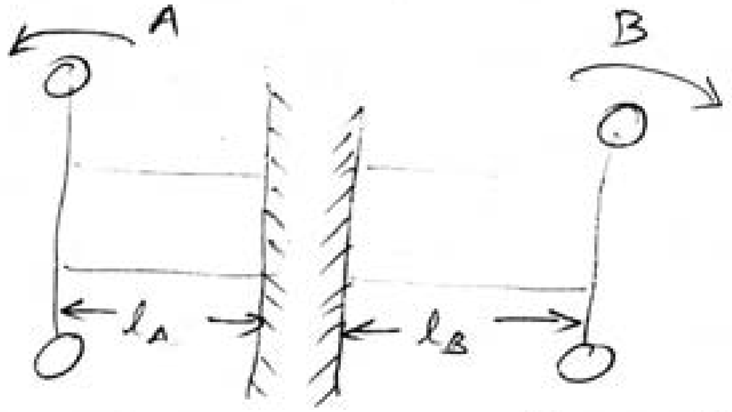


actual system

when rotor rotates same direction



When rotor rotates at different direction



equivalent two single rotor fixed at node

Location of Node

A node N, we can consider two seperate shaft AN and NB which are damped at N and each having a rotor at its end as shown in figure (3) in other words The two rotor system converted into two equivalent single rotor system. We know that natural frequency of torsional vibration of rotor

$$f_{n_A} = \frac{1}{2\pi}\sqrt{\frac{CJ}{I_A\, l_A}} \longrightarrow (1)$$

and neutral frequency of torsional vibration of rotor B.

$$f_{n_B} = \frac{1}{2\pi}\sqrt{\frac{CJ}{I_B l_B}} \longrightarrow (2)$$

Since nodepoint is common for both shafts therefore both the shafts should have same neutral circular frequency (ω_n) and hence natural frequency.

$$f_{n_A} = f_{n_B}.$$

$$(\text{or}) \quad \frac{1}{2\pi}\sqrt{\frac{CJ}{I_A l_A}} = \frac{1}{2\pi}\sqrt{\frac{CJ}{I_B l_B}}$$

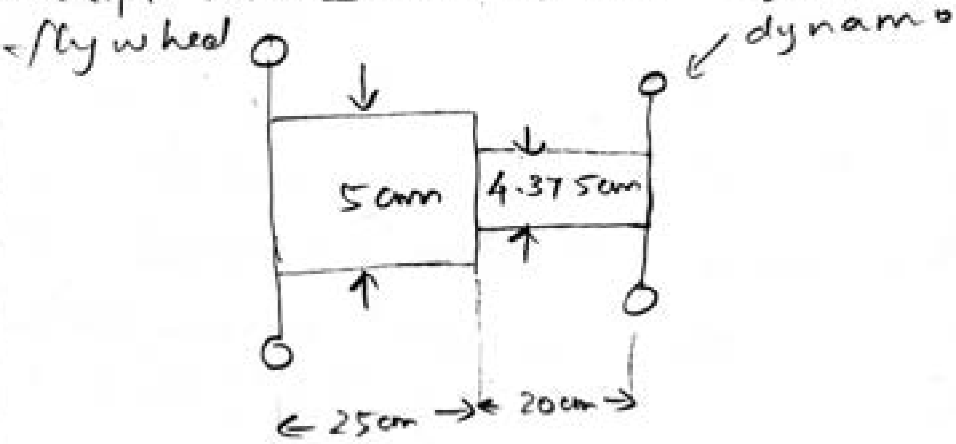
$$I_A l_A = I_B l_B$$

$$\text{WKT,} \quad l_A = \frac{I_B l_B}{I_A}$$

$$l_e = l_A + l_B$$

1. A Flywheel of an engine driving a dynamo has a mass of 200kg and has radius of gyration 30 cm. A shaft at the flywheel end has a effective length of 25 cm and 5.5 cm diameter. The armature mass is 225kg and has a radius of gyration 35.5 cm. The dynamo shaft has a dia 4.375 cm and equal length of 20 cm. Neglecting the inertia of the shaft and coupling. Calculate the frequency of the torsional vibration and position of

node . Take modulus of rigidity for shaft material is 80 GN/m².



Given Data:

$M_A = 200 kg$ $\quad$ $M_B = 225 kg$

$l_1 = 25 cm$
$= 0.25 m$

$l_2 = 20 cm$
$= 0.2 m.$

$d_1 = 5 cm$
$d_1 = 0.05 m$

$d_2 = 4.375 cm$
$d_2 = 0.04375 m$

$k_A = 30 cm$
$= 0.3 m$

$k_B = 25.5 cm$
$= 0.255 m$

$G = 80 \times 10^9 N/m^2.$

Solution:

$$l_e = l_1 + l_2 \left(\frac{d_1}{d_4}\right)^4$$

$$= 0.25 + 0.2 \left(\frac{0.05}{0.04375}\right)^4$$

$$l_e = 0.5912 \ m.$$

Location of node

$$I_A = m_A \times k_A^2.$$

$$= 200 \times 0.3^2.$$

$$= 18 \ kg m^2.$$

$$I_B = m_B \times k_B^2 = 225 \times 0.15$$

$$I_B = 14.63 \ kgm^2$$

WKT

$$I_A l_A = I_B l_B$$

$$18 l_A = 14.63 l_B$$

$$l_A = 0.8128 l_B$$

$$l_e = l_A + l_B$$

$$0.5912 = 0.8128 l_B + l_B$$

$$l_B = 0.3261 \ m$$

$$l_A = 0.2651 \ m$$

Hence the node lies at 0.2651m from flywheel (A) or 0.3261 from dynamo (B) on the equivalent shaft

$$f_{n_A} = \frac{1}{2\pi}\sqrt{\frac{CJ}{I_A l_A}}$$

$$J = \frac{\pi}{32} d^4 = \frac{\pi}{32} \times 0.05^4$$

$$J = 6.1359 \times 10^{-7} \ m^4$$

$C = G$

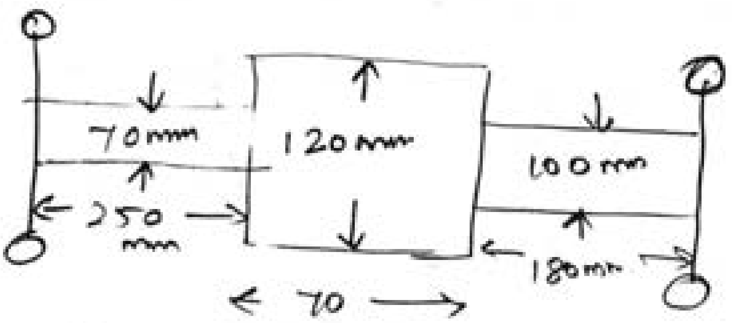
$$f_{n_A} = \frac{1}{2\pi}\sqrt{\frac{80 \times 10^9 \times 6.1359 \times 10^{-7}}{18 \times 0.2651}}$$

$$= 16.1422 \ Hz$$

for equivalent shaft

$$f_n = f_{n_A} = f_{n_B}$$

A Two rotor A and B are attache[d] to the two end of the shaft 500 mm lon[g]. The mass of the rotor A is 300 kg and its radius of gyration is 300 mm. The corresponding values of rotor B are 500 kg and 450 mm respectively. The shaft is 70 mm in dia for the first 250 mm. 120 mm for next 70 mm and 100 mm dia for the remaining length. The modulus of rigidity for the shaft material is 80 GN/m². Find the position of node, frequency of torsional vibratio[n]



Given Dada :

$m_A = 300\ kg$

$m_B = 500\ kg$

$k_A = 300\ mm$

$k_B = 450\ mm$

$d_1 = 0.07\ m$

$l_1 = 0.25\ m.$

$d_2 = 0.12\ m$

$l_2 = 0.07\ m$

$d_3 = 0.1\ m$

$l_3 = 0.18\ m.$

Solution:

$$I_A = m_A \times k_A^2$$

$$= 300 \times 0.3^2$$

$$I_A = 27 \text{ kg m}^2$$

$$l_e = l_1 + l_2 \left(\frac{d_1}{d_2}\right)^4 + l_3 \left(\frac{d_1}{d_3}\right)^4$$

$$= 0.25 + 0.07 \left(\frac{0.07}{0.12}\right)^4 + 0.18 \left(\frac{0.07}{0.1}\right)^4$$

$$= 0.25 + 0.0081 + 0.043218$$

$$l_e = 0.3013 \text{ m}$$

$$I_B = m_A k_B^2 = 500 \times 0.45^2.$$

$$= 131.25 \text{ kg m}^2.$$

WKT $\quad I_A l_A = I_B l_B.$

$$27 l_A = 101.25 l_B.$$

$$l_A = 3.75 l_B.$$

$$l_e = l_A + l_B$$

$$0.3013 = 3.75 l_B + l_B.$$

$$0.3013 = 4.75 l_B.$$

$$l_B = 0.0634 \text{ m}$$

$$l_A = 0.2379 \text{ m}$$

Hence the node lies at 0.2378 m from rotor (A) or 0.2379 m from rotor (B).

$$f_{n_A} = \frac{1}{2\pi}\sqrt{\frac{CJ}{I_A\, l_A}}$$

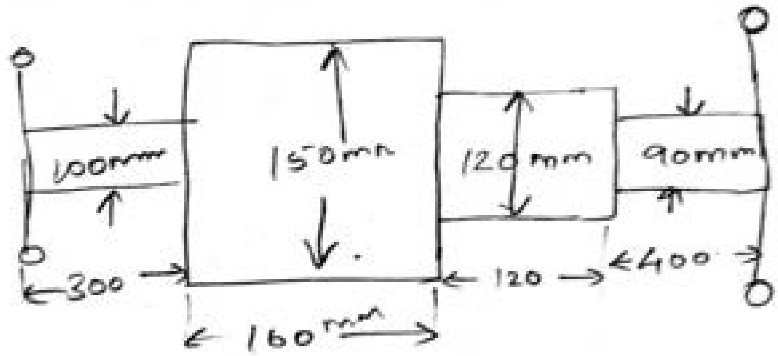
$$J = \frac{\pi}{32} \times d^4 = \frac{\pi}{32} \times 0.07^4$$

$$J = 2.3572 \times 10^{-6}\ m^4$$

$$f_{n_A} = \frac{1}{2\pi}\sqrt{\frac{80 \times 10^9 \times 2.3572 \times 10^{-6}}{27 \times 0.2379}}$$

$$f_{n_A} = 27.27\ Hz$$

A shaft shown in figure carries two masses the mass A is 350 kg with radius of gyration 0.8 m mass B is 550 kg with radius of gyration 0.95 m Determine the location of node and the natural frequency of free torsional vibration of the system. It is desired to have node at mid section of shaft of 120 mm diameter.



$$f_{n_A} = \frac{1}{2\pi}\sqrt{\frac{CJ}{I_A\, l_A}}$$

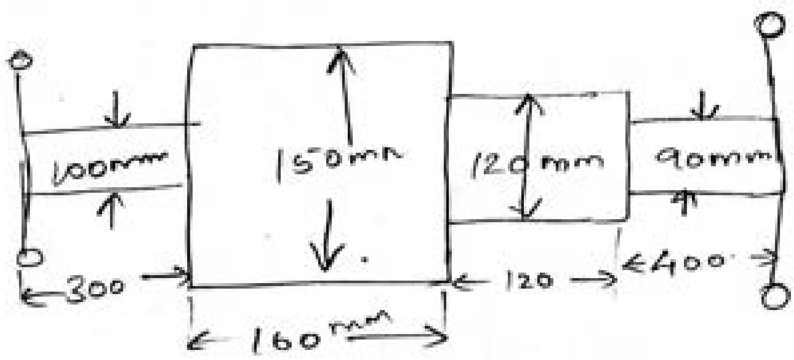
$$J = \frac{\pi}{32} \times d^4 = \frac{\pi}{32} \times 0.07^4$$

$$J = 2.3572 \times 10^{-6}\ m^4$$

$$f_{n_A} = \frac{1}{2\pi}\sqrt{\frac{80 \times 10^9 \times 2.3572 \times 10^{-6}}{27 \times 0.2379}}$$

$$f_{n_A} = 27.27\ Hz$$

A shaft shown in figure carries two masses the mass A is 350 kg with radius of gyration 0.8 m mass B is 550 kg with radius of gyration 0.95 m Determine the location of node and the natural frequency of free torsional vibration of the system. It is desired to have node at mid section of shaft of 120 mm diameter.



Given Data:

$M_A = 350 kg$ $d_1 = 0.1 m$ $l_1 = 0.3 m$

$k_A = 0.8 m$ $d_2 = 0.15 m$ $l_2 = 0.16 m$

$M_B = 550 kg$ $d_3 = 0.12 m$ $l_3 = 0.12 m$

$k_B = 0.95 m$ $d_4 = 0.09 m$ $l_4 = 0.4 m$

Solution:

$$l_e = l_1 + l_2\left(\frac{d_1}{d_2}\right)^4 + l_3\left(\frac{d_1}{d_3}\right)^4 + l_4\left(\frac{d_1}{d_4}\right)^4$$

$$= 0.3 + 0.16\left(\frac{0.1}{0.15}\right)^4 + 0.12\left(\frac{0.1}{0.12}\right)^4 + 0.4\left(\frac{0.1}{0.09}\right)^4$$

$$= 0.3 + 0.0316 + 0.0579 + 0.6097$$

$$l_e = 0.9992 m$$

$$I_A = m_A \times k_A^2 = 350 \times 0.8^2$$

$$= 224 kg m^2$$

$$I_B = 550 kg \times 0.95 m^2 = 496.375 kg m^2$$

WKT

$$I_A l_A = I_B l_B$$

$$224 l_A = 496.375 l_B$$

$$l_A = 2.2061 l_B$$

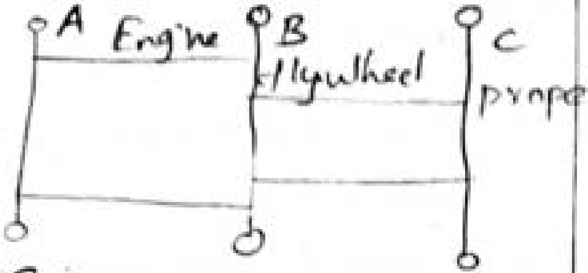
$$0.9992 l_e = 2.2061 l_B + l_B$$

$$l_B = 0.3116 m$$

$$l_A = 0.6875 m$$

Hence the node lies between 0.69005m from rotor A and 0.3114 from rotor B.

$$f_{n_A} = \frac{1}{2\pi}\sqrt{\frac{80\times10^9 \times 120}{224 \times 0.6875}} = 16.36 Hz.$$



Given:

$I_A = 800 kg m^2$

$I_B = 320 kg m^2$

$I_c = 20 kg m^2$

$d_1 = 5 cm = 0.05 m$

$d_2 = 2.5 cm = 0.025 m$

$l_1 = 200 cm = 2m$

$l_2 = 200 cm = 2m$

$l_e = l_1 + l_2 \left(\frac{d_1}{d_2}\right)^4$

$l_e = 34 m.$

WKT,

location of node

$I_A l_A = I_c l_c$

$800 l_A = 20 l_c$

$l_A = \frac{20 l_c}{800}$

$l_A = 0.025 l_c$

$$\frac{1}{I_c l_c} = \frac{1}{I_B}\left[\frac{1}{l_1 - l_A} + \frac{1}{l_2 - l_B}\right]$$

$$\frac{1}{20 l_c} = \frac{1}{320}\left[\frac{1}{2-0.025 l_c} + \frac{1}{2-l_c}\right]$$

$$\frac{1}{20 l_c} = \frac{1}{320}\left[\frac{(2-l_c)+(2-0.025 l_c)}{(2-0.02 l_c)(2-l_c)}\right]$$

$$\frac{1}{20 l_c} = \frac{1}{320}\left[\frac{4-1.025 l_c}{4-2l_c - 0.05 l_c + 0.025 l_c^2}\right]$$

$$320 = 20 l_c\left[\frac{4 l_c - 1.025 l_c}{4-2.05 l_c + 0.025 l_c^2}\right]$$

$320[4-2.05 l_c + 0.025 l_c^2]$

$= 20 l_c [4 (-1.025 l_c)]$

$1280 - 656 l_c + 8 l_c^2$

$= 80 l_c - 20.5 l_c^2$

$1280 - 736 l_c + 28.5 l_c^2 = 0$

$28.5 l_c^2 - 736 l_c = -1280$

$- 28.5 l_c^2 + 736 l_c = 1280$

$\div \quad -28.5$

$l_c^2 - 25.824 l_c = 44.912$

$l_c = 23.94 \ m$

$l_c = 1.875 \ m.$

$l_c = 23.94, \ l_A = 0.5985$

$l_c = 1.875 m, \ l_A = 0.0467 m$

Take $l_c = 23.94.$

$l_A = 0.5985$ this gives the position of single mode.

Take $k = 1.875, \ l_A = 0.0467 m$

thus given the position of double mode.

$$F_{n_c} = \frac{1}{2\pi}\sqrt{\frac{CJ}{I_c l_c}}$$

$J = \frac{\pi}{32} \times 0.05^4$

$J = 6.13 \times 10^{-7} m^4$

$$f_{n_{c_1}} = \frac{1}{2\pi}\sqrt{\frac{80\times10^{9}\times6.13\times10^{-7}}{800\times0.5985}}$$

$$F_n = 1.61\,Hz$$

$$f_{n_{c_2}} = \frac{1}{2\pi}\sqrt{\frac{80\times10^{9}\times6.13\times10^{-7}}{\times1.875}}$$

1. Different types of vibration
   (i) free vibration
   (ii) forced vibration
   (iii) damped vibration.

2. Hunting

Unit - IV.

Forced Vibration

When the body vibrates under the influence of external force the the body is set to be forced vibration

Forced vibration with constant harmonic excitation

Amplitude (Max. displacement)

$$x_{max} = \frac{F_0 / S}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}$$

r = frequency ratio $= \frac{\omega}{\omega_n}$

$$\phi = \tan^{-1}\left(\frac{2\xi r}{1-r^2}\right)$$

$$\omega = \omega_n = \sqrt{S/m} \text{ rad/s}$$

Amplitude of vibration

$$x_{max} = \frac{F_0}{2\xi S} \text{ (at resonance only)}$$

1. A mass of 50 kg is supported by a elastic structure of portal stiffness 20 kN/m The damping ratio of system is 0.2. The simple harmonic disturbing force acts on mass at any time t second the force face $F = 60 \sin 10 t$ N. Find the amplitude of the vibration and face angle caused by the damping.

Given Data:

$m = 50$ kg , $S = 20 \times 10^3$ N/m

$\varepsilon = 0.2$

$F = 60 \sin 10t \, N$

$\therefore F_0 = 60 \quad , \omega = 10$

Solution:

$$\omega_n = \sqrt{S/m} = \sqrt{\frac{20 \times 10^3}{50}}$$

$$\omega_n = 20 \text{ rad/s.}$$

$$r = 10/20 = 0.5 m.$$

$$x_{max} = \frac{F_0/s}{\sqrt{(1-r^2)^2 - (2\varepsilon r)^2}}$$

$$= \frac{60/20 \times 10^3}{\sqrt{(1-0.5^2)^2 - (2 \times 0.2 \times 0.5)^2}}$$

$$x_{max} = 4.07 \times 10^{-3} \, m.$$

$$x_{max} = 4.073 \text{ mm.}$$

$$\phi = \tan^{-1}\left(\frac{2\varepsilon r}{1-r^2}\right).$$

$$= \tan^{-1}\left(\frac{2 \times 0.2 \times 0.5}{1 - 0.5^2}\right).$$

$$\phi = 14.931.$$

A mass of 10kg is suspended from one end of a helical spring and other end is fixed. The stiffness of the spring is N/m the viscous damping causes the amplite to decreases to $\frac{1}{10}$th of initial value. In four complete oscillation. Its periodic force $F = 150 \cos 50t$ N is supplied at mass is vertical section. is the value of resonance and amplitude of forced vibration.

Given Data:

$$m = 10 kg$$

$$S = 10 \ N/mm = 10 \times 10^3 \ N/m$$

$$x_A = \frac{1}{10} \times x_0$$

$$\frac{x_0}{x_A} = 10$$

$$F = 150 \cos 50t \ N$$

$$F_0 = 150$$

$$\omega = 50$$

Solution:

$$r = \frac{\omega}{\omega_n}$$

$$\omega_n = \sqrt{\frac{S}{m}} = \sqrt{\frac{10^4}{10}} = 31.62 \ rad/s$$

$$r = \frac{50}{31.62} = 1.58$$

$$x_{max} = \frac{F_0/S}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}$$

$$S = \frac{1}{n} \ln \left[\frac{x_0}{x_n}\right]$$

$$S = \frac{1}{4} \ln [10] = 0.5756 \text{ m}.$$

$$\delta = \frac{2\pi \varepsilon}{\sqrt{1-\varepsilon^2}} = 0.091$$

$$x_{max} = \frac{150/10^4}{\sqrt{(1-1.58^2)^2 + (2\times 0.091 \times 1.58)^2}}$$

$$= 9.8216 \times 10^{-3} \text{ m}$$

$$x_{max} = 9.8216 \text{ mm}$$

Resonance $x_{max} = \frac{F_0}{2\varepsilon S} = \frac{150}{2\times 0.091 \times 10^4}$

$$x_{max} = 0.0824 \text{ m}$$

$$x_{max} = 82.4 \text{ mm}$$

A harmonic exciting force of 25N is acting of a machine part which is having a mass of 2kg and it vibrating in a viscous medium. This exciting force cause a resonance amplitude of 12.5mm with a period of 0.20 seconds Determine the damping coefficient (cc)

Given Data:

$F_0 = 25\text{ N}$

$m = 2\text{ kg}$

$R x_{max} = 12.5\text{ mm}$
$= 12.5\times 10^{-3}\text{ m}$

$t_p = 0.02\text{ s}$

Solution

$\omega_n = \frac{2\pi}{t_p} = \frac{2\pi}{0.02}$

$= 31.41 \text{ rad/s}.$

$\omega_n = \sqrt{\frac{S}{m}}$

$31.41 = \sqrt{\frac{S}{2}}$

$S = 1973.17 \text{ N/m}$

Then,

$$R_{x_{max}} = \frac{F_0}{2 \xi S}$$

$$0.0125 = \frac{25}{2 \times 1973.92 \times \xi}$$

$$\frac{1}{\xi} = 1.9739$$

$$\xi = 0.5066$$

$$C_c = 2 m \omega_n$$

$$= 2 \times 2 \times 31.41$$

$$C_c = 125.64 \ N/s/m$$

$$\frac{C}{C_c} = \xi$$

$$\frac{C}{125.64} = 0.5066$$

$$C = 63.6492 \ N/s/m$$

In the above problem if the system is excited from the harmonic force are frequently 4 Hz. Find the increased in amplitude for forced vibration when the damper is removed.

Given data:

$$f_n = 4 \ Hz$$

$$r = \frac{\omega}{\omega_n}$$

$$\omega_n = 31.41$$

$$\omega = 2 \pi f_n$$

$$= 2 \pi \times 4$$

$$\omega = 25.13 \ rad/s$$

$$r = \frac{25.13}{31.41} = 0.8 \ m$$

$$x_{max} = \frac{F_0 / s}{\sqrt{(1-r^2)^2 + (2 \xi r)^2}}$$

$$= 0.0142 \ m.$$

$$x_{max} = 14.28 \ mm$$

Amplitude without damper

$$x_{max} = \frac{F_0 / s}{(1 - r^2)}$$

$$= 0.0351 \ m$$

Increase in amplitude

$$= 0.0351 - 0.0142$$

$$= 0.02082 \ M$$

$$= 20.82 \ mm$$

4. (10.8) The body having a mass of 15kg is supended from a spring which deflects 12mm under the weight of the mass. Determine the frequency of the free vibration what is the viscous damping force needed to make the ~~periodic~~ motion a periodic at a speed of 1mm/s? If, when damped to this extent, a disturbing force having a maximum value of 100N and vibrating at 6Hz is made to act on the body, determine the amplitude of the ultimate motion.

Given Data:

$m = 15\,kg$, $\delta = 12\,mm = 0.012\,m$

$F_0 = 100N$, $f = 6Hz$

Solution:

(i) Frequency of free vibration (f_n).

$$f_n = \frac{1}{2\pi}\sqrt{g/\delta}$$

$$= \frac{1}{2\pi}\sqrt{\frac{9.81}{0.012}}$$

$$= 4.55Hz.$$

ii) When damping force needed to make the motion periodic at the speed of 1m/s.

WKT, Motion becomes periodic when the damped frequency ($\varepsilon = 1$)

$$C = ?$$

$$C = C_c$$

$$C_c = 2m\omega_n$$

$$\omega_n = \sqrt{S/m}$$

$$= \sqrt{g/\delta}$$

$$= 28.59\,m/s.$$

$$C_c = 2 \times 15 \times 28.59$$

$$= 857.7\,m/s.$$

Thus the force needed in point 0.857N at the speed of 1m/s.

(iii) $$x_{max} = \frac{F_0/S}{\sqrt{(1-r^2)^2 - (2\varepsilon r)^2}}$$

$$r = \frac{\omega}{\omega_n}$$

$$\omega = 2\pi f = 2\pi 6$$

$$\omega = 37.69$$

$$r = \frac{37.69}{28.59}$$

$$r = 1.3186$$

$$S = \frac{mg}{\delta} = \frac{15 \times 9.81}{0.012}$$

$$= 1226.25\,N/m$$

$$x_{max} = \frac{100/1226.5}{\sqrt{(1-1.3186^2)^2 (2 \times 1 \times 1.3186)^2}}$$

$$= 2.98 \times 10^{-3}\,m$$

The mass of 500 kg is mounted on support having a total stiffness of 100 kN/m which provides viscous damping. damping ratio is 0.4. Mass is constrained vertically and Subjected to vertical force of type $F_0 \sin \omega t$. Determine the frequency at which resonance will occur and maximum allowable value of F_0. If the amplitude at the resonance is to be restricted to 5 mm.

<u>G.D</u>

$$m = 500\,kg$$
$$S = 100 \times 10^3\,N/m$$
$$\varepsilon = 0.4$$
$$R_{x\,max} = 0.005\,m$$

<u>Solution</u>

$$\omega = \omega_n = \sqrt{S/m} = \sqrt{\frac{10^5}{500}}$$
$$= 14.14\ rad/s$$
$$\omega = 2\pi f$$

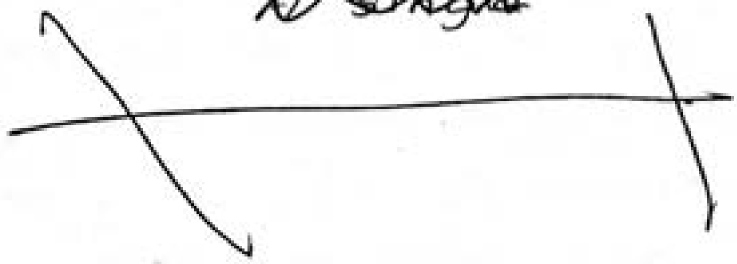
$(f = f_n)$
$$f = \frac{\omega}{2\pi} = \frac{14.14}{2\pi}$$
$$f = 2.25\ Hz$$

Maximum value of F_0.

$$x_{max} = \frac{F_0}{2\varepsilon S}$$
$$0.005 = \frac{F_0}{2 \times 0.4 \times 10^5}$$
$$F_0 = 400\,N$$

Forcing Caused by Unbalance

~~Drangle~~



$$\frac{x_{max}}{\left(\frac{m_u \cdot e}{m}\right)} = \frac{r^2}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}$$

where

m_u = unbalanced mass.

e = eccentricity.

m = vibratic mass.

1. A single cylinder vertical petrol engine of total mass of 200 kg is mounted upon a steel chasis frame. The vertical static deflection of the frame is 2.4 mm. Due to the weight of the engine. The mass of the reciprocating parts is 18 kg and the stroke of the piston is 160 mm with SHM. If dashpot of damping coefficient of 1 N/mm/s is used to dampen the vibration. Calculated the steady state. (i) The amplitude of the forced vibration at 500 rpm (ii) Speed of the driving shaft at which resonance occur.

G.D

$m = 200\,kg$

$\delta = 2.4\,mm$

$= 0.0024\,m$

$M_u = 18\,kg$

$N = 500\,rpm$

$C = 1\,N/mm/s.$

$C = 1000\,N/m/s.$

$l = 0.16\,m$

Solution:

$$\omega = \frac{2\pi N}{600} = \frac{2\pi \times 500}{600}$$

$$= 52.36 \text{ rad/s}$$

$$r = \frac{\omega}{\omega_n} \text{ where}$$

$$\omega_n = \sqrt{s/m} = \sqrt{g/\delta}$$

$$= \sqrt{\frac{200}{0.0024}}$$

$$\omega_n = 63.93 \text{ rad/s}$$

$$r = \frac{52.36}{63.93} = 0.819$$

Damping ratio

$$\mathcal{E} = \frac{c}{c_c}$$

$$\mathcal{E} = \frac{c}{2m\omega_n} =$$

$$= \frac{1000}{2 \times 200 \times 63.93}$$

$$= 0.039$$

$$e = \frac{L}{2} = \frac{0.16}{2}.$$

$$e = 0.08.$$

$$\frac{x_{max}}{\left(\frac{18 \times 0.08}{200}\right)} = \frac{0.819^2}{\sqrt{(1-0.819^2)^2 + (2 \times 0.819 \times 0.039)^2}}$$

$$\frac{x_{max}}{1.125} = 2$$

$$x_{max} = 14.4 \text{ mm}$$

The speed of driving shaft at which resonance will occur

$$\omega = \frac{2\pi N}{60} \quad \boxed{\omega = \omega_n}$$

$$N = \frac{\omega \times 60}{2\pi}$$

$$= \frac{63.93 \times 60}{2\pi}$$

$$N = 610.5 \text{ rpm}.$$

2. A vertical single stage air compressor having mass of 500 kg is mounted on spring, stiffness of 1.96×10^5 N/m and dashpots with a damping factor of 0.2m. The rotating parts are completely balanced and equivalent reciprocating parts weighs 20 kg. The stroke is 0.2 m. Determine the dynamic amplitude of vertical motion and phase difference between the motion and excitation force if the compressor is operated at 200 rpm.

G.D

$m = 500 kg$

$S = 1.96 \times 10^5 N/m$

$\mathcal{E} = 0.2 m$

$m_u = 20 kg$.

Stroke, $L = 0.2 m$

$N = 200$ rpm.

Solution :

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 200}{60}$$

$$\omega = 20.94 \text{ rad/s}.$$

$$\omega_n = \sqrt{S/m} = \sqrt{\frac{1.96 \times 10^5}{500}}$$

$$\omega_n = 19.8 \text{ rad/s}.$$

$$r = \frac{\omega}{\omega_n} = \frac{20.9}{19.8} = 1.057.$$

$$\mathcal{E} = \frac{c}{2 m \omega_n}$$

$$e = \frac{L}{2} = \frac{0.2}{2}$$

$$e = 0.1 m.$$

$$\frac{x_{max}}{\left(\frac{20 \times 0.1}{500}\right)} = \frac{1.057^2}{\sqrt{(1-1.057^2)^2 + (2 \times 0.2 \times 1.057)^2}}$$

$$\frac{x_{max}}{0.004} = 2.546.$$

$$x_{max} = 0.01018 m$$

$$x_{max} = 10.18 mm.$$

$$\phi = \tan^{-1}\left(\frac{2\mathcal{E}r}{1-r^2}\right) = \tan^{-1}\left(\frac{2 \times 0.2 \times 1.057}{1 - 1.057^2}\right)$$

$$\phi = -74.5$$

$$\phi = -74.5 + 180 = 105.499^\circ.$$

Forced Vibration due to excitation of the Support (Support motion)

$$\frac{x_{max}}{y} = \frac{\sqrt{1 + (2\mathcal{E}r)^2}}{\sqrt{(1-r^2)^2 + (2\mathcal{E}r)^2}}.$$

$$\phi = \tan^{-1}\left(\frac{2\mathcal{E}r}{1-r^2}\right).$$

5 The support of a spring mass system is vibrating with a amplitude of 6mm. and frequency of 20 Hz. If the mass is 1.1kg and the spring has a stiffness of 2000 N/m. Determine the amplitude of vibration of the mass. what amplitude will result if a damping factor of 0.25 is included in the system

G.D

$Y = 0.006\,m$

$f = 20\,Hz$

$m = 1.1\,kg$

$S = 2000\,N/m$

$\mathcal{E} = 0.25.$

Solution.

$r = \frac{\omega}{\omega_n} = \frac{2\pi f}{2\pi\omega}.$

$W = 2\pi f = 2\pi \times 20$

$= 125.6637\,rad/s$

$\omega_n = \sqrt{S/m}$

$= \sqrt{\frac{2000}{1.1}}$

$\omega_n = 42.64\,rad/s$

$r = \frac{125.66}{42.64}$

$r = 2.9469$

① Amplitud of Vibration

$\mathcal{E} = 0$

$\frac{x_{max}}{y} = \frac{\sqrt{1 + (2\mathcal{E}r)^2}}{\sqrt{(1-r^2)^2 + (2\mathcal{E}r)^2}}.$

$x_{max} = \frac{1 \times 0,006}{\sqrt{(1 - 2.9469^2)^2}}$

$x_{max} = 7.807 \times 10^{-4}\,m.$

(ii) Amplitude of Vibration.

$\mathcal{E} = 0.5$

$\frac{x_{max}}{y} = \frac{\sqrt{1 + (2\mathcal{E}r)^2}}{\sqrt{(1-r^2)^2 + (2\mathcal{E}r)^2}}.$

$\frac{x_{max}}{0.006} = \frac{\sqrt{1+(2 \times 0.25 \times 2.9)^2}}{\sqrt{(1-125.66)^2+(2 \times 0.25 \times 2.9)^2}}$

$x_{max} = 1.367 \times 10^{-3}\,m$

Amplitude vibration $\mathcal{E} = 0.25$

$\frac{x_{max}}{Y} = \frac{\sqrt{1 + (2\mathcal{E}r)^2}}{\sqrt{(1-r^2)^2 + (2\mathcal{E}r)^2}}$

$\frac{x_{max}}{0.006} = \frac{\sqrt{1+(2 \times 0.25 \times 2.946)^2}}{\sqrt{(1-2.946^2)^2 + (2 \times 0.25 \times 2.946)}}$

$\frac{x_{max}}{0.006} = \frac{\sqrt{1+2.169}}{\sqrt{58.96+2.169}}$

$\frac{x_{max}}{0.006} = 0.227$

$x_{max} = 1.365 \times 10^{-3}\,m$

A vehicle has a mass of 490 kg and the total spring constant of its suspension system, $S = 58800$ N/m. Profile of the road may be approximated to a sine wave of amplitude 40 mm and wave length 4 m. Determine the critical speed of the vehicle and amplitude of steady state motion when vehicle is driven at the critical speed, amplitude of steady state motion when vehicle is driven at 57 km/hr.

Given Data:

$m = 490$ kg

$S = 58,800$ N/m

$Y = 40$ mm

$\lambda = 4$ m.

$\varepsilon = 0.5$

Solution:

$$\omega_n = \sqrt{S/m} = \sqrt{\frac{58800}{490}}$$

$\omega_n = 10.95$ rad/s.

$$f_n = \frac{\omega_n}{2\pi} = \frac{10.95}{2\pi}$$

$f_n = 1.743$ Hz.

① Critical speed.

$$f_n = \frac{V_{cr}}{\lambda}$$

$$V_{cr} = f_n \times \lambda = 4 \times 1.743$$

$V_{cr} = 6.972.$

$$V_{cr} = \frac{6.972 \times 3600}{1000}$$

$= 25.10$ km/hr.

② Amplitude of steady state motion

$$\frac{x_{max}}{Y} = \frac{\sqrt{1 + (2\varepsilon r)^2}}{\sqrt{(1-r^2)^2 + (2\varepsilon r)^2}}$$

Take $r = 1$ (or) $\omega = \omega_n$

$= 0.056$ m

$$V = \frac{57 \times 10^3}{3600} = 15.8 \text{ m}$$

$$f_n = \frac{V}{\lambda} = \frac{15.8}{4}$$

$= 3.95$ Hz

Amplitude of steady state motion when the vehicle is driven at 57 km/hr

$V = 15.83\ m/s$.

$$f_n = \frac{V}{\lambda}$$

$$\frac{\omega}{2\pi} = \frac{15.83}{4}$$

$$\omega = 24.86\ rad/s.$$

Frequency ratio, $r = \frac{\omega}{\omega_n} = \frac{24.86}{10.95}$

$= 2.271$

The amplitude of the forced vibration is given by

$$\frac{x_{max}}{0.04} = \frac{\sqrt{1+(2\times0.5\times22.271)^2}}{\sqrt{(1-2.271^2)^2+(2\times0.5\times2.271)^2}}$$

$$x_{max} = 0.2095\ m$$

ISOLATION:

VIBRATION OSCILLATION:

The Process of reducing the vibration of machine and hence reducing the force which is transmitted to the foundation using vibration isolation

The material usually used for vibration isolation are rubberpad, belt, canvas, spring etc..

Un balanced exiting fore

$$F_0 = m_u \times \omega^2 \times e.$$

$$x_{max} = \frac{F_0/S}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}$$

$$\xi = \frac{F_T}{F_0} = \frac{\sqrt{1 + (2\xi r)^2}}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}$$

$$\phi = \tan^{-1}\left(\frac{2\xi r}{1-r^2}\right).$$

A machine has a 100 kg and 20 kg rotor with 0.5 mm eccentricity. The mounting spring have 85×10^3 Nm The damping ratio is 0.02. The operatin[g] system is 600 rpm and the unit is constrained to move vertically. Find dynamic amplitude of the machine and the force transmitted to the shaft.

Given Data:

$m = 100$ kg

$m_u = 20$ kg

$e = 0.5$ mm

$\xi = 0.02$

$N = 600$ rpm.

$S = 85 \times 10^3$ N/m

Solution:

$$\omega = \frac{2\pi N}{60}$$

$$= 62.83 \text{ rad/s}.$$

$$\omega_n = \sqrt{S/m}$$

$$= \sqrt{\frac{85 \times 10^3}{100}}$$

$$\omega_n = 29.15 \text{ rad/s}$$

$$r = \frac{\omega}{\omega_n} = \frac{62.83}{29.15}$$

$$= 2.155 \text{ m}$$

$$F_0 = m_u \times \omega^2 \times e$$

$$= 20 \times 62.83^2 \times 0.0005$$

$$= 39.47.$$

Maximum amplitude

$$x_{max} = \frac{F_0/S}{\sqrt{(1-r^2)^2+(2\xi r)^2}}$$

$x_{max} = 1.274 \times 10^{-4} m$

$$\frac{F_T}{F_0} = \frac{\sqrt{1+(2\xi r)^2}}{\sqrt{(1-r^2)^2+(2\xi r)^2}}$$

$F_T = 10.87 N$

2. A mass of machine is 75 kg is mounted on spring of stiffness of 12×10^5 N/m and with an assumed damping factor 0.2 and piston within a machine of mass 2 kg has a reciprocating motion with a stroke of 80 mm. and speed of 3000 cycle per /minute (rpm). Assume the motion to be SHM (i) The amplitude motion of machine (ii) phase angle with respect to the exciting force. (iii) force transmitted to the vibration. (iv) Phase angle of transmitted force with respect to exciting force.

Given Data:

$m = 75 kg$

$S = 12 \times 10^5 N/m$

$\xi = 0.2$

$m_u = 2 kg$

$L = 80 mm$

$= 80 \times 10^{-3} m$

Solution:

$e = 0.04 m \; (L/2)$

$$\omega = \frac{2\pi N}{60}$$

$= 314.16$ rad/s

$$\omega_n = \sqrt{S/m}$$

$= 126.49$ rad/s

$$r = \frac{\omega}{\omega_n} = \frac{314.16}{126.49} = 2.48$$

$F_0 = m_u \times \omega^2 \times e$

$= 7895.72N$

$x_{max} = \dfrac{F_0/s}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}$

$= 1.25 \times 10^{-3} m$

$\dfrac{F_T}{F_0} = \dfrac{\sqrt{1+(2\xi r)^2}}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}$

$F_t = 2120.405N$

$\phi = \tan^{-1}\left(\dfrac{2\xi r}{1-r^2}\right)$

$\phi = 169.09$

$\alpha = \tan^{-1}(2\xi r)$

$\alpha = 44.7698$

$\therefore \phi - \alpha = 124.32°$

3. A single cylinder engine has a out of balance of 500 N at the engine speed of 300 rpm. Total mass of the engine is 150 kg and It carries a set of spring of total stiffness 300 N/cm. Find the amplitude of steady state motion of the mass and the maximum oscillating force transmitted to the foundation (ii) If the viscous damping interupted between mass and foundation The damping force being 1000 N at 1 m/s. find amplitude of the forced damped oscillation of mass of angle of lag with

disturbing force?

Given data:

$F_0 = 500\ N$

$N = 300\ rpm$

$m = 150\ kg$

$S = 300\ N/cm$

$= 300 \times 10^2\ N/m$.

$\omega_n = \sqrt{\frac{300 \times 10^2}{150}}$

$= 14.14\ rad/s$.

$r = \frac{\omega}{\omega_n} = \frac{\frac{2 \times \pi \times 300}{60}}{14.14}$

$r = 2.22$.

Amplitude of the steady state motion

$\varepsilon = 0$

$x_{max} = \frac{F_0/s}{\sqrt{(1-r^2)^2 + (2\varepsilon r)^2}}$

$= \frac{500/300 \times 10^2}{\sqrt{(1-2.22^2)^2 + (2\varepsilon r)^2}}$

$x_{max} = 4.2357 \times 10^{-3}$.

$\frac{F_T}{F_0} = \frac{\sqrt{1+(2\varepsilon r)^2}}{\sqrt{(1-r^2)^2 + (2\varepsilon r)^2}}$.

$F_T = 127.271\ N$

When there is a damper.

$C = 1000\ N/m/s$.

$C_C = 2m \times \omega_n$

$= 2 \times 150 \times 14.14$

$C_C = 4242.6\ m$

$\varepsilon = 0.235$

$x_{max} = \frac{F_0/s}{\sqrt{(1-r^2)^2 + (2\varepsilon r)^2}}$

$= \frac{500/300 \times 10^2}{\sqrt{(1-2.22^2)^2 + (2 \times 0.235 \times 2.22)^2}}$

$= 4.10 \times 10^{-3}\ m$

$\phi = \tan^{-1}\left(\frac{2\varepsilon r}{1-r^2}\right)$.

$= -14.87 + 180$

$\phi = 165.13\ rd$

$\frac{F_T}{F_0} = \frac{\sqrt{1+(2\varepsilon r)^2}}{\sqrt{(1-r^2)^2 + (2\varepsilon r)^2}}$

$= 176.94\ N$

A machine of mass 75 kg is mounted on spring of stiffness 12×10^5 N/m and with an assumed damping factor of 0.2. A piston within the machine of mass 2 kg has a reciprocating motion with a stroke of 80 mm and a speed of 3000 cycle/min. Assume the motion to be SHM. Find (i) amplitude of motion of the machine (ii) phase angle with respect to the exciting force (iii) The force transmitted to the foundation. (iv) The phase angle of transmitted force with respect to the exciting force (v) The phase lag of the transmitted force.

Given Data:

$m = 75\,kg$

$s = 12\times10^5\ N/m.$

$\xi = 0.2.$

$m_u = 2\,kg$

Stroke $= 0.08\ m.$

$N = 3000\ rpm$

Solution

$\omega = \frac{2\pi N}{60}$

$\omega = 314.16\ rad/s$

$\omega_n = \sqrt{\frac{s}{m}}$

$= \sqrt{\frac{12\times10^5}{75}}$

$\omega_n = 126.49\ rad/s.$

$r = \frac{\omega}{\omega_n} = \frac{314.16}{126.49}$

$r = 2.48$

$e = \frac{L}{2} = \frac{0.08}{2} = 0.04$

(i) Amplitude of motion of the machine.

$$x_{max} = \frac{F_0/s}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}$$

$F_0 = m_u \times \omega^2 \times e.$

$= 2\times 314.16^2 \times 0.04$

$F_0 = 7895.72\ N$

$$x_{max} = \frac{7895.72/12\times10^5}{\sqrt{(1-2.48^2)^2 + (2\times2.48\times0.2)^2}}$$

$x_{max} = 1.25\times10^{-3}\ m.$

(ii) phase angle with respect to exciting force.

$$\phi = \tan^{-1}\left(\frac{2\xi r}{1-r^2}\right)$$

$= \tan^{-1}\left(\frac{2\times0.2\times2.48}{1-2.48^2}\right)$

$\phi = -10.90 + 180.$

$\phi = 169.1^\circ$

$\phi = 2.95$ radians.

(iv) phase angle

$\alpha = \tan^{-1}(2\varepsilon r)$

$= \tan^{-1}(2\times0.2\times2.48)$

$= 44.81$

(iii) Force transmitted to the foundation

$$\frac{F_T}{F_0} = \frac{\sqrt{1+(2\varepsilon r)^2}}{\sqrt{(1-r^2)+(2\varepsilon r)^2}}$$

$$\frac{F_T}{7895.72} = \frac{\sqrt{1+(2\times0.2\times2.48)^2}}{\sqrt{(1-2.48)^2+(2\times0.2\times2.48)^2}}$$

$F_T = 2114.62\ N$

$\simeq 2114.62\ N$

(v) phase lag

$\phi - \alpha = 169.1 - 44.81$

$= 124.31^\circ$

An industrial machine weighing 445 kg is supported on a spring with a static deflection of 0.5 cm. If the machine has rotating imbalance (e) of 25 kg cm. Determine the force transmitted at 1200 rpm, and the dynamic amplitude at that speed.

Given Data:

$m = 445\ kg$

$\delta = 0.5\ cm$

$= 0.5\times10^{-2}\ m.$

$e = 25\times10^{-2}\ kg\,m.$

$N = 1200\ rpm.$

Solution:

$\omega = \frac{2\pi N}{60}$

$= 125.66\ rad/s.$

$$\omega_n = \sqrt{s/m} = \sqrt{g/\delta}$$

$$= \sqrt{\frac{9.81}{0.5 \times 10^{-2}}}$$

$$= 44.29 \text{ rad/s}.$$

$$r = \frac{\omega}{\omega_n} = \frac{125.66}{44.29}$$

$$= 2.84$$

$$F_0 = m_u \times \omega^2 \times e$$

$$= (125.66)^2 \times 0.25$$

$$F_0 = 3947.60 \text{ N}.$$

WKT, force transmitted, when there is no damper $\varepsilon = 0$

$$\frac{F_T}{F_0} = \frac{\sqrt{1 + (2\varepsilon r)^2}}{\sqrt{(1-r^2)^2 + (2\varepsilon r)^2}}$$

$$\frac{F_T}{3947.6} = \frac{\sqrt{1}}{\sqrt{(1-2.84^2)^2}}$$

$$F_T = 558.71 \text{ N}$$

Dynamic amplitude at 1200 rpm when there is no damper

$$x_{max} = \frac{F_0/s}{(1-r^2)}$$

when $r > 1$

$$\omega_n = \sqrt{s/m}.$$

~~s = 8r~~

$$44.71 = \sqrt{\frac{s}{445}}$$

$$s = 8.73 \times 10^5 \text{ N/m}$$

$$x_{max} = \frac{\frac{3947.6}{8.73 \times 10^5}}{(1 - 2.84^2)}$$

$$x_m = -6.4 \times 10^{-4} \text{ m}$$

The mass of an electric motor is 120 kg and it runs at 1200 rpm. The armature mass is 35 kg and its C.G is lies 0.5 mm for the axis of rotation the motor is mounted on five spring of negligible $\varepsilon=0$ damping. So that the.

Given data:

$m = 10\,kg$

(7) $S = 10\,N/mm = 10^4\,N/m$

$x_4 = \frac{x_0}{10}$ $n = 4$.

$\frac{x_0}{x_4} = 10$

$F = 150 \cos 50t\ N$

Solution:

$\omega_n = \sqrt{S/m}$

$= \sqrt{\frac{10^4}{10}}$

$\omega_n = 31.62\ rad/s$

$\delta = \frac{1}{n} \ln \left[\frac{x_0}{x_4}\right]$

$= \frac{1}{4} \ln [\cancel{\ }10]$

$\delta = 0.5756$

$\delta = \frac{2\pi \varepsilon}{\sqrt{1-\varepsilon^2}}$.

$0.5756 = \frac{2\pi \varepsilon}{\sqrt{1-\varepsilon^2}}$.

$\frac{\sqrt{1-\varepsilon^2}}{\varepsilon} = \frac{2\pi}{0.5756}$

$\frac{1-\varepsilon^2}{\varepsilon^2} = 119.137$

$\varepsilon_1^2 = 120.137\varepsilon^2$

$\varepsilon = 0.0912$

$F = 150 \cos 50t$

$\boxed{F = F_0 \cos \omega t}$.

$x_{max} = \frac{F_0/S}{\sqrt{(1-r^2)^2 + (2\varepsilon r)^2}}$.

$r = \frac{\omega}{\omega_n} = \frac{50}{31.62}$

$r = 1.58$.

$x_{max} = \frac{150/10^4}{\sqrt{(1-1.58^2)^2 + (2\times0.0912\times1.58)}}$

$= \frac{0.015}{\sqrt{(\quad)}}$

$x_{max} = 9.84 \times 10^{-3} m$

At resonace.

$x_{max} = \frac{F_0}{2\varepsilon \cdot S}$

$= \frac{150}{2\times0.091\times10^4}$

$x_{max} = 0.0821\ m$

x ———— x

Dynamics of Machines

Deals with the study of various force acting on various machine elements. The forces may be static or kinematic

Unit - V.

Gyroscope:

Whenever a rotating body changes the direction (axis of rotation). A couple is formed on the rotating body. This couple is known a gyroscopic couple.

Application:

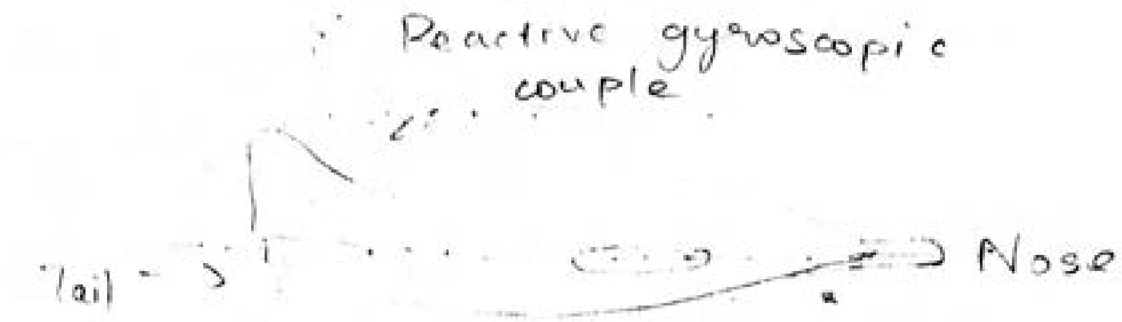
Aeroplane, ~~naval ship~~ and Missiles, space to sense the angular motion of the body

Navalship, it is used as navigation system

It is used in two wheeler and four wheeler moving in carved path.

Aeroplane

Effect of Gyroscopic couple:



1\. An Aeroplane a complete halfcircle 60 m radius to the left when flying at 200km/hr. The rotatary engine and the propeller of plane weighs 4000N with a radius of gyration 30cm And the engine runs at 2250rpm clockwise. when view from the rear Find the gyroscopic couple and state its effect.

Solution:

Radius, $R = 60\,m$.

velocity, $v = \dfrac{200 \times 1000}{3600} = 55.55\,m/s$

weight, $W = 4000\,N$

radius of gyration, $k = 30\,cm = 0.3\,m$.

$N = 2250\,rpm$.

Solution:

$$C = I\omega\omega$$

$$I = mk^2$$

$$m = \frac{W}{g} = \frac{4000}{9.81}$$

$$= 407.75 \text{ kg}$$

$$I = 407.75 \times 0.3^2$$

$$= 36.7 \text{ kgm}^2$$

$$\omega = \frac{2\pi N}{60} = \frac{2250 \times 2 \times \pi}{60}$$

$$= 235.62 \text{ rad/s}$$

$$\omega_p = \frac{V}{R} = \frac{55.55}{60}$$

$$= 0.926 \text{ rad/s}$$

$$C = I\omega\omega_p$$

$$= 36.7 \times 235.62 \times 0.926$$

$$C = 8005.9 \text{ Nm}$$

Gyroscopic effect:

Nose will raise and tail will dip.

~~When aeroplane takes left when seen from the rear and~~

2\. The rotator of a turbo jet engine has a mass of 210kg and radius of gyration 250 mm. The engine rotates at a speed of 9500 rpm in the clockwise direction. If viewed from the front of the aeroplane. The aeroplane while flying a 975 km/hr turns with a radius of 2.25 km to the right. Find the gyroscopic couple and gyroscopic effect.

Given Data:

$$m = 210 kg$$

$$k = 250 mm$$

$$N = 9500 rpm$$

$$V = \frac{975 \times 1000}{3600} \text{ m/s}$$

$$= 270.8 \text{ m/s}$$

$$R = 2.25 km$$

Solution:

$$= 2250 m.$$

$$I = mk^2$$

$$= 210 \times 25^2$$

$$= 13.125 \text{ kg m}^2$$

$$\omega = \frac{2\pi N}{60} = \frac{2 \times \pi \times 9500}{60}$$

$$\omega = 994.8 \text{ m/s}$$

$$\omega_p = \frac{v}{R} = \frac{270.8}{2250}$$

$$\omega_p = 0.12 \text{ Rad/s}$$

$$\therefore C = I \omega \omega_p$$

$$= 13.125 \times 994.84 \times 0.12$$

$$= 1572. \text{ Nm}$$

Gyroscopic Effect: when the plane take righ turn Depress the tail and nose raises.

Gyroscopic Effect of Naval Ship

Types of movement

1. Steering
2. Pitching
3. Rolling

1. Steering:

It is the turing of a ship in curved either to the right or the left

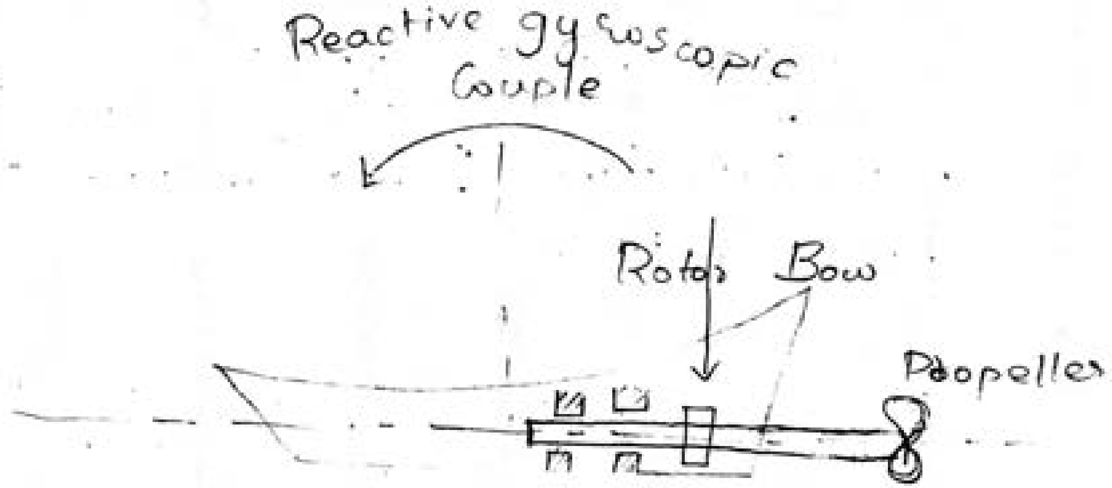
2. Pitching

The up and down movement of the ship is pitching

3. Rolling

The sideway movement of the ship along the longitudinal axis is called rolling.

Effect of Gyroscopic Couple



Effect of gyroscopic couple

Gyroscopic effects.

Problem on steering:

Gyroscopic effect chart for ship during steering:

S.No	View point	Direction of rotor rotation	Turn	Effect
1.	Stern	Clockwise	Left	Bow raised. Stern depressed.
2.	Stern	Clockwise	Right	Bow depressed Stern raised
3.	Stern	Anticlockwise.	Left.	Bow depressed Stern raised
4.	Stern	Anti clockwise	Right	Bow raised Stern depressed.
5.	Bow	Anti clockwise	left	Bow raised Stern depressed
6.	Bow	Anti Clockwise	Right	Bow depressed Stern raised
7.	Bow	Clockwise	left	Bow depressed Stern raised
8.	Bow	Clockwise	Right	Bow raised Stern depressed

A ship sails at a speed of 125 km/hr The mass of its turbine rotor is 600 kg having a radius of gyration 0.6 m. It rotates at 1600 rpm in a clockwise direction when looking from its stern. When the ship stern to the left with a radius of curvature of 110 m. What is the gyroscopic couple and the effect.

Given Data:

$$V = 125 \text{ km/hr}$$

$$= \frac{125 \times 1000}{3600}$$

$$= 34.72 \text{ m/s}$$

$$m = 600 \text{ kg}$$

$$k = 0.6 \text{ m}$$

$$N = 1600 \text{ rpm}$$

$$R = 110 \text{ m}$$

Solution:

$$C = I \omega \omega_P$$

$$I = mk^2$$

$$= 600 \times (0.6)^2$$

$$= 216 \text{ kgm}^2$$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 600}{60}$$

$$= 167.5 \, m/s.$$

$$\omega_p = \frac{v}{R} = \frac{34.72}{110}$$

$$= 0.316 \, m/s$$

$$C = I \omega \omega_p$$

$$= 216 \times 167.55 \times 0.316$$

$$= 11436.3 \, Nm.$$

Gyrosopic Effect:

When the rotates in clockwise direction looking from stern. The gyroscopic effect is raise the bow lower the stern.

2. A high speed ship is driven by a turbine rotor which has a inertia of $20 kg m^2$ and is running at 3000 rpm in clockwise direction when viewed from the bow. The ship is moving with the velocity of 72 km/hr. ~~vel~~ taking a right turn a curve of 600 m radius.
Determine the gyroscopic couple applied to the ship and its effect.

Given Data:

$$I = 20 \ kg m^2$$

$$N = 3000 \ rpm$$

$$V = 72 \ km/hr = \frac{72 \times 1000}{3600}$$

$$= 20 \ m/s$$

$$\omega = \frac{2\pi N}{60} = \frac{2 \times \pi \times 3000}{60}$$

$$= 314.16 \ rad/s$$

$$R = 600 \ m$$

Solution:

$$\omega_p = V/R = \frac{20}{600}$$

$$= 0.033 \ r/s$$

$$C = I \omega \omega_p$$

$$= 20 \times 314.16 \times 0.033$$

$$= 209.44 \ Nm$$

∴ when the rotor rotated in clockwise direction viewed from the bow. and the ship steers to the right. The effect of gyroscopic is raise the bow and lower the stern.

3. Each pedal wheel of a steamer have a mass of 1600kg and of a radius of gyration of 1.2 m. The steamer turns to port in a circle of 160 m radius at 24 km/hr. The speed of the pedal is 90 rpm. Find the magnitude and the effect of gyroscopic couple acting on the steamer

Given Data:

$$m = 1600 \text{ kg}$$
$$k = 1.2 \text{ m}$$
$$R = 160 \text{ m}$$
$$V = 24 \text{ km/hr} = 6.67 \text{ m/s}$$
$$N = 90 \text{ rpm}$$

Solution:

$$I = mk^2$$
$$= 1600 \times 1.2^2$$
$$= 2304 \text{ kg m}^2.$$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 90}{60}$$

$$= 9.42 \text{ rad/s}.$$

$$\omega_p = \frac{v}{R} = \frac{6.66}{160}$$

$$= 0.42$$

$$C = I \omega \omega_p$$

$$= 2304 \times 9.42 \times 0.42$$

$$C = 903.45 \text{ Nm}$$

Gyroscopic effect:

In this problem, the direction of viewing and the sense of rotation of the rotor are not specified. Only the turn i.e., towards the port is given. So the gyroscopic effect on ship during steering are summerised as

Viewpoint	Sense of rotor Rotation	Turn	Effect
Stern	Clockwise	left	Bow raised stern depressed
Stern	Anticlockwise	left	Bow depressed stern raised
Bow	Clockwise	Right	Bow raised stern depressed
Bow	Anticlockwise	Right	Bow depressed stern raised.

Problem on Pitching of Ship

1. The rotor of a turbine installed in a boat with its axis longitudinal axis of the boat makes 1500 rpm clockwise when viewed from the stern. The rotor has a mass of 750 kg and a radius of rotation of 300 mm. If at an instant the boat pitches in the longitudinal vertical plane so that bow raises from the horizontal plane with an angular velocity of 1 rad/s. Determine the torque acting in the boat and the direction in which it tends to turn the boat at the instant.

Given Data:

$$N = 1500 rpm$$

$$m = 750 kg$$

$$k = 300 mm$$

$$k = 0.3 m.$$

$$\omega_p = 1 rad/s.$$

Solution:

$$C = I \omega \omega_p$$

$$I = mk^2$$

$$= 750 \times 0.3^2.$$

$$I = 67.5 kgm^2.$$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 1500}{600}$$

$$\omega = 157.1 rad/s$$

$$C = 67.5 \times 157.1 \times 1$$

$$= 10604.25 Nm$$

when the rotor rotates in clockwise direction looking from stern and the ship pitches upward and the bow raised is to turn the boat star board side.

2. A ship is pitching through a total angle of 15°. The oscillation may be taken as simple harmonic and the complete period is 32 second. The turbine rotor weighs 6 ton. Its radius of gyration is 45 cm and it is rotating at 2400rpm. Calculate the maximum value of gyroscopic couple. ~~set up by the~~ stepup by the rotor and its effect when the bow is ~~descending~~ descending and the rotor is rotating clockwise looking from AFT. What is the maximum angular acceleration to which the ship is subjected while pitching

Given Data:

$$2\phi = 15°$$

$$\phi = 7.5°$$

$$\phi = 0.13 \text{ radians}$$

$$M = 6000 \text{ kg}$$

$$t_p = 32 \text{ s}$$

$$k = 0.45 \text{ m}$$

$$N = 2400 \text{ rpm}.$$

Solution:

Maximum Gyroscopic caple

$$C_{max} = I \cdot \omega \cdot \omega_{P_{max}}.$$

$$I = mk^2.$$

$$= 6000 \times 0.45^2.$$

$$I = 1215 \ kgm^2$$

$$\omega = \frac{2\pi N}{60} = \frac{2 \times \pi \times 2400}{60}$$

$$= 251.32 \ rad/s$$

$$\omega_{P_{max}} = \phi \times \omega_0$$

Ang velocity of SHM, $\omega_0 = \frac{2\pi}{t_p} = \frac{2\pi}{32} = 0.196 \ rad/s$

$$\omega_P = 0.13 \times 0.196 = 0.0255 \ rad/s$$

$$C_{max} = I \omega \omega_{P_{max}}$$

$$= 1215 \times 251.32 \times 0.025$$

$$C_{max} = 7786.5 \ Nm$$

Gyroscopic effect

when the bow is desending and the rotor is rotating clockise. looking from AFT is to turn the ship toward port side maximum angular acceleration $\alpha_{max} = \phi \times \omega_0^2$.

$$= 0.13 \times 0.196^2.$$

$$= 5.012 \times 10^{-3} \ rad/s^2$$

3. Marine turbine rotor of inertia 700 kg m² rotates at 2800rpm clockwise when viewed from left if ship pitches with angular SHM with a period of 6s and amplitude of 0.2rad. Find (i) Maximum angular velocity of rotor (ii) Maximum gyroscopic couple and gyroscopic effect as the bow dips.

Given Data:

$I = 700 \, kg m^2$.

$N = 2800$ rpm.

$t_p = 6$ s

$\phi = 0.2$ rads.

Solution:

(i) $\omega = \frac{2\pi N}{60} = \frac{2\pi \times 2800}{60}$

$= 293.22$ rad/s.

(ii) $\omega_{p\,max} = \phi \times \omega_0$

$\omega_0 = \frac{2\pi}{t_p} = \frac{2\pi}{6} = 1.047$ r/s

$\omega_{p\,max} = 0.2 \times 1.042$

$= 0.209$ rad/s.

$C_{max} = I . \omega . \omega_{p\,max}$

$= 42988.29$ Nm

When the rotor rotate clockwise viewing from the stern and the bow dips is to turn the ship towards the port side

A. The turbine rotor of ship has a mass of 20 tons and radius of gyration of 0.75 m. Its speed is 2000 rpm. The ship pitches 6° above and below the horizontal position. One complete oscillation takes 18 seconds and the motion is SHM. Determine C_{max}, α_{max} and gyroscopic effect:

Given Data:

$$m = 20000\ kg$$

$$k = 0.75\ m$$

$$N = 200\ rpm$$

$$\phi = 6^\circ$$

$$\phi = 0.1047\ radian$$

$$t_p = 18\ seconds$$

Solution:

$$I = m \times k^2$$

$$= 20000 \times 0.75^2$$

$$I = 11250\ kg\,m^2.$$

$$\omega = \frac{2\pi \times 2000}{60} = 209.44\ rad/s$$

$$\omega_0 = \frac{2\pi}{t_p} = \frac{2\pi}{18} = 0.34\ rad/s$$

$$\omega_p = 0.1047 \times 0.349$$

$$= 0.0367\ rad/s$$

$$C = I\,\omega\,\omega_p$$

$$= 11250 \times 209.44 \times 0.0367$$

$$C = \cancel{[illegible]}\ 84819.15 \text{ Nm}$$

Gyroscopic effect:

When the rotor rotates with clockwise direction looking from the rear and the ship pitching upward i.e., star board side.

Problem on Steering, Pitching and Rolling

1. A turbine rotor of a ship has ~~2.4~~ 2.2 ton and rotates at ~~1750~~ 1800 rpm clockwise when viewed from the AFT. The radius of gyration of rotor is ~~300~~ 320 mm. Determine gyroscopic couple and its effect. when

(i) The ship turns at an radius of 250m with a speed ~~22~~ 25 km/hr

(ii) The ship pitches with the bow rising at an angular velocity of ~~0.85~~ 0.8 radians/sec.

(iii) Ship rolls at an angular velocity of ~~0.15~~ 0.1 rad/s

Given Data:

$$M = 2.4 \text{ tonnes} = 2400 \, kg$$

$$N = 1750 \, rpm$$

$$k = 300 \, mm = 0.3 \, m$$

$$V = 22 \times \frac{1000}{3600} = 6.11 \, m/s.$$

$$R = 250 \, m.$$

Solution:

(i) The ship turns at an radius of 250 m and sp. velocity 22 km/hr.

$\frac{22 \times 1000}{3600}$

$$I = m \times k^2.$$

$$= 2400 \times 0.3^2.$$

$$I = 216 \, kgm^2$$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 1750}{60}$$

$$\omega = 183.26 \, rad/s.$$

$$\omega_p = \frac{v}{R} = \frac{6.11}{250} = 0.024 \, rad/s$$

$$C = I \omega \omega_p.$$

$$= 216 \times 183.26 \times 0.024$$

$$\boxed{C = 967.61 \, Nm.}$$

when the rotor rotates in clockwise direction viewed from stern end in the ship turns right i.e., lower the bow and rise the stern.

(ii) Pitches with bow rising at an angular velocity of 0.85 rad/s.

$$C = I \omega \omega_p.$$

$$\omega_p = 0.85 \text{ rad/s}.$$

$$I = 216 \text{ kg m}^2$$

$$\omega = 183.26 \text{ rad/s}$$

$$C = 0.85 \times 216 \times 183.26$$

$$\boxed{C = 33646.54 \text{ Nm}}$$

When the rotor rotate clockwise viewing from stern (AFT) and the bow rises (i.e.) turn the ship towards star board end.

(iii) When ship rolls at an angular velocity 0.15 rad/s.

$$\omega_p = 0.15 \text{ rad/s}$$

$$C = 0.15 \times 216 \times 183.26 \text{ rad/s}$$

$$\boxed{C = 5937.62 \text{ Nm}}$$

During rolling as the axis of spin is always parallel to the axis of precession. There is no gyroscopic effect.

2. The rotor of the turbine of a ship has a mass of 2400 kg and rotates at a speed of ~~300~~ 3100 rpm Anticlockwise direction when viewed from the stern. The rotor has a radius of gyration of 0.4 m. Determine the gyroscopic couple and its effect.

(i) The ship steers to the left in a curve of 78 m radius at a speed of 14 knot [1 knot = 1860 m/hr]

(ii) The ship pitches 5° above and below the normal position and the bow is descending with its maximum velocity. The pitching motion is SHM with a periodic time 35 s

(iii) The ship rolls and at the instant its angular velocity is 0.03 rad/s clockwise when viewed from the stern and also find the maximum angular acceleration.

Given Data:

$$R = 78\ m.$$

$$V = 14\ knots.$$

$$= 14 \times \frac{1880}{3600}$$

$$V = 7.233\ m/s.$$

(i) $I = mk^2$

$= 2400 \times 0.4^2$

$= 384 \ kg \ m^2$.

$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 3100}{60} = 324.63 \ rad/s$

$\omega_p = V/R = 7.23/78 = 0.092 \ rad/s$.

$C = I \omega \omega_p$

$= 384 \times 324.63 \times 0.092$.

$C = 11468.53 \ Nm$

When the rotor rotates anticlockwise direction looking from stern and the ship steers to the left. i.e., the lower the bow and rises the stern.

(ii) $\phi = 5^\circ = 5 \times \frac{\pi}{180} = 0.087$ radians

$t_p = 35 s$.

$C = I \omega \omega_p$.

$I = 384 \ kgm^2$

$\omega = 324.63 \ rad/s$

$\omega_p = \phi \times \omega_0$

$\omega_0 = \frac{2\pi}{t_p} = \frac{2 \times \pi}{35}$

$= 0.1795 \ rad/s$.

$\omega_p = 0.087 \times 0.1795$

$\omega_p = 0.0156 \ rad/s$.

$C = 384 \times 324.63 \times 0.0156$

$C = 1944.6 \ Nm$.

As the bow descends during pitching, the ship would turn towards right (or) star board side.

(ii) $\omega_p = 0.03$ rad/s.

$C = I\omega\omega_p$.

$I = 364\ kgm^2$

$\omega = 324.63$ rad/s.

$C = 364 \times 324.63 \times 0.03$

$C = 3739.7\ Nm$

During rolling, as the axis of spin is always parallel to the axis of precission so there is no gyroscopic effect.

Four Wheeler.

Stability of a four wheeler when moving in curved path

(i) Reaction due to the weight of the vehicle.

$$\frac{W}{4} = \frac{m \cdot g}{4.}$$

(ii) Reaction due to gyroscopic effect

$$\omega_p = \frac{V}{R}, \quad C = \omega_w \omega_p (4 I_w \pm G \cdot I_E)$$

vertical reaction at each outer or inner wheels

$$\frac{P}{2} = \frac{C}{2x}$$

(iii) Reaction due to centrifugal effect.

$$F_c = \frac{mv^2}{R.}$$

The couple tending to overturn the vehicle. (or) Overturning couple,

$$\frac{mv^2 h}{R} = F_c \times h = C_o$$

Vertical reaction at each inner or outer wheels.

$$\frac{Q}{2} = \frac{mv^2 h}{2Rx}$$

Total vertical reaction at each outer wheels.

$$P_o = \frac{W}{2} + \frac{P}{2} + \frac{Q}{2}.$$

11'y at inner wheels

$$P_i = \frac{W}{2} - \frac{P}{2} - \frac{Q}{2}.$$

1. A rear engine automobile is travelling along a track of 100m radius each of the four wheeler has a moment of inertia of 2.5 kgm^2 and the effective dia of 0.6m. The rotating parts of the engine has a moment of inertia of 1.2 kgm^2. The engine axis is parallel to the rear axial and the crank shaft rotates in the same sense as the road wheel. Gear ratio of the engine speed to the back wheel is 3:1. The automobile has a mass of 1600kg and centre of gravity 0.5m above road level. the width of the crank and the vehicle is 1.5m. Determine the limiting speed of the vehicle along

the curve for allow four vehic wheel to maintain contact with the road surface. Assume that the road surface is not compared and centre of gravity of automobil lies centrally with respect to the four wheel

Given Data:

$$R = 100\,m$$

$$I_w = 2.5\ kgm^2$$

$$d_w = 0.6\,m$$

$$I_E = 1.2\ kgm^2$$

$$G = \frac{\omega_E}{\omega_w} = \frac{3}{1} = 3$$

$$m = 1600\,kg$$

$$h = 0.5\,m$$

$$x = 1.5\,m$$

Solution:

Road reaction over each wheel $= \frac{W}{4} = \frac{mg}{4} = \frac{1600 \times 9.81}{4}$

$= 3924\,N.$

Let, V = limiting speed of the wheel.

WKT, angular velocity of wheel

$$\omega_w = \frac{V}{R_w}$$

$$N_\omega = \frac{1}{0.3} V$$

$$N_\omega = 3.33 V$$

Angular velocity of $\omega_P = \frac{V}{R} = \frac{V}{100}$

$$\omega_P = 0.01 V$$

Gyroscopic couple for wheel.

$$C_\omega = \omega_\omega \cdot \omega_P [4 I_\omega \pm G I_E]$$

$$= 3.33 V \times 0.01 V [4 \times 2.5 \pm 3 \times 1.2]$$

$$= 0.0333 V^2 [10 \pm 3.6]$$

(+) $C_\omega = 0.45288 V^2$ (−) $C_\omega = 0.2131 V^2$.

Gyroscopic effect of Engine parts

$$C_E = 4 \cdot I_E \cdot \omega_E \, \omega_P$$

$$= 4 \times 1.2 \times \omega_E \times 0.01 V$$

$$= 4 \times 1.2 \times 3 \times 3.33 V \times 0.01 V$$

$$= 0.47 V^2$$

Due to gyroscopic couple the vertical reaction rails will be produced. The reaction will be vertically upward on the outer wheel, downward and vertically downward of the inner wheel.

the magnitude of the reaction at each of the inner and outer wheel will be

$$\frac{P}{2} = \frac{C}{2x}$$

$$C = C_E + C_W$$

$$= 0.48V^2 + 0.448V^2$$

$$C = 0.928V^2.$$

$$\frac{P}{2} = \frac{0.928V^2}{2 \times 1.5}$$

$$\frac{P}{2} = 0.309V^2$$

WKT, centrifugal force, $\boxed{F_c = \frac{M \times V^2}{R}}$

$$F_c = \frac{1600 \times V^2}{100}$$

$$F_c = 16V^2$$

Overturning couple, $\boxed{C_o = F_c \times h}$

$$C_o = 16V^2 \times 0.5$$

$$C_o = 8V^2$$

Let the magnitude of this reaction at each inner or outer wheel.

$$\frac{Q}{2} = \frac{mV^2h}{2Rx}.$$

$$\frac{Q_1}{2} = \frac{1600 \times V^2 \times 0.5}{2 \times 100 \times 1.5}$$

$$\frac{Q_1}{2} = 2.6 v^2$$

$$\frac{P}{2} + \frac{Q_1}{2} \leq \frac{W}{4}.$$

$$0.309 v^2 + 2.6 v^2 \leq \frac{1600 \times 9.81}{4}.$$

$$2.97 v^2 \leq 3924$$

$$V \leq 36.35 \, m/s.$$

2. A rear engine automobile is travelling along a curved track of 120 m radius each of the four wheel has a moment of inertia of 2.3 kgm². and a effective diameter of 600mm. The rotating parts of the engine has a moment of inertia of 1.5 kgm². The gear ratio of engine ~~to~~ the backwell is 3:2. The engine axial is ||el to the rear axis and the crank shaft rotates in the same sense as the road wheel. The mass of the vehicle is 2050 kg and the centre of mass 520 mm above the road level. The width of the track is 1.6 m

What will be the limiting speed of the vehicle? If all four wheel maintain contact with road surface.

Given Data:

$R = 120\,m$

$I_w = 2.3\,kg\,m^2$

$d = 600\,mm$

$r = 300\,mm = 0.3\,m$

$I_E = 1.5\,kg\,m^2$

$G = 3:2 = \frac{3}{2}$

$M = 2050\,kg$

$h = 520\,mm$
$= 0.52\,m$

$x = 1.6\,m$

Solution:

Road Reaction over each wheel

$$= \frac{W}{4} = \frac{mg}{4} = \frac{2050 \times 9.81}{4}$$

$$= 5027.63\ N.$$

Let, v = limiting speed of the wheel

WKT, angular velocity of wheel

$$\omega_w = \frac{v}{R_w}$$

$$\omega_w = \frac{1}{0.3}\,v.$$

$\omega_w = 3.33\,V$

Angular velocity of $\omega_P = \frac{V}{R} = \frac{V}{120}$

$\omega_P = 0.0083 \times V$

$C_w = \omega_w \cdot \omega_P (4 I_w \pm G \cdot I_E)$

$= 3.33V \times 0.0083V (4 \times 2.3 \pm 1.5 \times 1.5)$

$= 0.316 V^2$

$C_E = 4 I_E \times \omega_E \times \omega_P$

$= 4 \times 1.5 \times 1.5 \times 3.33V \times 0.0083 V^2$

$C = C_E + C_w$

$= 0.2496 V^2 + 0.316 V^2$

$= 0.569 V^2$.

Due to gyroscopic couple the vertical reaction raile will be produced. The reaction will be vertically upward on the outer wheel, doownward and vertically downward of the inner wheel.

Then the magnitude of the reaction at each of the inner and outer wheel will be

$$\frac{P}{2} = \frac{C}{2x}$$

$$\frac{P}{2} = \frac{0.569V^2}{2\times1.0}$$

$$\frac{P}{2} = 0.176V^2$$

$$F_c = \frac{m\times V^2}{R} = \frac{2050\times V^2}{120}$$

$$F = 17.08V^2$$

$$C_0 = F_c\times h$$

$$= 17.08\times0.52$$

$$= 8.88V^2$$

Let the magnitude of this reaction at each inner or outer wheel

$$\frac{Q}{2} = \frac{mV^2h}{2R\cancel{x}}$$

$$\frac{Q}{2} = \frac{2050\times V^2\times0.52}{2\times120\times1.6}$$

$$\frac{Q}{2} = 2.77V^2$$

$$\frac{P}{2} + \frac{Q}{2} \leq \frac{W}{4}.$$

$$0.176V^2 + 2.77V^2 \leq 5027.6$$

$$V = 41.25\,m/s.$$

Stability of Two Wheel.

$$C_1 = \frac{v^2}{r_w R}(2 I_\omega + G I_E)\cos\theta$$

Centrifugal couple.

$$C_2 = \left(\frac{mv^2}{R}\right)\cos\theta \times h$$

Total Overturning Couple.

$$C_o = \frac{v^2}{R}\left[\frac{2 I_\omega \pm G I_E}{r_\omega} + mh\right]\cos\theta$$

1. Each rod wheel of a motor cycle has a moment of inertia of $1.5 kgm^2$. The rotating parts of the engine of the motorcycle have a mass moment inertia of 0.25 kgm^2. The speed of the engine is 5 times the speed of the wheel and is in the same sense. The mass of the motor cycle with its rider is 250kg and centre of gravity is 0.6m above the ground level. Find the angle of heal if the cycle is travelling at 50 km/hr and is taking a turn of 30m radius. The wheel dia is 0.6m.

Given Data:

$$I_\omega = 1.5\,kgm^2$$

$$I_E = 0.25\,kgm^2$$

$$G = \frac{\omega_E}{\omega_w} = 5$$

$$m = 250\,kg.$$

$$h = 0.6\,m$$

$$v = \frac{50\,km}{hr} = 13.89\,m/s.$$

$$R = 30\,m.$$

$$d_\omega = 0.6\,m$$

$$r_w = 0.3\,m$$

Solution:

WKT, Gyroscopic effect due to two wheel and due to rotating mass of engine

$$C_1 = \frac{v^2}{r_\omega R}\left(2I_\omega + GI_E\right)\cos\theta.$$

$$C_1 = \frac{13.89^2}{0.3\times 30}\left(2\times1.5 + 5\times0.25\right)\cos\theta$$

$$C_1 = 91.11\cos\theta$$

Centrifugal couple

$$C_2 = \left(\frac{mv^2}{R}\right)\cos\theta\times h$$

$$C_2 = \left(\frac{250 \times 13.84^2}{30}\right) \cos\theta \times 0.6$$

$$C_2 = 964.66 \cos\theta$$

Total overturing couple.

$$C_0 = \frac{V^2}{R}\left[\frac{2I_w \pm GI_E}{r_w} + mh\right]\cos\theta$$

$$= \frac{13.89^2}{30}\left[\frac{2 \times 1.5 \pm 5 \times 0.25}{0.3} + 250 \times 0.6\right] \cos\theta.$$

$$= 6.43107\left[\frac{4.25}{0.3} + 150\right]\cos\theta$$

$$C_0 = 1055.767 \cos\theta$$

Balancing couple:

$$= mgh\sin\theta$$

$$= 250 \times 9.81 \times 0.6 \times \sin\theta$$

$$= 1471.5 \sin\theta$$

$$\left.\begin{array}{l}\text{Overturning} \\ \text{couple}\end{array}\right\} = \left\{\begin{array}{l}\text{Balancing} \\ \text{couple.}\end{array}\right.$$

$$1055.767 \cos\theta = 1471.5 \sin\theta$$

$$\theta = \tan^{-1}(0.717)$$

$$\theta = 35.61^\circ.$$

2. The racing motorcycle travels 140 km/h around a curve of 120 m radius measured horizontally. The cycle and rider has a mass of 150 kg and centre of gravity lies a 0.7 m above the ground level. When the motor cycle is vertical. Each wheel is 0.6 m diameter and has a moment of inertia of 1.5 kgm². The engine has rotating parts where moment of inertia about there axis of rotation is 0.25 kgm² and it rotates as 5 times the wheel mean speed in same direction. Find the angle of balancing of the track.

Given Data:

$$v = 140\ km/hr = 38.89\ m/s$$

$$R = 120\ m$$

$$M = 150\ kg$$

$$h = 0.7\ m$$

$$r_w = 0.3\ m$$

$$I_w = 1.5\ kgm^2$$

$$I_E = 0.25\ kgm^2$$

$$G = \frac{\omega_E}{\omega_{W}} = 5$$

Solution: $\theta = 55.55^\circ$

$$C_1 = \frac{V_E^2}{r_w R}\left(2I_w + G\,I_E\right)\cos\theta$$

$$= \frac{38.84^2}{0.3\times 120}\left(2\times 1.5 + 5\times 0.25\right)\cos\theta$$

$$= 42.01 \times 4.25 \cos\theta$$

$$C_1 = 178.55 \cos\theta.$$

$$C_2 = \left(\frac{mv^2}{R}\right)\cos\theta \times h$$

$$= \left(\frac{150\times 38.84^2}{120}\right)\cos\theta \times 0.7$$

$$= 1323.38 \cos\theta.$$

Total overturning couple

$$C_0 = \frac{v^2}{R}\left[\frac{2I_w \pm G\,I_E}{r_w} + mh\right]\cos\theta$$

$$= \frac{38.89^2}{120}\left[\frac{2\times 1.5 \pm 5\times 0.25}{0.3} + (150\times 0.7)\right]\cos\theta$$

$$= 12.6036\left[15 \pm .105\right]\cos\theta$$

$$C_0 = 1512.43 \cos\theta.$$

Balancing couple $= mgh \sin\theta$.

$$= 150\times 9.81\times 0.7 \sin\theta.$$

$$= 1030.05 \sin\theta.$$

$$\left.\begin{array}{c}\text{Total}\\ \text{Overturning}\\ \text{couple}\end{array}\right\} = \left\{\begin{array}{c}\text{Balaning}\\ \text{couple.}\end{array}\right.$$

$$1512.43 \cos\theta = 1030.09 \sin\theta$$

9 $$1.4683 = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

$$\theta = \tan^{-1}(1.4683)$$

$$\boxed{\theta = 55.74^\circ}$$

Governor 7

Function of Governor

The function of a governor is to automatically
10 maintain speed of an engine within specified limit. Whenever there is a variation load. 9.

9. 1. WATT governor :-

$$h = \frac{g}{\omega^2} = \frac{g}{\left(\frac{2\pi N}{60}\right)^2}.$$

$$h = \frac{895}{N^2}.$$

1. Calculate the vertical height of a watt governor which rotates at 60 rpm. Also find (i) change in vertical height when the speed increases to 61 rpm

(i) Vertical height of a Watt Governor

Given Data: (ii) Change in vertical height

$$N_1 = 60 \text{ rpm}$$

$$N_2 = 61 \text{ rpm}$$

1. $$h_1 = \frac{895}{N_1^2}$$

$$= \frac{895}{60^2} = 0.2486$$

2. $$h_2 = \frac{895}{N_2^2} = \frac{895}{61^2}$$

$$= 0.2405$$

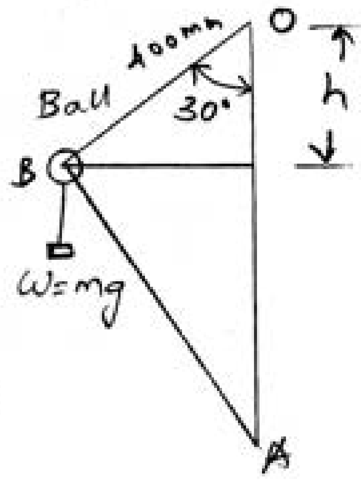
$$h = h_2 - h_1$$

$$= 0.2405 - 0.2486$$

$$= -0.081 \text{ m}.$$

2. The length of the upper arm of a watt governor is 400 mm and its inclination to the vertical in 30°. Find the percentage increase speed if the ball raised by 20mm

Given Data:



$OB = 400 \text{mm}$

$OB = 0.4 \text{ m}$

$\angle AOB = 30°$

$h = 20 \text{mm} = 0.02 \text{ m}$

From the geometry

$$h_1 = OB \cos\theta = 0.4 \times \cos 30^\circ$$
$$= 0.346$$

WKT, $h_1 = \frac{895}{N_1^2}$

$$0.346 = \frac{895}{N_1^2}$$

$$N_1 = 50.8295 \text{ rpm}$$

The speed of governor when the ball raised 20mm is given by

$$h_2 = h_1 - 0.02$$
$$= 0.346 - 0.02$$
$$h_2 = 0.326 \text{ m}$$

$$h_2 = \frac{895}{N_2^2}$$

$$0.326 = \frac{895}{N_2^2}$$

$$N_2 = 52.3965 \text{ rpm}$$

% increase in speed $= \frac{N_2 - N_1}{N_1}$

$$= 3.08\ \%$$

Porter Governor.

Let, m = mass of each ball.

M = Mass of the central load.

$W = m \times g$ = mass × gravity

r = radius of rotation

h = height of governor

N = speed of the ball

ω = angular speed.

Centrifugal force, $F_c = m\omega^2 r$

α = Angle of inclination upper link

β = Angle of inclination of lower link (arm)

Formula:

$$h = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{895}{N^2}$$

$$h = \frac{(m+M)}{m} \times \frac{895}{N^2}.$$

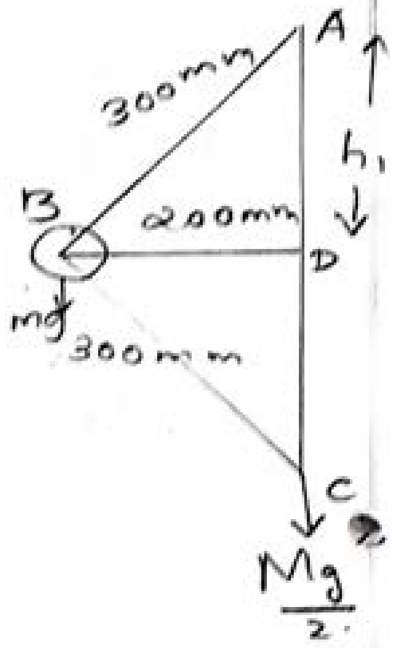
$$N_1^2 = \frac{mg + (Mg - F)}{mg} \times \frac{895}{h_1}$$

$$N_2^2 = \frac{mg + (Mg - F)}{mg} \times \frac{895}{h_2}$$

$$h_1 = AD = \sqrt{AB^2 - BD^2}$$

$$h_2 = AD = \sqrt{AB^2 - BD^2}$$

1. The arm of a porter governor are each 300mm long are hinged on the axis of rotation. The mass of each ball is 5kg. The radius of rotation of the ball is 200mm. When the governor begins to lift and 250m when the governor is at maximum speed. Determine the minimum and maximum speed and range of ~~speed~~ the governor. The mass of the sleeve is 15kg



Given Data:

$AB = BC = 300mm = 0.3m$

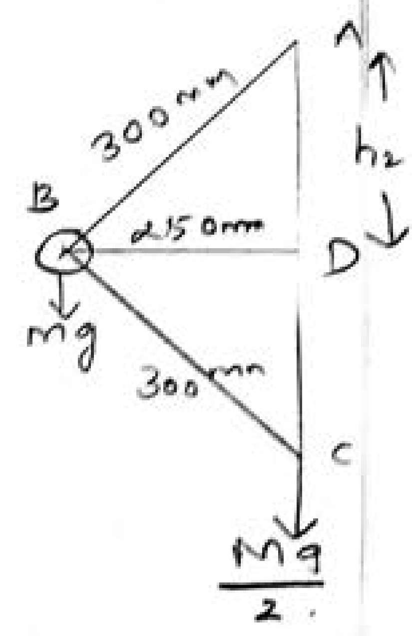
$m = 5kg$

$r_1 = 200mm = 0.2m$

$r_2 = 250mm = 0.2m.$

$M = 15\ kg.$

Solution:



$$h_1 = AD = \sqrt{AB^2 - BD^2}$$

$$= \sqrt{0.3^2 - 0.2^2}$$

$$h_1 = 0.223m$$

WKT, speed of the porter governor

$$N_1^2 = \left(\frac{m+M}{m}\right) \times \frac{895}{h_1}$$

$$\left(\frac{5+15}{5}\right) \times \frac{895}{0.223}$$

$$N_1 = 1267\ rpm$$

The find Maximum speed

$$h_2 = AD = \sqrt{AB^2 - BD^2}$$

$$= \sqrt{0.3^2 - 0.25^2}$$

$$h_2 = 0.165\,m$$

$$N_2^2 = \left(\frac{m+M}{m}\right) \times \frac{895}{h_2}$$

$$= \left(\frac{5+15}{5}\right) \times \frac{895}{0.165}$$

$$N_2 = 147.2989 \text{ rpm}$$

Range speed = Max. speed - Min speed

$$= 147.3 - 126.7$$

$$= 20.6 \text{ rpm}.$$

Range of speed considering the friction at the sleeve

To find minimum speed.

Take frictional force, $F = 30N$

$$N_1^2 = \frac{mg + (Mg - F)}{mg} \times \frac{895}{h_1}$$

$$= \frac{(5 \times 9.81) + [(15 \times 9.81) - 30]}{5 \times 9.81} \times \frac{895}{0.223}$$

$$= \frac{49.05 + [117.15]}{49.05} \times 4013.45$$

$$N_1 = 116.62 \text{ rpm}$$

$$N_2^2 = \frac{mg + (Mg + F)}{mg} \times \frac{895}{h_2}$$

$$= \frac{5 \times 9.81 (15 \times 9.81 + 30)}{5 \times 9.81} \times \frac{895}{0.165}$$

$$N_2 = 158.15 \text{ rpm}.$$

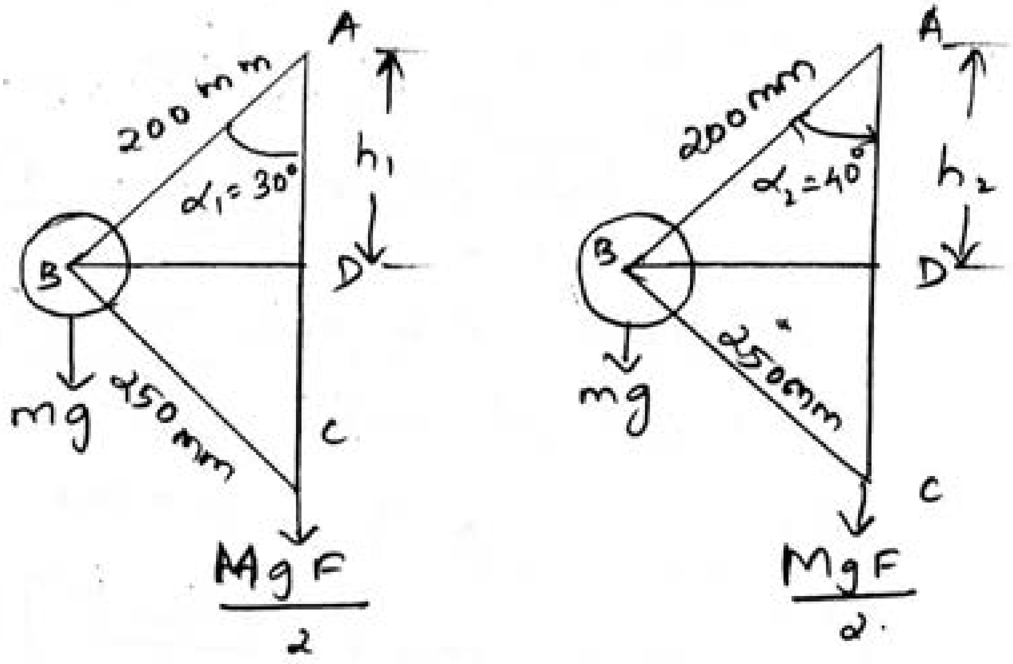
Range of speed $= N_2 - N_1$

$= 158.15 - 116.62$

$= 41.53$ rpm.

2. The length of the upper & lower arm of a porter governor 200mm and 250mm. Both the arms are pivoted on the axis of rotation. The central load is 150N. The weight of each ball is 20N and the friction of the sleeve together with the resistance of operating gear is equivalent to a force of 30N at the sleeve. If the limiting inclination of the upper arm to the vertical are 30° and 40°. Taking friction into consideration. Find the range of speed of the governor.

Given Data:



Solution:

$$AB = 200\,mm = 0.2\,m$$

$$BC = 250\,mm = 0.25\,m$$

$$w = mg$$
$$= 20\,N$$

$$W = Mg$$
$$= 150\,N$$

$$F = 30\,N$$
$$\alpha_1 = 30°$$
$$\alpha_2 = 40°$$

Solution:

$$r_1 = BD = AB \sin\alpha' = 0.2 \sin 30°$$
$$= 0.1\,m$$

$$h_1 = AD = AB\cos\alpha_1 = 0.2\cos 30°$$
$$= 0.173\,m$$

WKT, $\tan\beta_1 = \frac{BD}{CD}$

$$CD = \sqrt{BC^2 - BD^2} = \sqrt{0.25^2 - 0.1^2}$$

$$= 0.229$$

$$\tan\beta_1 = \frac{BD}{CD} = \frac{0.1}{0.229} = 0.436$$

WKT, $$q_1 = \frac{\tan\beta_1}{\tan\alpha_1} = \frac{0.436}{0.577}$$

$$q_1 = 0.756$$

$$N_1^2 = \frac{mg + \left[\frac{Mg - F}{2}\right](1+q_1)}{mg} \times \frac{895}{h_1}$$

$$= \frac{20 + \left[\frac{150-30}{2}\right](1+0.756)}{20} \times \frac{895}{0.173}$$

$$= \frac{20 + 105.36}{20} \times 5173.41$$

$$N_1 = 180.07 \text{ rpm}$$

To find Maximum Speed:

$$r_2 = AB \sin\alpha_2$$

$$= 0.129$$

$$h_2 = AD = AB\cos\alpha_2$$

$$= 0.153$$

30 $$CD = 0.214\,m$$

[illegible] = [illegible]

$$\tan \beta_2 = \frac{0.128}{0.214}$$

$$\tan \beta_2 = 0.5981$$

$$q_2 = \frac{\tan \beta_2}{\tan \alpha_2}$$

$$\tan \alpha_2 = \tan 40$$

$$\tan \alpha_2 = 0.839$$

$$q_2 = \frac{0.5981}{0.839}$$

$$q_2 = 0.7128.$$

$$N_2^2 = \frac{mg + \left[\frac{Mg+F}{2}\right](1+q_2)}{mg} \times \frac{895}{h_2}.$$

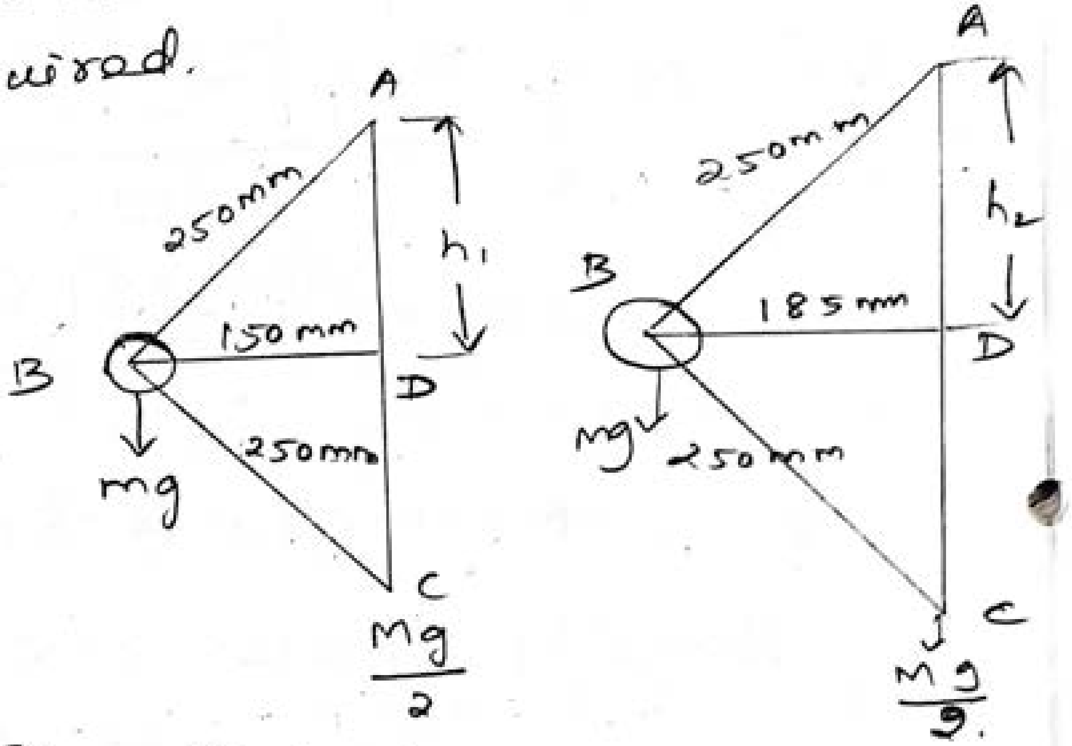
$$= \frac{20 + \left[\frac{150+30}{2}\right](1+0.7128)}{20} \times \frac{895}{0.133}$$

$$N_2 = 225.69 \text{ rpm}$$

Range of speed. $= 225.69 - 180.67$

$= 45.62$ rpm

A loaded governor of the porter type has equal arm and link each 250 mm long. The mass of each ball is 2kg and the centre mass is 12 kg. when the ball radius is 150 mm. The valve is fully open and the radius is 185 mm. The value is closed. Find the maximum speed and range of speed. If the maximum speed is to be increased 20% by a addition of mass to the central load. Find what additional mass is required.



Given Data:

$AB = BC = 250 mm = 0.25 m$

$m = 2 kg$ | $r_1 = 150 mm = 0.15 m$

$M = 12 kg$ | $r_2 = 185 mm = 0.185 m.$

Solution:

$$h_1 = AD = \sqrt{AB^2 - BD^2}$$

$$= 0.2\ m$$

~~$N_1 = 176.99$ rpm~~

$$h_2 = AD = \sqrt{AB^2 - BD^2}$$

$$= \sqrt{0.25^2 - 0.185^2}$$

$$h_2 = 0.168$$

$$N_1^2 = \left(\frac{m+M}{m}\right) \times \frac{895}{h_1}$$

$$= \left(\frac{2+12}{2}\right) \times \frac{895}{0.2}$$

$$N_1 = 176.98\ rpm.$$

$$N_2^2 = \left(\frac{m+M}{m}\right) \times \frac{895}{h_2}$$

$$= \left(\frac{2+12}{2}\right) \times \frac{895}{0.168}$$

$$N_2 = 193.1105\ rpm$$

Range, $N = 193.1105 - 176.98$

$$= 16.12\ rpm.$$

ii) Addition mass required.

$$N_2' = N_2 + 20\%$$

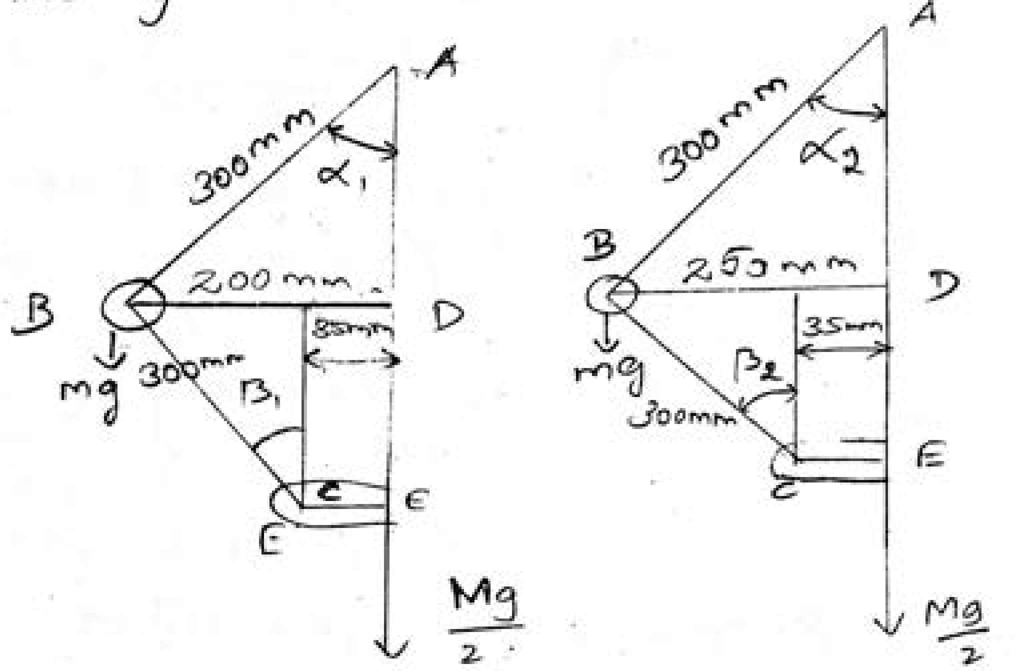
$$= 193.1105 \times 1.2$$

$$N_2' = 231.7326\ rpm.$$

$$(N_2')^2 = \left(\frac{m+M_t}{m}\right) \times \frac{895}{h_2}$$

$$M_t = 18.15\ kg.$$

4. A porter governor has all four arm 300 mm long. The upper arm are pivoted on the axis of rotation and the lower arm are attached to the sleeve at a distance of 35 mm from the axis each ball has a mass of 7 kg and mass of the sleeve is 54 kg If the extreme radii of rotation of the ball are 200 mm and 250 mm. Determine the range of speed of the governor



Given Data:

$AB = BC = 300mm = 0.3m$

$CE = 35\ mm = 0.035m$

$m = 7\ kg$

$M = 54\ kg$

$r_1 = 200\ mm = 0.2m$

$r_2 = 250\ mm = 0.25m$

Solution :

$r_1 = 200 mm$

$h_1 = AD = \sqrt{AB^2 - BD^2}$

$= \sqrt{0.3^2 - 0.2^2}$

$h_1 = 0.224 m$

$BF = BD - FD$

$= 200 - 35$

$BF = 165 mm$

$CF = \sqrt{BC^2 - BF^2}$

$= \sqrt{0.3^2 - 0.165^2}$

$CF = 0.25 m$

$$\tan\alpha_1 = \frac{BD}{AD} = \frac{0.2}{0.224} = 0.89$$

$$\tan\beta_1 = \frac{BF}{CF} = \frac{0.165}{0.25} = 0.66$$

$$q_1 = \frac{\tan\beta_1}{\tan\alpha_1} = \frac{0.66}{0.89} = 0.739$$

$$N_1^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{895}{h_1}$$

$$= \frac{7 + \frac{54}{2}(1+0.739)}{7} \times \frac{895}{0.224}$$

$N_1 = 175.57$ rpm

Maximum speed when $r_2 = 250 mm$

$h_2 = AD = \sqrt{AB^2 - BD^2}$

$= 0.166 m$

$BF = BD - FD$

$= 0.250 - 0.35 = 0.215 m$

$$CF = \sqrt{BC^2 - BF^2}$$
$$= \sqrt{0.32^2 - 0.215^2}$$
$$= 0.209m.$$

$$\tan \alpha_2 = \frac{BD}{AD} = \frac{0.25}{0.166} = 1.50$$

$$\tan \beta_2 = \frac{BF}{CF} = \frac{0.215}{0.209} = 1.029$$

$$q = \frac{\tan \beta_2}{\tan \alpha_2} = \frac{1.029}{1.506} = 0.683$$

$$N_2^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{895}{h_2}$$

$$= \frac{7 + \frac{54}{2}(1+0.683)}{7} \times \frac{895}{0.166}$$

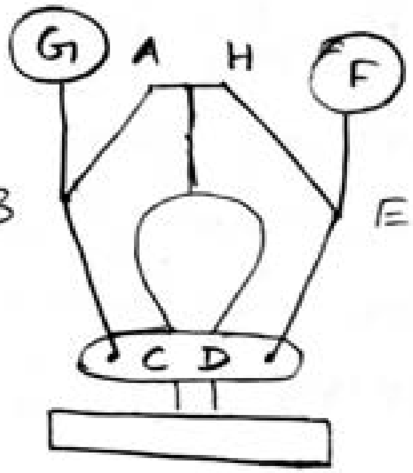
$$N_2 = 200.97 \text{ rpm}$$

Range = Max - Min. speed.

$$= 200.97 - 175.5$$
$$= 25.49 \text{ rpm}$$

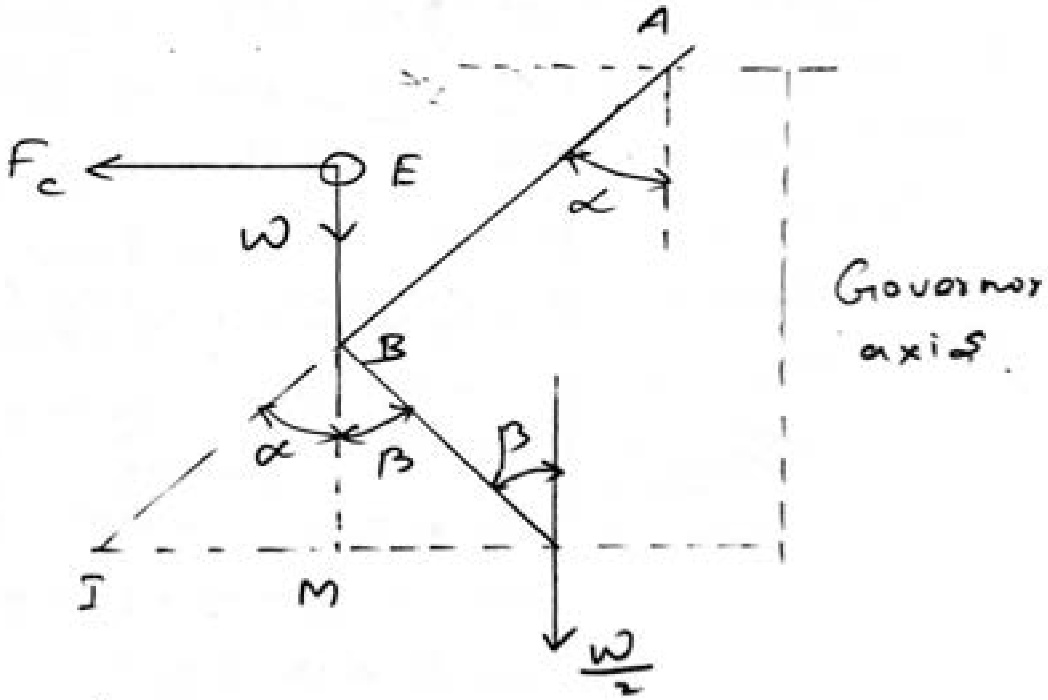
Proell Governor

A porter governor is also known as Proell governor, if the two ball are mounted on the extension of the link CB & DG

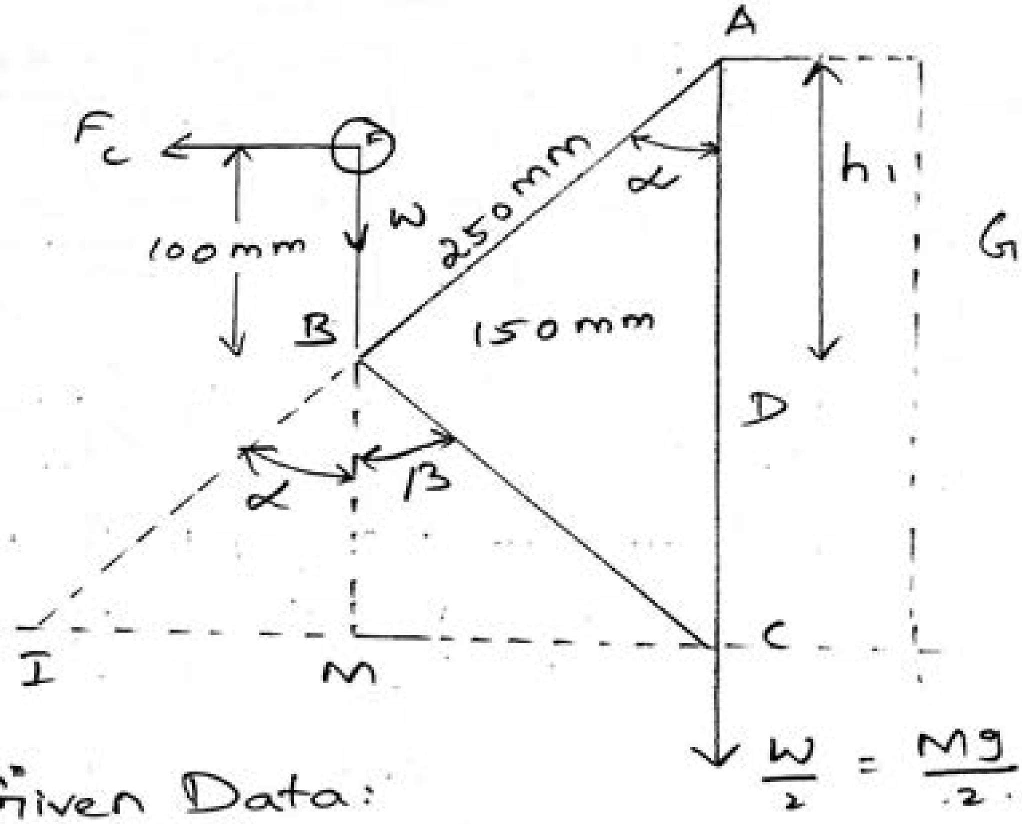


$$N^2 = \frac{BM}{EM}\left[\frac{m+M}{m}\right]\frac{895}{h}$$

$$= \frac{BM}{EM}\left[\frac{m+\frac{M}{2}(1+q)}{m}\right]\frac{895}{h}.$$



1. The proell governor has all the four arm of length 250 mm. The upper and lower ends of the arm are pivoted on the axis of governor. The extension of the lower are each 100 mm long and parallel to the axis. when the radius of the ball path is 150 mm. The mass of each ball is 4.5 kg and the mass of central load is 36 kg Determine the equilibrium speed of the governor.



Given Data:

$AB = BC = 250 mm = 0.25 mm$

$BE = 100 mm = 0.1 m$

$m = 4.5 kg$

$M = 36 kg$

$r = 150 mm = 0.15 m$

Solution :

$$h = AD = \sqrt{AB^2 - BD^2}$$
$$= \sqrt{0.25^2 - 0.15^2}$$
$$= 0.2\,m$$

$$BM = DC = AD = 0.2\,m$$

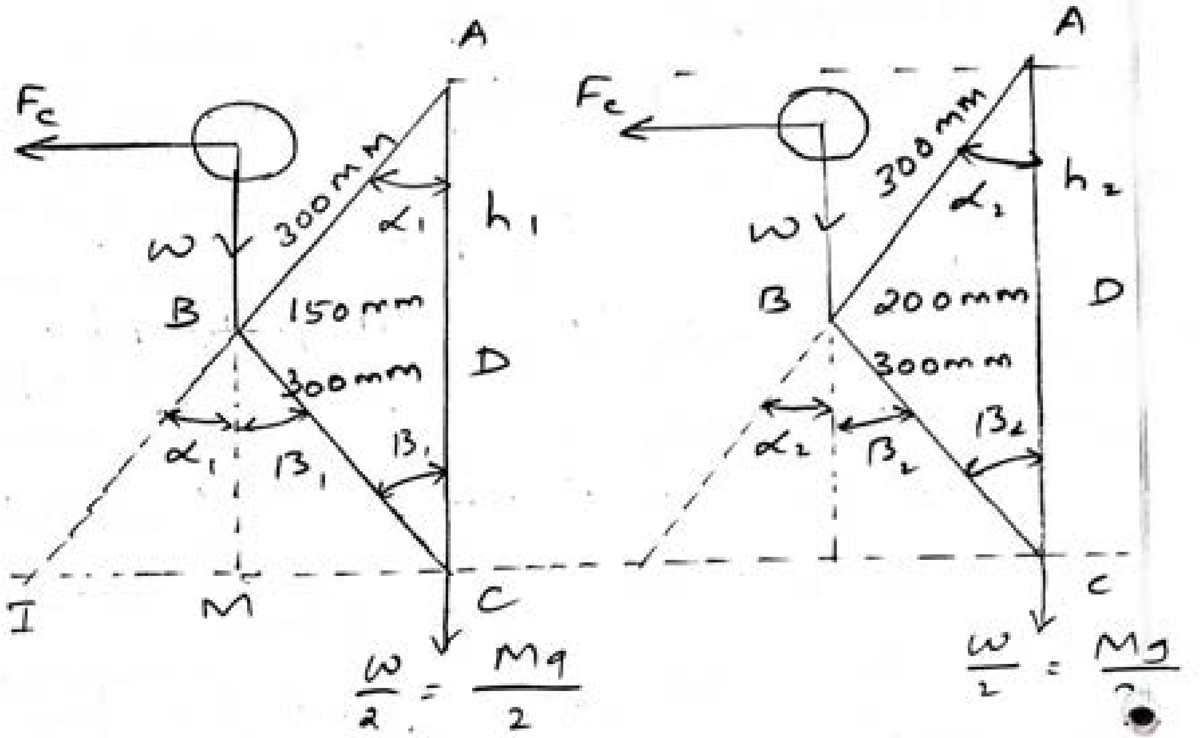
$$EM = EB + BM$$
$$= 0.1 + 0.2$$
$$= 0.3\,m$$

$$N = \frac{0.2}{0.3}\left[\frac{4.5 + 36}{4.5}\right] \times \frac{895}{0.2}$$

$$N = 163.86\ rpm.$$

2. A proell Governor has equal arm of length 300mm. The upper + lower end of the arm are pivoted at the axis of the governor. The extension of the lower link are 80mm. Length and. parallel to the axis when the radii of rotation on the ball of 150 mt 200 mm. The mass of each ball is 10Kg and the mass of central load is 100 kg Calculate. the range of the governor.

Given Data:



Solution:

$AB = BC = 300\,mm = 0.3\,m$

$BE = 80\,mm = 0.08\,m$

$m = 10\,kg$

$M = 100\,kg$

$h = AD = \sqrt{AB^2 - BD^2}$

$= \sqrt{0.3^2 - 0.15^2}$

$= 0.26\,m$

$BM = DC = AD = 0.26\,m$

$EM = EB + BM$

$= 0.08 + 0.26$

$= 0.34\,m$

$N_1^2 = \frac{0.26}{0.34}\left[\frac{10+100}{10}\right] \times \frac{895}{0.25}$

$N_1 = 173\,rpm$

$r_2 = 0.2\,m.$

$h = AD = \sqrt{AB^2 - BD^2}$

$= \sqrt{0.3^2 - 0.2^2}$

$= 0.2236.$

$BM = DC = AD = 0.2236$

$EM = EB + BM$

$= 0.08 + 0.2236$

$EM = 0.3036\,mm.$

$N_2^2 = \frac{BM}{EM}\left[\frac{m+M}{m}\right]\frac{895}{h_2}$

$N_2 = 180.07\,rpm$

$Range = N_2 - N_1$

$= 180.07 - 170.415$

$= 9.659\,rpm.$

Hartnell Governor

In this type of governor the balls are controlled by a spring. The spring is fitted in compression, so that a force is applied to the sleeve. The roller is fitted to groove in the sleeve.

Analysis of Hartnell Governor

Let m - mass of each ball in kg

M - mass of sleeve in kg.

r_1 & r_2 - Min & Max radius of rotation (m)

r - radius of rotation at mid position.

ω_1 & ω_2 - angular speed of the governor at min & max radius (rad/s)

S - Spring force exerted on the sleeve (N).

S_1 & S_2 - Spring force exerted on the sleeve at min and max radius

s - Stiffness of spring (Nm)

a - length of the vertical (or) ball arm of the lever (m)

b - length of the horizontal (or) the sleeve arm (m)

x - Sleeve lift (m)

Formula:

1. Min radius, $r_1 = Mg + S_1 = 2\left(\frac{a}{b}\right) \cdot F_{c_1}$
2. Max radius, $r_2 = Mg + S_2 = 2\left(\frac{a}{b}\right) F_{c_2}$
3. $F_c = m\omega^2 r$.
4. Sleeve lift, $x = \left(\frac{b}{a}\right)(r_2 - r_1)$
5. Spring Stiffness, $S = \frac{S_2 - S_1}{x}$

$$= 2\left(\frac{a}{b}\right)^2 \left(\frac{F_{c_2} - F_{c_1}}{r_2 - r_1}\right)$$

6. Initial Compression of spring

$$\delta = \frac{S_1}{S_2}.$$

1. A hartnell governor has equal mass of ball 3kg. set initially at a radius of 200mm. The arm of the bell crank lever are 110mm vertically and 150mm horizontally. Find initial compression force of the spring at a radius of 200mm and 250 rpm.

(ii) Stiffness of spring required to permit sleeve moment of ~~four~~ 4mm on a fluctuation of 7.5% in the engine speed.

Given data:

$m = 3kg$

$r_1 = 0.2m$ $\qquad N_1 = 250$ rpm.

$a = 0.11\ m$ $\qquad x = 0.004$

$b = 0.15\ m$ $\qquad C_S = 0.075$

Solution:

$$F_{c_1} = m\omega_1^2 r_1^2$$

$$\omega_1 = \frac{2\pi N}{60} = 26.18$$

$$F_{c_1} = 3 \times 26.18^2 \times 0.2$$

$$\boxed{F_{c_1} = 411.2\ N}$$

WKT,
for minimum radius

$$r_1 = Mg + S_1 = 2\left(\frac{a}{b}\right) \cdot F_{c_1}$$

$$M_0 = 0$$

$$\Rightarrow S_1 = S_1 = 2\left(\frac{0.11}{0.15}\right) \times 411.2$$

$$\boxed{S_1 = 603.14\ N}$$

$$C_S = \frac{2(N_2 - N_1)}{N_1 + N_2}$$

$$0.075 = \frac{2(N_2 - 250)}{N_2 + 250}$$

$$0.075N_2 + 18.75 = 2N_2 - 500$$

$$518.75 = 1.925N_2$$

$$\boxed{N_2 = 269.48\ rpm}$$

Sleeve lift,

$$x = \frac{b}{a}(r_2 - r_1)$$

$$0.004 = \frac{0.15}{0.11}(r_2 - 0.2)$$

$$\boxed{r_2 = 0.203\ m}$$

$$F_{c_2} = m\omega_2^2 r_2$$

$$= 3 \times \left(\frac{2\pi \times 269.48}{60}\right)^2 \times 0.203$$

$$\boxed{F_{c_2} = 484.98\ N}$$

WKT,
For maximum radius

$$Mg + S_2 = 2\left(\frac{a}{b}\right) \times F_{c_2}.$$

$$S_2 = 2\left(\frac{0.11}{0.15}\right) \times 484.98$$

$$S_2 = 711.31$$

$$S = \frac{S_2 - S_1}{x}$$

$$= \frac{711.31 - 603.14}{0.004}$$

$$\boxed{S = 27042.5\ N}$$

(484.93 2. A hartnell governor having a centre sleeve spring and two right angle bellcrank . moves between 250rpm and 300 rpm. For a sleeve lift of 14.5mm. The sleeve arm and the b arm are 85mm and 125mm. The lever are pivot at 125mm from the governor axis and the mass of each ball is 2.8kg. The ball arm ar parallel to the governor axis at the lowest equilibrium speed. Determine the load on th spring and the lowest and highest equilibriu speed (ii) stiffness of spring (iii) initial compr of the spring:

14

N

Given data:

$N_1 = 280\,rpm$

$N_2 = 300\,rpm$

$x = 14.5\,mm = 0.0145\,m$

$b = 0.085\,m$

$a = 0.125\,m$

$m = 2.8\,kg$

$r = r_1 = 0.125\,m$

Solution

$x = \frac{b}{a}(r_2 - r_1)$

$0.0145 = \frac{0.085}{0.125}(r_2 - 0.125)$

$\boxed{r_2 = 0.146\,m}$

WKT, For min radius.

$r_1 = Mg + S_1 = 2\left(\frac{a}{b}\right) \times Fc_1$

$Fc_1 = m\omega_1^2 \cdot r_1$

$= 2.8 \times \left(\frac{2 \times \pi \times 280}{60}\right)^2 \times 0.125$

$\boxed{Fc_1 = 300.91\,N}$

$M = 0$

$\Rightarrow S_1 = 2\left(\frac{0.125}{0.085}\right) \times 300.9$

$\boxed{S_1 = 885.04\,N}$

$Fc_2 = 2.8\left(\frac{2\pi \times 300}{60}\right)^2 \times 0.146$

$\boxed{Fc_2 = 403.47\,N}$

WKT, for max radius.

$M = 0$

$\Rightarrow S_2 = 2\left(\frac{0.125}{0.085}\right) \times 403$

$S_2 = 1186.67$

$s = \frac{S_2 - S_1}{x}$

$= \frac{1186.67 - 885}{0.0145}$

$s = 20805.9\,N/m$

$\delta = \frac{S_1}{s} = \frac{885}{20806}$

$\delta = 0.0425\,m$

3. The arm of a Hartnell governor are of equal length when the sleeve is in the midpoint in the ~~midpos~~. The mass of rotating circle of a diameter of 20mm. The arm are vehicle in midposition neglecting friction, equilibrium of this position is 300rpm. Maximum variation of speed, taking friction into account is to be ± 5% of mid position speed ~~for~~ a maximum sleeve movement of 25mm. The sleeve mass is 5kg and the friction of skeve 30N. Assuming the power of governor is sufficient to overcome 1% change of speed on each side of the mid position. Find (a) Neglecting effect of arm)(i) mass of each rotating arm (ii) spring stiffness, Initial compression of sleeve.

Given Data:

$a = b$

$d = 0.2m \Rightarrow r = 0.1m$

$N_1 = 300rpm$

$x = 25mm = 0.025m$

$M = 5kg$

$F = 30N$

Solution:

$$\omega_1 = \frac{2\pi N_1}{60}$$

$$= 2\pi \times \frac{300}{60}$$

$$\omega_1 = 31.4 \text{ rad/s}.$$

Mass of each rotating ball

Since the change of speed at midposition to overcome is 1%.

Min. speed $\omega_1 = \omega - 0.01\omega$

$= 0.99 \times 31.4$

$\omega_1 = 31.09$ rad/s

Max speed $\omega_2 = \omega + 0.01\omega$

$= 1.01 \times 31.4$

$\omega_2 = 31.72$ rad/s

$F_{c_1} = m \times \omega_1^2 \times r_1$

$= m \times 31.09^2 \times 0.1$

$F_{c_1} = 96.65m$

$F_c = m \times \omega_2^2 \times r$

$F_{c_2} = m \times (31.72)^2 \times 0.1$

$F_{c_2} = 100.61m$

W.K.T, For min. speed of mid position

$S + (Mg - F) = 2F_{c_1} \times \frac{a}{b}$

$S + (5 \times 9.81 - 30) = 2 \times 96.65m$

~~S 193.3m~~

$S - 193.3m + 19.05 = 0$

For max speed

$S + (Mg + F) = 2 \times F_{c_2} \times \frac{a}{b}$

$S - 201.2m + 79.05 = 0.$

Solving ① + ②.

$m = 7.57$ kg

Spring Stiffness.

Since the max. variation of speed for minimum speed.

$\omega_1 = \omega - 0.05\omega$

$= 31.4 - 0.05(31.4)$

$\omega_1 = 29.83$ rad/s

For maximum speed

$\omega_2 = \omega + 0.05\,\omega$

$= 31.4 + 0.05(31.4)$

$= 32.97\ rad/s.$

$F_{c_1} = m\omega_1^2 r_1$

$x = \frac{b}{a}(r_2 - r_1)$

$r_1 = r - \left(\frac{x}{2}\right)\left(\frac{a}{b}\right)$

$= 0.1 - \left(\frac{0.025}{2}\right)$

$r_1 = 0.0875$

$r_2 = 0.1 + \left(\frac{0.025}{2}\right)$

$r_2 = 0.1125$

$F_{c_1} = 590N$

$F_{c_2} = m\omega_2^2 \times r_2.$

$= 7.57 \times 32.97^2 \times 0.1125$

$F_{c_2} = 931.1147N$

For min. speed of mid position.

$F_{c_1} = 7.614 \times (29.83)^2 \times 0.0875$

$F_{c_1} = 592.82N.$

$S_1 + Mg = 2\left(\frac{a}{b}\right) \times F_{c_1}$

$S_1 + (5 \times 9.81) = 2 \times 592.828$

$S_1 = 1131.31N$

$S_2 + Mg = 2\left(\frac{a}{b}\right) \times F_{c_2}.$

$S_2 + (5 \times 9.81) = 2 \times 931.1147$

$S_2 = 1804N.$

$S = \frac{S_2 - S_1}{x}.$

$S = \frac{1804 - 926.29}{0.025}$

$S = 26895.2N$

$\delta = \frac{S_1}{S}$

$= \frac{1131.15}{26895.2}$

$\delta = 0.042.$