STELLA MARY'S COLLEGE OF ENGINEERING

(Accredited by NAAC, Approved by AICTE - New Delhi, Affiliated to Anna University Chennai)

Aruthenganvilai, Azhikal Post, Kanyalumari District, Tamilnadu - 629202.

ME8593 DESIGN OF MACHINE ELEMENTS

(Anna University: R2017)

Prepared By

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2021-2022 (ODD)

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Aruthenganvilai, Kallukatti Junction Azhikal Post, Kanyakumari District-629202, Tamil Nadu.

DEPARTMENT OF MECHANICAL ENGINEERING

COURSE MATERIAL

REGULATION	**2017**
YEAR	**III**
SEMESTER	**05**
COURSE NAME	**DESIGN OF MACHINE ELEMENTS**
COURSE CODE	**ME8593**
NAME OF THE COURSE INSTRUCTOR	**Mr. J. STARLIN DEVA PRINCE**

SYLLABUS:

UNIT I STEADY STRESSES AND VARIABLE STRESSES IN MACHINE MEMBERS 9

Introduction to the design process - factors influencing machine design, selection of materials based on mechanical properties - Preferred numbers, fits and tolerances – Direct, Bending and torsional stress equations – Impact and shock loading – calculation of principle stresses for various load combinations, eccentric loading – curved beams – crane hook and ‘C’ frame- Factor of safety - theories of failure – Design based on strength and stiffness – stress concentration – Design for variable loading.

UNIT II SHAFTS AND COUPLINGS 9

Design of solid and hollow shafts based on strength, rigidity and critical speed – Keys, keyways and splines - Rigid and flexible couplings.

UNIT III TEMPORARY AND PERMANENT JOINTS 9

Threaded fastners - Bolted joints including eccentric loading, Knuckle joints, Cotter joints –Welded joints, riveted joints for structures - theory of bonded joints.

UNIT IV ENERGY STORING ELEMENTS AND ENGINE COMPONENTS 9

Various types of springs, optimization of helical springs - rubber springs – Flywheels considering stresses in rims and arms for engines and punching machines- Connecting Rods and crank shafts.

UNIT V BEARINGS 9

Sliding contact and rolling contact bearings - Hydrodynamic journal bearings, Sommerfeld Number, Raimondi and Boyd graphs, -- Selection of Rolling Contact bearings.

TEXT BOOKS:

1. Bhandari V, "Design of Machine Elements", 4th Edition, Tata McGraw-Hill Book Co, 2016.
2. Joseph Shigley, Charles Mischke, Richard Budynas and Keith Nisbett "Mechanical Engineering Design", 9th Edition, Tata McGraw-Hill, 2011.

REFERENCES:

1. Alfred Hall, Halowenko, A and Laughlin, H., "Machine Design", Tata McGraw-Hill, BookCo.(Schaum's Outline), 2010
2. Ansel Ugural, "Mechanical Design – An Integral Approach", 1st Edition, Tata McGraw-Hill Book Co, 2003.
3. P.C. Gope, "Machine Design – Fundamental and Application", PHI learning private ltd, New Delhi,2012.
4. R.B. Patel, "Design of Machine Elements", MacMillan Publishers India P Ltd., Tech-Max Educational resources, 2011.
5. Robert C. Juvinall and Kurt M. Marshek, "Fundamentals of Machine Design", 4th Edition, Wiley, 2005
6. Sundararajamoorthy T. V. Shanmugam .N, "Machine Design", Anuradha Publications,

Course Outcome Articulation Matrix

	Program Outcome												***PSO***		
Course Code / CO No	***1***	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***	***10***	***11***	***12***	***1***	***2***	***3***
ME8097 / C426.1	3	2	0	3	0	0	0	0	2	3	0	3	3	3	0
ME8097 / C426.2	3	3	0	3	3	0	0	0	2	3	0	3	3	3	0
ME8097 / C426.3	3	3	0	3	3	0	0	0	2	3	0	3	3	3	0
ME8097 / C426.4	3	3	0	3	3	0	0	0	2	3	0	3	3	3	0
ME8097 / C426.5	3	3	0	3	3	0	0	0	2	3	0	3	3	3	0
Average	3	3	0	3	2	0	0	0	2	3	0	3	3	3	0

⇒ Type ~~tion~~ to the Design Process

Design:

A plan or drawing Produced to show the Look and function or working of a building garment or other object before it is made.

(or)

Design is the creation of plan (or) convention for the construction of an object or a system.

Machine Design.

Machine design is defined as the use of scientific Principles, technical information and imagination in the description of a machine (or) a mechanical ~~type~~ system to perform specific functions with Maximum economy and Efficiency.

New ideas (or) Designers Ideas } → Design Process → Real Product.

⇒ Types of Design:

1. Adaptive Design
2. Developed Design.
3. New Design.

1. Adaptive Design:

1. Adaptive Design.

Adaptation of existing designs with Minor Modification.

2. Developed design

Designer Starts from an existing Method.

3. New Design

Find outcome may differ

⇒ Design Process

Recongnition of Need

↓

Definition of Problem

↓

Synthesis

↓

Analysis and optimization

↓

Evaluation

↓

~~Presentation~~

Presentation

* **Recognition of Need:**

The First phase of activity which refers to the identification of demand for any design (or) Modification of any existing design.

* **Definition of Problem**

It is nothing ~~by~~ but specifying the actual requirement and Present stage of the Problem.

* **Synthesis: and Analysis**

Some group of activities ~~is carried out on different models by changing~~ like model formation, applying physical and technical Principles, computation and checking the result etc.

In this phase, a small model (Prototype) is formed and it is tested under various working conditions and the results are gathered for making final design.

* **Optimization:**

This is the next step in which the above analysis is carried out on different models by changing the variable like Material, loads, Methods, operating temp etc.

*** Evaluation:**

Evaluation is a significant phase of the total design process. Evaluation is the final proof of the ~~succes~~ successful design and usually involves the testing of a prototype in the laboratory.

*** Presentation:**

The proper communication of design details like correct dimensions, machining process, tolerance details and other working condition.

⇒ Factors Influencing Machine Design:

* Type of Loading
* Size and shape of the object.
* Material Properties required
* Environmental conditions
* place of usage
* Human safety
* cost
* Service Life
* Appearance
* Quantity Required
* Handling Provisions

workshop facilities and Mfg methods.

* Type of Loading

-> Steady Load -> dynamic Load -> Impact Load

* Size and shape of the Object.

-> small or big size, simple or intricate shape

* Material Properties.

Such as hard, soft, rigid, transparent opaque, conductive, ductile and brittle.

* Environmental condition.

Such that the components is to be operated in corrosive (or) Non-corrisive atmosphere, cool or hot conditions etc.

* Place of usage:

Such that the machine is employed in land or ~~waste~~ on water or air (ie space) etc.

* Human safety

For which the parts should have provisions for safe handling and easy maintenance etc.

* Cost

Cost is the another predominant factor for which machine component should be designed.

* Service ~~Factor~~ Life:

For Long service Life the machine part should be very strong and for service Life, comparatively less strong item is sufficient.

* Appearance

This factor in ~~so~~ mainly meant for fast sales Promotion.

* Quantity Required

This factor is decided by the Material and place for usage.

* Handling Provisions:

Any component sholld be designed such that it must have some Provisions for easy handling during shifting from one places and ~~Proper Methods~~ to another.

* Workshop Facilities and Manufacturing Method:

Unless there are suitable workshops in the nearb places and Proper Manufacturing method.

⇒ Selection of Material based on Mechanical Properties

* Strength
* Hardness
* Toughness
* Ductility
* Malleability
* Elasticity
* Plasticity
* brittleness
* stiffness
* Creep
* Fatigue.
* Resilience

* Strength

Ability of Material to applied load without failure.

* Hardness

Ability of a material to reach abraion.

* Toughness

Ability of Material to reach Shock Loads.

* Ductility

Property of Material which enable is to be drawn into thin wires.

* Malleability

Property by which Material can be rolled in to thin sheets.

* Elasticity

If any Material specimen is Loaded, its dimension changes, when the Load is removed, the specimen gets itital shape.

* plasticity

Plasticity is the property of a material which retains the deformation produced under Load permanently.

* brittleness

It is the Property of a material opposite to ductility.

The Property of breaking of a material with little permanent distortion.

* Stiffness

Load to the deflection.

* Creep When a part is subjected to a constant stress ~~variable loads~~, at high temp for a long period of time, it will undergo a slow and permanent deformation is called Creep

* Fatigue: (or) elastic limit

When any machine element subjected to variable loads, it may fail before the stipulated time which has been calculated by treating the machine element under static or steady Load and that failure due to variable load is called fatigue failure.

* Resilience

In the ability of Material to resist absorb energy and to resist shock and Impact Load.

⇒ <u>Fits and Tolerances:</u>

~~The~~

* <u>Fits</u>

The degree of tightness or looseness b/w two mating parts known as the fit of the Part.

This fits can be classified as running [(or)sliding] fit, drive fit, force fit, and shrink fit.

* Tolerance

The total Permitted variation in a Partick dimension of a single part in termed as the tolerance.

* Preffered Numbers

When a machine in to be made in several size having different power (or) Capacit If in necessory to decide what capacities will cover a certain range efficiently with min Number of sizes.

⇒ <u>Direct Stress:</u> in Machine Part

* <u>Load:</u>

It is defined as any external force acting upon a machine part

* <u>Types of Load</u>

→ Dead or steady Load

When it doesnot change in

16

28 magnitdide ~~or~~ ~~and~~ (or) direction.

11

24 → Live (or) Variable Load

19

A Load is said to be a Live or variable load, when it changes continually.

⇒ Suddenly applied or shock Load

When it is suddenly applied (or) removed.

→ Impact Load

when it is applied some ~~impact~~ initial velocity.

→ Stress (σ)

When some external system of forces or Loads act on a body, the internal force (equal and opposite] are set up at various sections of the body, which resist the External force. This internal ~~force~~ force per unit area at any section of the body is known as unit stress (or) simply a stress. (σ)

$$\text{Stress } \sigma = \frac{P}{A}$$

P = Force (or) Load acting on a body

A = Cross-sectional area of the body

$$1 \text{ Pa} = 1 \text{ N/m}^2$$

$$1 \text{ MPa} = 1\times10^6 \text{ N/m}^2 = 1 \text{ N/mm}^2$$

$$1 \text{ GPa} = 1\times10^9 \text{ N/m}^2 = 1\times10^3 \text{ N/mm}^2$$

→ Strain (ε)

When a ~~force~~ system of forces or Loads act on a body, it undergoes some deformation. This deformation per unit length is known as unit strain (or) simply a strain.

$$\varepsilon = \frac{\text{change in Length}}{\text{original Length.}}$$

$$\varepsilon = \frac{\delta l}{l}$$

→ compressive stress

$$\sigma_c = P/A$$

→ compressive strain

$$\varepsilon_t = \frac{\delta l}{l}$$

→ Tensile stress

$$\sigma_t = P/A$$

→ Tensile strain

$$\varepsilon_t = \frac{\delta l}{A}$$

→ Young's Modulus (or) Modulus of Elasticity

'Hooke's Law', States that when a material is Loaded within elastic Limit, the stress is directly Propositional to strain

$$\sigma \propto \varepsilon$$

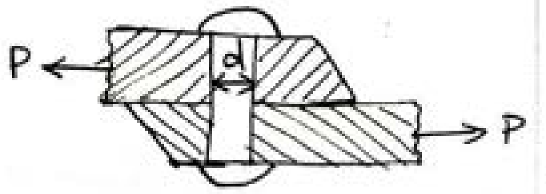
$$\sigma = E \times \varepsilon$$

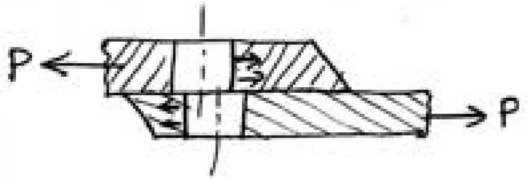
$$E = \frac{\sigma}{\varepsilon} = \frac{P/A}{\delta l / l}$$

$$\boxed{E = \frac{P l}{A \times \delta l}}$$

→ Shear stress and strain

When a body is subjected to two equal and opposite force acting tangentially accross the resisting section, as a result of which the body tends to shear off the section, then the stress induced is called shear stress.





The corresponding strain is known as shear strain and it is measured by the angular deformat accompanying the shear stress.

$$\text{Shear stress } \tau = \frac{\text{Tangential force}}{\text{Resisting Area}}$$

$A = \pi/_4 \times d^2$ [Area of hole]

$$\tau = \frac{P}{A} = \frac{P}{\pi/_4 \times d^2} = \frac{4P}{\pi d^2}$$

Double shear stress

$$\tau = \frac{P}{2 \times A} = \frac{P}{2 \times \frac{\pi}{4} \times d^2} = \frac{2P}{\pi d^2}$$

$A = \pi d t$ [Area of plate]

→ Shear Modulus (or) Modulus of Rigidity

It has bean found experimentally that with in the ~~to~~ elastic limit, the shear stress is directly Proportional to shear strain

$$\tau \propto \phi$$

$$\tau = C\phi \qquad \phi = \text{shear strain.}$$

$$C = \frac{\tau}{\phi}$$

→ Working stress

when designing machine parts, it is describle to keep the stress Lower than the maximum or ultimate stress at which failure of the material take place. This stress known as working stress (or) Design stress.

→ Factor of safety.

As the ratio of the maximum stress to the working stress.

$$FOS = \frac{\text{Maximum stress}}{\text{Working stress.}}$$

→ Torsional shear stress

When a machine member is subjected to the action of two equal and opposite couples acting in parallel planes (or torque or twisting moment), then the machine ~~me~~ member is said to be subjected to torsion. The stress set up by torsion is known as torsional shear stress

$$\boxed{\frac{T}{J} = \frac{C\theta}{l} = \frac{\tau}{r}}$$

$J = \pi/_{32} \times D^4$

τ = Torsional shear stress $\qquad T_{max} = \pi/_{16} \times \tau d^3$

r = Radius of shaft $\qquad P = \frac{2\pi NT}{60}$

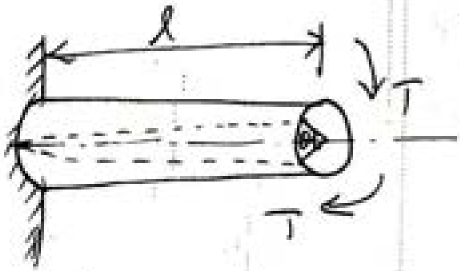
J = Polar moment of inertia

C = Modulus of Rigidity

l = Length of the shaft

T = Torque or twisting Moment

θ = Angle of twist in radius on a length l



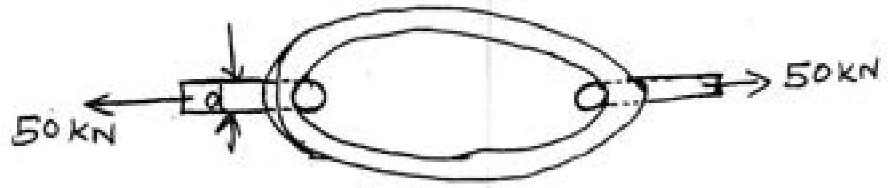
→ Bending stress

The machine part of structural members may be subjected to static or dynamic Load which cause bending stress in the sections besides other types of stresses such as tensile, compressive and shearing stresses.

$$\frac{M}{I} = \frac{\sigma_b}{y} = \frac{E}{R}$$

Problem based on Direct stress, bending stress & Torsional stress:

1. A coil chain of a crane required to carr a maximum load of 50KN is shown in fig.



Find the diameter of the link stock, if the permiss tensile stress in the link material is not exceed 75MPa

Given:

Load $P = 50\text{ KN} = 50\times10^3\text{ N}$

tensile stress $\sigma_t = 75\text{ MPa} = 75\times10^6\text{ N/m}^2$

$$= \frac{75\times10^6}{(10^3)^2}$$

$$\sigma_t = 75\text{ N/mm}^2$$

To find

diameter of the Link d

Soln

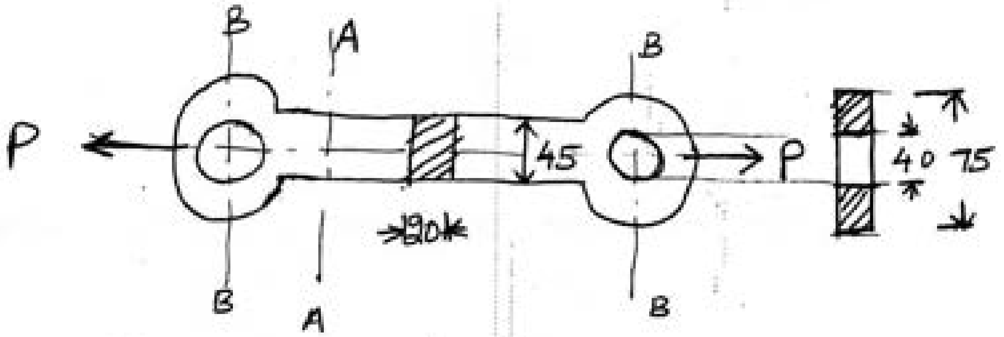
Stress $\sigma = \frac{P}{A}$

$$P = \sigma \times A$$

$$50\times10^3 = 75 \times \pi/4 \times d^2$$

$\boxed{d = 29.13\text{ mm}}$ say 30 mm.

2. A cast iron link, as shown in Fig ~~4.4~~ is required to transmit a steady tensile load of 45 kN. Find the tensile stress induced in the link material at sections A-A and B-B.



315, 9, 10, 11, 15, 27, 304, 305, 307.

Given:

Tensile Load $P = 45\text{ kN} = 45\times10^3\text{ N}$

TOFind

Tensile stress at A-A

Tensile stress at B-B

Soln

Tensile stress at section A-A

$$\sigma_t = \frac{P}{A_1} =$$

A_1: Area at section A-A

$A_1 = 20\times45$ [Rectangle section bd]

$$\boxed{A_1 = 900\text{ mm}^2}$$

$$\sigma_t = \frac{45\times10^3}{900}$$

$$\boxed{\sigma_t = 50\text{ N/mm}^2}$$

Tensile stress induced at section B-B

$$\sigma_t = \frac{P}{A_2}$$

A_2 = Area of section BB

$A_2 = 20 \times [75 - 40]$

$\boxed{\cancel{A_2 = 64.3\ N/mm^2}}$ $A_2 = 700\,mm^2$

$$\sigma_t = \frac{45 \times 10^3}{700}$$

$$\boxed{\sigma_t = 64.3\ N/mm^2}$$

3) A hydraulic press exerts a total Load of 3.5 MN. This Load is carried by two steel rods, supporting a upper head of the press. If the safe stress is 85 Mpa and $E = 210\ kN/mm^2$ Find 1. diameter of the rods, and 2. extension in each rod in a length of 2.5m.

Given:

Load $P = 3.5\,MN = 3.5 \times 10^6\ N$

Stress $\sigma = 85\,MPa = 85 \times 10^6\ N/m^2$

$\sigma = 85\ N/mm^2$

Young's Modulus $E = 210 \times 10^3\ N/mm^2$

Length of each rod $L = 2.5m.$

To Find:

1. Diameter of the rods d

2. Extension in each rod δl

Soln

1. Diameter of the rod d

If $\sigma_t = \frac{P_1}{A}$

P_1 = Load carried by each rod.

$$P_1 = \frac{P}{2} = \frac{3.5\times10^6}{2}$$

$A = \pi/4 \times d^2$

$$P_1 = 1.75\times10^6\ N$$

$$\sigma_t = \frac{1.75\times10^6}{\pi/4\times d^2}$$

~~1.75×10~~

$$85 = \frac{1.75\times10^6}{\pi/4\times d^2}$$

$$d^2 = 26213$$

Ans $\boxed{d = 162\ mm}$

2. Extension in each rod δl

$$\delta l = \frac{Pl}{AE}$$

$$\delta l = \frac{\sigma\times l}{E} = \frac{85\times2.5\times10^3}{210\times10^3}$$

Ans $\boxed{\delta l = 1.012\ mm}$

4. A Cantilever of span 500mm carries a vertical downward load of 6 kN at its free end. Assume yield value of 350 MPa and Factor of safety as 3. Find the Economical section for Cantilever among (19)

a. circular cross-section of diameter 'd'

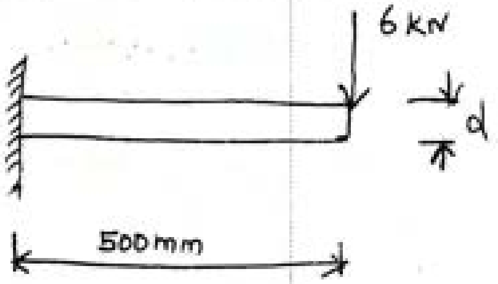
b. Rectangular cross section of depth 'h' and width t with $h/t = 2$

c. I section section of depth 7t and flange width 5t where t is the thickn

Specify the dimension and cross sectionc area of the economical section.

Soln

1. circular cross-section.



Given

$P = 6\,kN$ $l = 500\,mm$

To find:

Area.

Load: $P = 6\,kN = 6\times10^3\,N$

Length $l = 500\,mm$

Yield stress $\sigma_y = 350\,MPa = 350\,N/mm^2$

FoS $n = 3$

For ~~bed~~ bending Eqn

$$\boxed{\frac{M}{I} = \frac{\sigma_b}{y} = \frac{E}{R}}$$

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

M: bending Moment = Load x Distance = $\cancel{6}\times P \times l$

$M = 6\times10^3 \times 500$

$$\boxed{M = 3\times10^6 \text{ N.mm}}$$

I = Moment of inertia = $\pi/64 \times d^4$

y = Distance of extrame fibre from neutral Axis

$$y = \frac{d}{2}$$

$$\sigma_b = \frac{\sigma_y}{3}$$

$$\sigma_b = \frac{\cancel{3\times10}\ 350}{3}$$

$$\sigma_b = 116.7 \text{ N/mm}^2$$

$$\frac{M}{I} = \frac{\sigma_b}{y} >$$

$$\frac{3\times10^6}{\pi/64 \times d^4} = \frac{116.7}{d/2}$$

$$\frac{3\times10^6 \times 64}{\pi \times d^{\cancel{4}3}} = \frac{116.7\times2}{\cancel{d}}$$

$$\frac{61.115\times10^6}{d^3} = 116.7\times2$$

$$\frac{61.115\times10^6}{116.7\times2} = d^3$$

$$d^3 = 261848.74$$

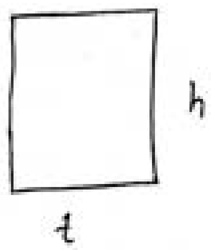
$$d = [261848.74]^{1/3}$$

$$\boxed{d = 64\,mm}$$

$$A = \pi/4 \times d^2 = \pi/4 \times 64^2$$

$$\boxed{A = 3217\ mm^2}$$

b. For Rectangular section of depth h and width t



Area = $t\times h$

$h/t = 2$. $h = 2t$

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

$$I = \frac{bd^3}{12} \quad \text{[For Rectangular Section]}$$

$$I = \frac{t\times h^3}{12} = \frac{t\times(2t)^3}{12} = \frac{8t^4}{12}$$

$$y = \frac{h}{2} = \frac{\cancel{2}t}{\cancel{2}}$$

$$\boxed{y = t}$$

$$\frac{3\times10^6}{\frac{8t^4}{12}} = \frac{116.7\times}{t}$$

$$\frac{3\times10^6\times12}{8t\cancel{4}^3} = \frac{116.7\cancel{t}}{\cancel{t}}$$

$$\frac{4.5\times10^6}{t^3} = 116.7$$

$$t^3 = \frac{4.5\times10^6}{116.7}$$

$$t^3 = 38560.41$$

$$\boxed{t = 33.78\ mm}$$

$$h = 2t = 2\times 33.78$$

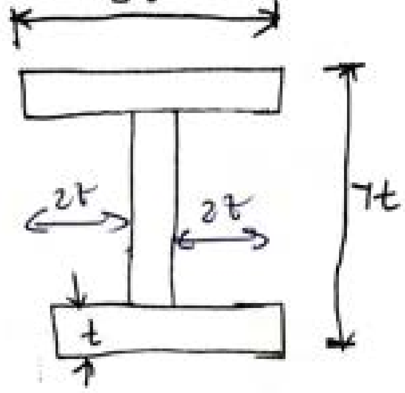
$$\boxed{h = 67.56\ mm}$$

$$A = \cancel{b}h\times t = 67.56\times 33.78$$

$$\boxed{A = 2282.46\ mm}$$

$b_1 = 5t$
$h = 7t$

c. For I section



$$\frac{M}{I} = \frac{\sigma_b}{y}$$

$I = I_1 - I_2$

$I_1 = \frac{bh^3}{12}$

$I_2 = \frac{b_1 h_1^3}{12}$

$M = 3\times10^6\ N/mm^2$

$I = \frac{bd^3}{12}$ [General Formula]

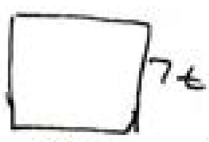
$$I = \frac{1}{12}\left[5t\times(7t)^3 - 4t\times(5t)^3\right]$$

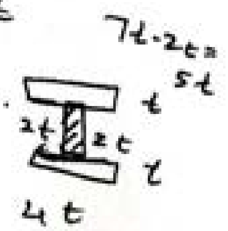
$$I = \frac{1715t^4 - 500t^4}{12}$$

$$I = \frac{1215t^4}{12}$$

$b_1 = 2t+2t$
$= 4t$

$h_1 = 7t-t-t$
$= 5t$





$y = \frac{7t}{2} = 3.5t$

$\frac{M}{I} = \frac{\sigma_b}{y}$

$\frac{3\times10^6}{\frac{1215t^4}{12}} = \frac{116.7}{3.5t}$

$\frac{3\times10^6\times12}{1215t^{\cancel{4}3}} = \frac{116.7}{3.5\cancel{t}}$

$\frac{29.629\times10^3}{t^3} = 33.34$

$\frac{29.629\times10^3}{33.34} = t^3$

$t^3 = 888.61$

$\boxed{t = 9.61\ mm}$

Area $= [5t\times7t] - [4t\times5t]$

$= 35t^2 - 20t^2$

Area $= 15t^2 = 15\times9.61^2$

Ans $\boxed{\text{Area} = 1386.43\ mm^2}$

*:

$I = I_1 - I_2$

$I = \frac{bh^3}{12} - \frac{b_1h_1^3}{12}$

$I = \frac{1}{12}[bh^3 - b_1h_1^3]$

$I = \frac{1215}{12}t^4$

$I = 101.25t^4$

$y = 7t/2 = 3.5t$

⇒ Impact & shock Loading

Machine members are subjected to the load with impact. The stress Produced in the member due to the falling Load is known as impact stress.

Types of Impact Load:

which is goes on increasing over a period of time till the max. value reached. ← 1. Gradually Load

which is applied suddenly (or) with some initial velocity ← 2. Suddenly applied (or) impact or shock Load.

Consider a bar carrying a Load W at a height h and falling on the collar provided at the Lower end as shown in fig.

Let

A = Cross-sectional Area of the bar

E = Young's Modulus

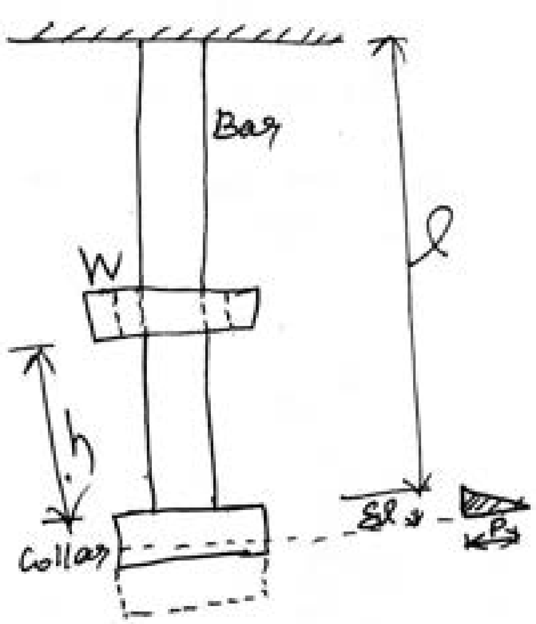
l = Length of the bar.

δl = Deformation of the bar

P = Force at which the deflection δl is Produced,

σ_i = stress induced in the bar due to impact Load.

h = Height through the load falls.



Kinetic ~~at~~ energy gained by the system of

Strain energy $= \frac{1}{2} \times P \times \delta l$ — Due to the falling load the energy is gained,

Potential energy lost by the weight — At the same time the weight 'W' falls down

$= W [h + \delta l]$ — ($\because$ Potential energy is lost)

Kinetic Energy = Potential Energy

$$\frac{1}{2} \times P \times \delta l = W [h + \delta l]$$

$$\sigma = \frac{P}{A} \Rightarrow P = \sigma_i \times A$$

$$\delta l = \frac{\sigma_i \times l}{E}$$

$$\sigma = e \times E \Rightarrow \frac{\delta l}{l} \times E$$

$$\frac{1}{2} \times \sigma_i \times A \times \frac{\sigma_i \times l}{E} = W [h + \delta l]$$

$$\frac{1}{2} \times \sigma_i \times A \times \frac{\sigma_i}{E} \times l = W \left[h + \frac{\sigma_i \times l}{E}\right]$$

$$\frac{Al}{2E} (\sigma_i)^2 = Wh + \frac{Wl}{E} \times \sigma_i$$

$$\frac{Al}{2E} (\sigma_i)^2 - Wh - \frac{Wl}{E} \times \sigma_i = 0$$

From this quadratic Eqn, we find ~~out~~ that

$$\boxed{\sigma_i = \frac{W}{A}\left[1 + \sqrt{1 + \frac{2hAE}{Wl}}\right]}$$

$ax^2 + bx + c = 0$

$x = \sigma_i$

ie, $$\boxed{\sigma_i = \sigma + \sigma\left[1 + \sqrt{\frac{2h}{e}}\right]}$$

$\sigma = W/A$

$\sigma/E = e$

Relation b/w suddenly applied and gradually applied Load

Putting $h = 0$

$$\boxed{\sigma_i = \frac{2W}{A}}$$

When the same Load W is gradually applied the stress produced

$$\boxed{\sigma = \frac{W}{A}}$$

For this, it is clear that the stress due to suddenly load is double that of the gradually applied load

$$\sigma_i = 2\sigma$$

Problem

An Unknown weight falls through 10mm on a collar rigidly attached to the Lower end of the vertical bar 3 m long and $600 mm^2$ in section. If the maximum instantaneous extension is known to be 2mm, what is the corresponding stress and the value of unknown weight? Take $E = 200 kN/mm^2$

Given

height $h = 10mm$

Length $l = 3m = 3000 mm$

Area $A = 600 mm^2$

$\delta l = 2mm$

Young's Modulus $E = 200 kN/mm^2 = 200\times10^3 N/mm^2$

To Find:

1. Stress σ
2. weight W

Soln

1. Stress σ

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{\sigma}{\varepsilon}$$

$$\sigma = E \times \varepsilon$$

$$\sigma = E \times \frac{\delta l}{l} = \frac{200\times10^3 \times 2}{3000}$$

$$\boxed{\sigma = 133.3 N/mm^2}$$

2. Weight: (W)

W.K.T $\sigma = \frac{W}{A}\left[1 + \sqrt{1 + \frac{2hAE}{Wl}}\right]$

Potential Energy = kinetic Energy

$$W[h+\delta l] = \frac{1}{2} \times P \times \delta l$$

P = static Load = Stress × Area

$$P = 133.3 \times 600$$

$$P = 79.98 \times 10^3 \ N$$

$$W[10+2] = \frac{1}{2} \times 79.98 \times 10^3 \times 2$$

$$W = \frac{79980}{12}$$

$$\boxed{W = 6665 \ N}$$

2) An I section beam of depth 250mm is supported at two points 4m apart. It is loaded by a weight of 4KN falling through a height h and striking the beam at mid span. Moment of inertia of a section is $8 \times 10^7 \ mm^4$. Modulus of elasticity is $210 \ KN/mm^2$. Determine the permissible value of h if the stress is limited to $120 N/mm^2$

Given Data:

Length of beam $l = 4m = 4000 mm$

Depth of beam $d = 250mm$

Impact Load (falling) $W = 4KN = 4 \times 10^3 N$

Moment of inertia $I = 8 \times 10^7 \ mm^4$

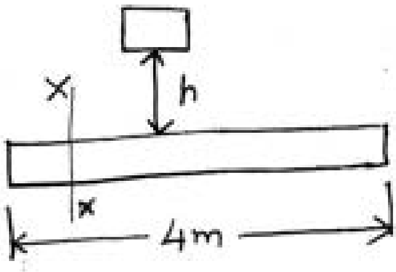
Modulus of elasticity $E = 210 \ KN/mm^2$

$E = 210 \times 10^3 \ N/mm^2$

Allowable bending stress $\sigma_b = \cancel{210} \ 120 \ N/mm^2$

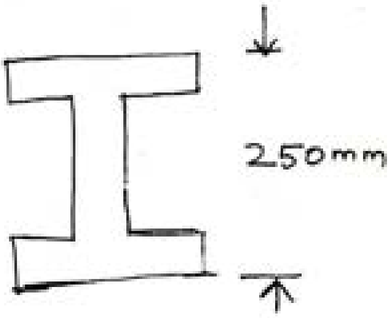
<u>To find</u>

Value of h



<u>Soln</u>

The first find out 'P'



From bending stress Eqn

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

$$\sigma_b = \frac{M \times y}{I}$$

The maximum bending moment $M = \frac{Pl}{4}$

$$\sigma_b = \frac{Pl \times y}{4I}$$

$$120 = \frac{P \times 4000 \times 125}{4 \times 8 \times 10^7}$$

$$\boxed{P = 76.8 \times 10^3 \ N}$$

$\sigma_b = 120 \ N/mm^2$

$l = 4000 \ mm$

$y = d/2 = \frac{250}{2} = 125 \ mm$

$I = 8 \times 10^7 \ mm^4$

Kinetic Energy = Potantial Energy.

$$\frac{1}{2} \times P \times \delta l = W [h + \delta l]$$

To find δl

$$\delta l = \frac{P l^3}{48 E I} = \frac{76.8 \times 10^3 \times 4000^3}{48 \times 210 \times 10^3 \times 8 \times 10^7}$$

$$\boxed{\delta l = 6 mm}$$

$$\frac{1}{2} \times 76.8 \times 10^3 \times 6 = 4 \times 10^3 [h + 6]$$

$$h + 6 = \frac{0.5 \times 76.8 \times 10^3 \times 6}{4 \times 10^3}$$

$$h + 6 = 57.6$$

$$h = 57.6 - 6$$

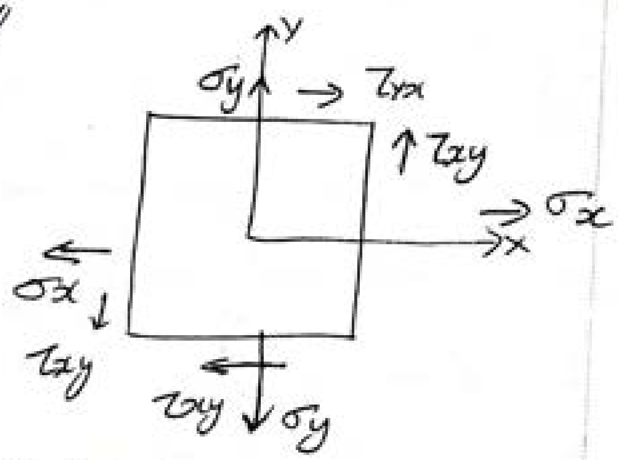
$$\boxed{h = 51.6 \ mm}$$

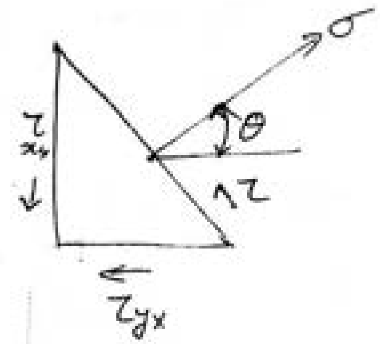
⇒ Principal planes and Principal stress

When a system of forces act on a body, all the particles of that body are disturbed and their dimensions and Locations are varied due to straining action by the Force.

At any strained particle, there are three planes mutually Perpendicular to each other which carry only normal stresses and no shear stress. These planes which are having only stress are called Principal planes and these normal stress are called Principal stresses.

Among these principal stress one is having maximum value and other is having minimum value. There are some planes which are 45° to Principal planes and carry only maximum shear stresses and hence known as shearplane.





<PSG DB 7.2>

Maximum Principal stress $\sigma_1 = \frac{\sigma_x+\sigma_y}{2} + \sqrt{\left[\frac{\sigma_x+\sigma_y}{2}\right]^2 + (\tau_{xy})^2}$

(or) $\sigma_1 = \frac{1}{2}\left[\sigma_x+\sigma_y\right] + \sqrt{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}$

Minimum Principal stress $\sigma_2 = \frac{\sigma_x+\sigma_y}{2} - \sqrt{\left[\frac{\sigma_x+\sigma_y}{2}\right]^2 + \left[\tau_{xy}\right]^2}$

(or) $\sigma_2 = \frac{1}{2}\left[\sigma_x+\sigma_y\right] - \sqrt{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}$

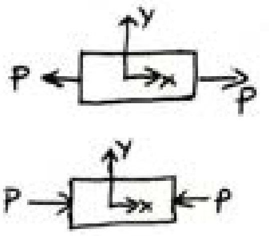
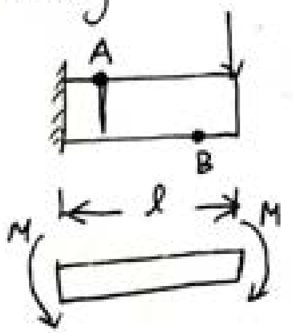
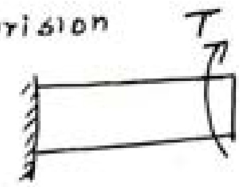
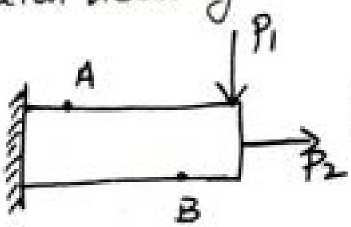
Maximum shear stress $\tau_{max} = \frac{1}{2}\sqrt{\left[\sigma_x-\sigma_y\right]^2+4\tau_{xy}^2}$

Location ~~of~~ 'θ_1':

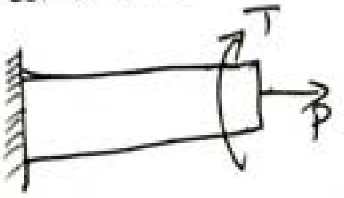
$$\tan 2\theta_1 = \frac{2\tau_{xy}}{\sigma_x-\sigma_y}.$$

$$\theta_2 = 90+\theta_1$$

→ Principal stresses for different loading condition (19)

SL NO	Loading	Stress[es] Produced	Manipulation for Principal stresses
1.	Axial 	Direct tensile stress σ_d.	(i) Use $\sigma_x = \sigma_d$ ~~is~~ $\sigma_y = 0$, $\tau_{xy} = 0$ (ii) $\sigma_x = -\sigma_d$ $\sigma_y = \tau_{xy} = 0$
2	Bending 	Bending stress, σ_b Tensile at point A, compressive at point B	(i) $\sigma_b = \frac{M_b}{Z}$ → section modulus of the cross section (ii) $\sigma_x = \sigma_b$ (point A) $\sigma_x = -\sigma_b$ (point B) (iii) Use $\sigma_y = \tau_{xy} = 0$
3	Torision 	shear stress τ_{xy}	(i) $\tau_{xy} = \frac{M_t \times r}{J}$ (ii) $\sigma_x = \sigma_y = 0$
4)	Axial & Bending 	(i) Direct tensile stress $\sigma_d = P_2$ ~~tr~~ (ii) Bending stress σ_b due to P Point A (σ_b is +) Point B (σ_b is −)	(i) σ_a is σ_b (ii) At point A $\sigma_x = \sigma_a + \sigma_b$ At point B $\sigma_x = \sigma_a - \sigma_b$

5 Axial & Torsion



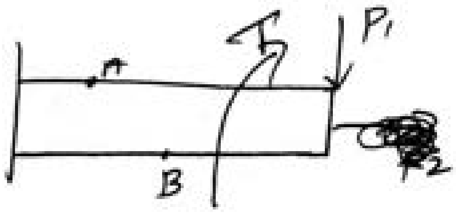
(i) Direct tensile stress σ_d due to P

(ii) Shear stress τ_{xy} due to Mt

(i) σ_d & τ_{xy} find

(ii) $\sigma_x = \sigma_d$, $\sigma_y = 0$

6. Bending & Torsion



(i) Bending stress σ_b due to P [At A it is tensile, B it is compressive]

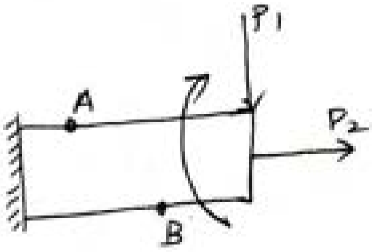
(i) find σ_b and τ_{xy}

(ii) At point A $\sigma_x = \sigma_b$

at point B $\sigma_x = -\sigma_b$

(iii) $\sigma_y = 0$

7. Axial bending and Torsion



(i) σ_d due to P_2 (Tensile)

(ii) σ_b due to P [Tensile at A, compressive at B]

(iii) τ_{xy} due to T

(i) At point A

$\sigma_x = \sigma_d + \sigma_b$ (both tensile)

(ii) τ_{xy} is Present

(iii) $\sigma_y = 0$

At point B

i) $\sigma_x = \sigma_d - \sigma_b$

[σ_b is compressive]

(ii) τ_{xy} is present

(iii) $\sigma_y = 0$

NOTE:

① Find Direct / Bending / shear stress as per the loading given.

② Algebraically add Direct stress (σ_d) & Bending stress (σ_b) gives σ_x.

$\sigma_x = \sigma_d + \sigma_b$

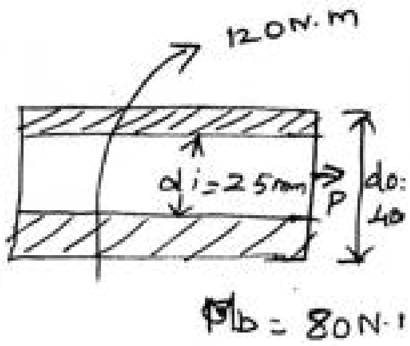
③ Treat τ_{xy} separately. DO NOT Added with σ_d (or) σ_b.

④ Use σ_x and τ_{xy} in the principal stress eqn.

Problem

1. A hollow shaft of 40mm outer diameter and 25 mm inner diameter is subjected to a twisting moment of 120 N.m, simultaneously, it is subjected to an axial thrust of 10 KN and a bending moment of 80 N.m Calculate the maximum compressive and shear stress

Given



outer dia of Shaft $d_o = 40mm$

Inner dia of shaft $d_i = 25mm$

Twisting Moment $T = 120 N.m = 120 \times 10^3 N.mm$

Axial Load $P = 10 KN = 10 \times 10^3 N$

Bending Moment $M_b = 80 N.m = 80 \times 10^3 N.mm$

To Find:

1. Maximum compressive stress (σ_1)
2. Shear stress τ_{xy}.

Soln

1. Maximum compressive stress 'σ_1'

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left[\frac{\sigma_x + \sigma_y}{2}\right]^2 + \tau_{xy}^2}$$

<DB.7.2>

or

$$\sigma_1 = \frac{1}{2}\left[\sigma_x + \sigma_y\right] + \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}\Big]$$

From table $\boxed{\sigma_x = \sigma_d + \sigma_b}$ Tensile.

$\boxed{\sigma_x = \sigma_d - \sigma_b}$ compressive

σ_d = Direct compressive stress. due to axial Load.

$$\boxed{\sigma_x = \sigma_d + \sigma_b}$$

$$\sigma_d = \frac{P}{A} = \frac{P}{\pi/4\,[d_o^2 - d_i^2]} \quad \langle 7.1 \rangle$$

$$\sigma_d = \frac{10\times10^3}{\pi/4\,[40^2 - 25^2]}$$

$$\boxed{\sigma_d = 13.05 \ N/mm^2}$$

σ_b = bending stress due to bending.

$$\cancel{\sigma_b} \quad \frac{M}{I} = \frac{\sigma_b}{y} \quad \langle 7.1 \rangle$$

$$\sigma_b = \frac{M}{I} \times y$$

Moment of inertia $I = \pi/64 \times [d_o^4 - d_i^4]$

$$I = \frac{\pi}{64} \times [\cancel{60}\,40^4 - 25^4]$$

$$\boxed{I = 106.48\times10^3 \ mm^4}$$

$$y = \frac{d_o}{2} = \frac{40}{2}$$

$\boxed{y = 20mm}$

$$\cancel{\sigma_b} = \frac{80 \times \cancel{10^3}}{106.48 \times 10^3} = \cancel{80}$$

$$\sigma_b = \frac{80 \times 10^3 \times 20}{106.48 \times 10^3}$$

$$\boxed{\sigma_b = 15.02\ N/mm^2}$$

$$\sigma_x = \sigma_d + \sigma_b = 13.05 + 15.02$$

$$\boxed{\sigma_x = 28.07\ N/mm^2}$$

$$\boxed{\sigma_y = 0}$$

To find τ_{xy}

From Twisting Moment Eqn <7.1>

$$\frac{M_t}{J} = \frac{C\theta}{l} = \frac{\tau}{r}$$

$$\frac{M_t}{J} = \frac{\tau}{r}$$

$$\tau = \frac{M_t}{J} \times r$$

$$\tau = \frac{120 \times 10^3}{\cancel{6.43 \times 10^{11}}\ 235.61 \times 10^3} \times 20$$

$$\boxed{\tau = 10.18\ N/mm^2}$$

J = Polar Moment of inertia

$$J = \pi/32 \times [d_o^4 - d_i^4]$$

$$J = \pi/32 \times [40^4 - 25^4]$$

$$J = \cancel{6.43 \times 10^{4}}\ 235.61 \times 10^3\ mm^4$$

$$r = \frac{d_o}{2} = \frac{40}{2} = 20mm$$

$$M_t = 120 \times 10^3\ N.m$$

$$\boxed{\tau_{xy} = 10.18\ N/mm^2}$$

$$\sigma_1 = \cancel{\sigma}\ \frac{1}{2}\left\{[\sigma_x + \sigma_y] + \sqrt{[\sigma_x - \sigma_y]^2 + 4\tau_{xy}^2}\right\}$$

$$\sigma_1 = \frac{1}{2}\left\{\left[28.07 + 0\right] + \sqrt{\left[28.07 - 0\right]^2 + 4 \times (10.18)^2}\right\}$$

$$\sigma_1 = \frac{1}{2} \times \left\{[28.07] + [34.676]\right\}$$

Ans $\boxed{\sigma_1 = 31.37\ N/mm^2}$

Maximum shear stress τ_{max}

$$\tau_{max} = \frac{1}{2}\sqrt{[\sigma_x - \sigma_y]^2 + 4\tau_{xy}^2}$$

$$= \frac{1}{2}\sqrt{[28.07 - 0]^2 + 4 \times (10.18)^2}$$

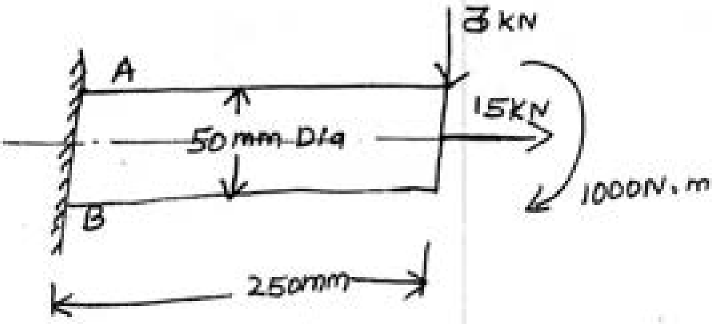
Ans $\boxed{\tau_{max} = \overset{17.99}{\cancel{10.51}}\ N/mm^2}$

Result.

$\sigma_1 = 31.37\ N/mm^2$

$\tau_{max} = 10.51\ N/mm^2$

2. A shaft as shown in Fig. is subjected to a bending Load of 3 KN, Pure torque of 1000 N.m and an axial pulling force of 15 KN. Calulate the stress at A and B



Given

Bending Load ~~Distance~~ = $P_b = 3\,KN = 3\times10^3\,N$

Torque (or) Twisting Moment $M_t = 1000\,N.m$
$= 1000\times10^3\,N.mm$

Axial Load $P = 15\,kN = 15\times10^3\,N$

Diameter of shaft $d = 50\,mm$

Length of shaft $l = 250\,mm$

To Find:

The Principal stresses at A & B

Soln

At point A

Maximum Principal stress $\sigma_1 = \frac{1}{2}\left\{(\sigma_x+\sigma_y) + \sqrt{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}\right\}$

both

$\sigma_x = \sigma_d + \sigma_b$ [Tensile]

σ_d: direct stress due to Axial Load

<7.1> $\sigma_d = \frac{P}{A} = \frac{15\times10^3}{\pi/4 \times d^2} = \frac{15\times10^3}{\pi/4 \times (50)^2}$

$\boxed{\sigma_d = 7.64\ N/mm^2}$

σ_b: bending stress due to bending

<7.1> $\frac{M_b}{I} = \frac{\sigma_b}{y}$

$\sigma_b = \frac{M_b}{I} \times y$

$\sigma_b = \frac{750\times10^3}{306.796\times10^3} \times 25$

$\boxed{\sigma_b = 61.11\ N/mm^2}$

I: MOI

$I = \pi/64 \times d^4 = \pi/64 \times 50^4$

$I = 306.796\times10^3\ mm^4$

$y = d/2 = \frac{50}{2} = 25\ mm$

M_b = Load x Distance

$M_b = 3\times10^3 \times 250 = 750\times10^3\ N.mm$

$\sigma_x = \sigma_d + \sigma_b = 7.64 + 61.11$

$\boxed{\sigma_x = 68.75\ N/mm^2}$

$\boxed{\sigma_y = 0}$

τ_{xy} = shear stress

$\tau_{xy} = \frac{M_t \times r}{J}$ <7.1>

$\tau_{xy} = \frac{1000\times10^3 \times 25}{613.592\times10^3}$

* $\boxed{\tau_{xy} = 40.74\ N/mm^2}$

$r = d/2 = \frac{50}{2} = 25mm$

J = PMOI

$J = \pi/32 \times d^4$

$J = \pi/32 \times 50^4$

$J = 613.592 \times10^3\ mm^4$

$$\sigma_1 = \frac{1}{2}\left[(\sigma_x+\sigma_y) + \sqrt{(\sigma_x-\sigma_y)^2 + 4\tau_{xy}^2}\right] \quad \langle 7.2 \rangle$$ (23)

$$= \frac{1}{2}\left[(68.75+0) + \sqrt{(68.75-0)^2 + 4\times(40.74)^2}\right]$$

$$= \frac{1}{2}\left[(68.75) + 106.60\right]$$

$$\boxed{\sigma_1 = 87.67\ N/mm^2}$$

$$\sigma_2 = \frac{1}{2}\left[(\sigma_x+\sigma_y) - \sqrt{(\sigma_x-\sigma_y)^2 + 4\tau_{xy}^2}\right] \quad \langle 7.3 \rangle$$

$$= \frac{1}{2}\left[(68.75+0) - \sqrt{(68.75-0)^2 + 4\times(40.74)^2}\right]$$

$$= \frac{1}{2}\left[68.75 - 106.60\right]$$

$$\boxed{\sigma_2 = -18.92\ N/mm^2}$$ compressive.

~~τ_{max}~~

Maximum shear stress τ_{max}

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_x-\sigma_y)^2 + 4\tau_{xy}^2}$$

$$= \frac{1}{2}\times\sqrt{(68.75-0)^2 + 4\times(40.74)^2}$$

$$= \frac{1}{2}\times\sqrt{(68.75)^2 + 4(40.74)^2}$$

$$= \frac{1}{2}\times 106.60$$

$$\boxed{\tau_{max} = 53.30\ N/mm^2}$$

At point B

$\sigma_x = \sigma_d - \sigma_b$ [is combressive]

$\sigma_x = 7.64 - 61.11$

$$\boxed{\sigma_x = -53.47\ N/mm^2}$$

$\sigma_y = 0$

$\tau_{xy} = 40.74\ N/mm^2$ [Previous value]

$$\sigma_1 = \frac{1}{2}\left[[-53.47+0] + \sqrt{[-53.47-0]^2 + 4\times(40.74)^2}\right]$$

$$\sigma_1 = \frac{1}{2}\left[-53.47 + 97.45\right]$$

$$\boxed{\sigma_1 = 21.99\ N/mm^2}$$

$$\sigma_2 = \frac{1}{2}\left[(\sigma_x + \sigma_y) - \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}\right]$$

$$\sigma_2 = \frac{1}{2}\left[(-53.47) - \sqrt{[-53.47-0]^2 + 4(40.74)^2}\right]$$

$$\sigma_2 = \frac{1}{2}\times\left[(-53.47) - 97.45\right]$$

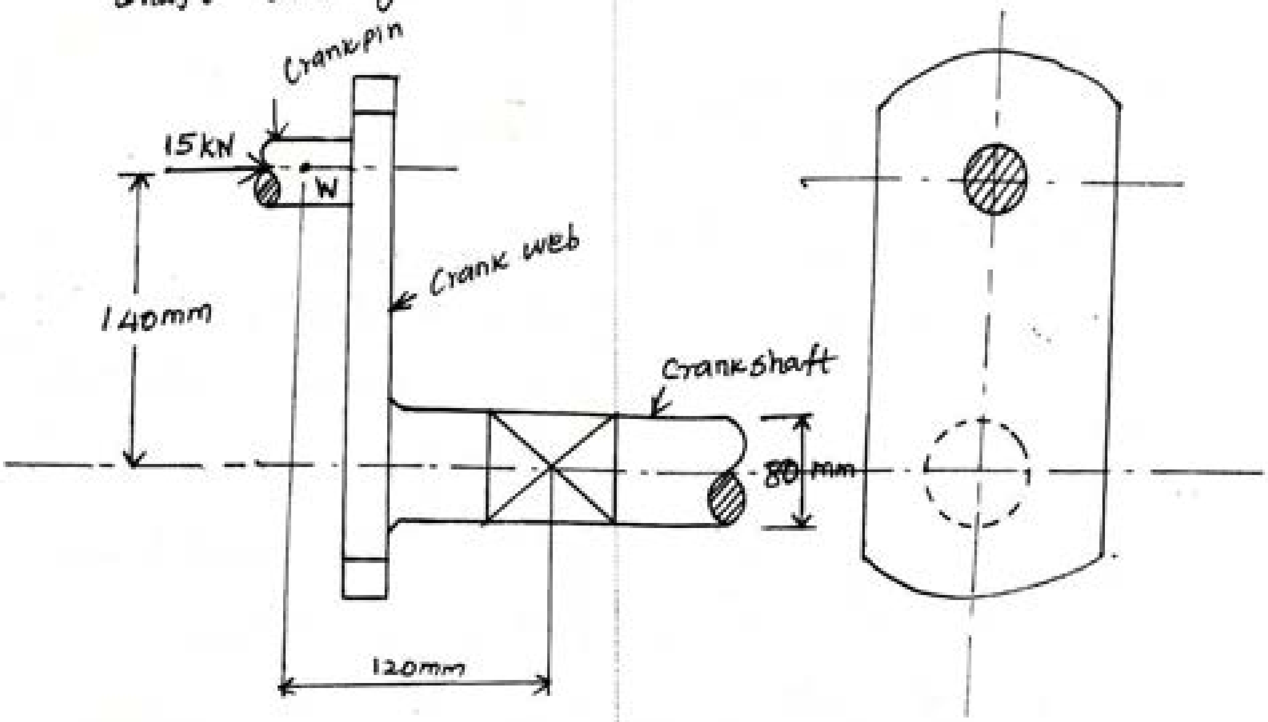
$$\boxed{\sigma_2 = -75.46}\ N/mm^2 \text{ compressive}$$

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

$$= \frac{1}{2}\sqrt{(-53.47-0)^2 + 4\times(40.74)^2}$$

$$\boxed{\tau_{max} = 48.72\ N/mm^2}$$

3. An overhang crank pin and shaft is shown in fig. (24)
A tangential Load of 15 kN acts on the crank pin. Determine the maximum principal stress and the maximum shear stress at the centre of the crank shaft bearing.



Given data:

Tangential ~~Axial~~ Load $P = 15\,kN = 15 \times 10^3\,N$

diameter $d = 80\,mm$

Vertical height $h = 140\,mm$

Length (horizontal) $l = 120\,mm$

To Find:

1. Maximum Principal stress σ_1
2. Maximum shear stress τ_{max}.

Soln

1. Maximum Principal stress σ_1 ⟨7.2⟩

$$\sigma_1 = \frac{1}{2}\left[(\sigma_x+\sigma_y) + \sqrt{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}\right]$$

$\sigma_x = \sigma_d + \sigma_b$

$\boxed{\sigma_d = 0}$

$\sigma_x = \sigma_b$

$\sigma_b = \frac{M_b \times y}{I}$ ⟨7.1⟩

M_b = Bending Moment at the centre of crankshaft bearing

$M_b = P \times l = 15\times10^3 \times 120$

$M_b = 1.8\times10^6$ N.mm

$y = d/2 = \frac{80}{2} = 40$mm

$I = \pi/64 \times d^4 = \pi/64 \times 80^4$

$I = 2.01\times10^6$ mm4

$\sigma_b = \frac{1.8\times10^6 \times 40}{2.01\times10^6}$

$\sigma_b = 35.82$ N.mm

$\boxed{\sigma_x = \sigma_b = 35.82 \text{N.mm}}$

$\boxed{\sigma_y = 0}$

τ_{xy} = shear stress due to torque transmitted

M_t = torque transmitted to the Axial of the shaft

$M_t = P \times h = 15\times10^3 \times 140$

$M_t = 2.1\times10^6$ N.mm

$r = d/2 = 80/2 = 40$mm

$J = \pi/32 \times d^4 = \pi/32 \times (80)^4$

$J = 4.02\times10^6$ mm4

$\tau_{xy} = \frac{M_t \times r}{J}$

$= \frac{2.1\times10^6 \times 40}{4.02\times10^6}$

$\boxed{\tau_{xy} = 20.89 \text{ N/mm}^2}$

$$\sigma_1 = \frac{1}{2}\left[(35.82+0) + \sqrt{(35.82-0)^2 + 4\times(20.89)^2}\right.$$

$$\sigma_1 = \frac{1}{2}\{35.82 + 55.033\}$$

Ans $\boxed{\sigma_1 = 45.42 \ N/mm^2}$

Maximum shear stress τ_{max} <7.2>

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

$$\tau_{max} = \frac{1}{2}\times\sqrt{[35.82-0]^2 + 4\times(20.89)^2}$$

$$\tau_{max} = \frac{1}{2}\times 55.033$$

Ans $\boxed{\tau_{max} = 27.51 \ N/mm^2}$

4. The stress ~~due to~~ state in a machine member is given as follows. $\sigma_x = 20$ Mpa, $\sigma_y = 7$ Mpa, $\tau_{xy} = 4$ Mpa, Find the Principal normal and shear stresses. Locate the angle of σ_1 and σ_2 from x axis

H.W

⇒ Theories of Failure: <7.3>

A given machine member may fail [ie, it will no longer be able to perform its intended function] due to various reasons in various ~~methods~~ modes.

It necessary to known the various conditions of failure of machine members. some failure theores as follows.

(I) Maximum Principal Stress [Rankine theory]

Normal stress on the m/e member ← σ_1 or σ_2 or σ_3 [which is Maximum] $= \sigma_y$

For design Purpose $\boxed{\sigma_1 = \sigma_y}$

✲ σ_1 or σ_2 or σ_3 [Which is Maximum] $= \frac{\sigma_y}{n}$ For ductile Material

σ_1 or σ_2 or σ_3 [Which is Maximum] $= \frac{\sigma_u}{n}$ For brittle Material

→ Factor of Safety

n = FOS

(ii) Maximum shear theory [Guest's or Couloumb's]

✲ $(\sigma_1 - \sigma_2)$ (or) $(\sigma_2 - \sigma_3)$ (or) $(\sigma_3 - \sigma_1) = \frac{\sigma_y}{n}$

(iii) Maximum Strain Theory (or) [St Venant's theory]

$$\sigma_1 - \nu(\sigma_2+\sigma_3) \text{ (or) } \sigma_2 - \nu[\sigma_3+\sigma_1] \text{ (or) } \sigma_3 - \nu[\sigma_1+\sigma_2] = \frac{\sigma}{1}$$

ν = poisson's ratio

(iv) Maximum Strain Energy Theory :

$$\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu[\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1] = \left(\frac{\sigma_y}{n}\right)^2$$

(v) Octahedral (or) Distortion Energy Theory [Von Mises - Hencky]

$$\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1\sigma_2 - \sigma_2\sigma_3 - \sigma_3\sigma_1 = \frac{\sigma_y^2}{n}$$

ν = Poisson's Ratio

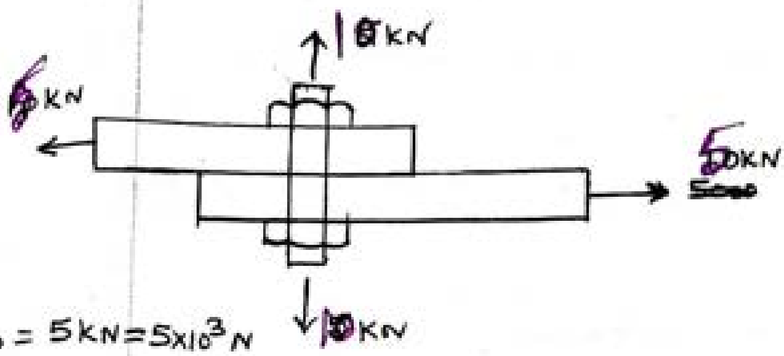
Problem 1

1. The Load on a bolt consists of an axial pull of 10 kN together with a transverse shear force of 5 kN. Find the diameter of bolt required according to.

   1. Maximum Principal Stress Theory
   2. Maximum shear stress Theory
   3. Maximum Principal strain theory
   4. Maximum Strain energy Theory
   5. Maximum distortion energy Theory

Take Permissible tensile stress at elastic limit = 100 MPa and poisson's ratio = 0.3

Given data:



Axial tensile load

$P_t = 10KN = 10\times10^3 N$

Transverse ~~Axial~~ shear force $P_s = 5KN = 5\times10^3 N$

Permissible tensile stress $\sigma_t = 100MPa = 100\times10^6 N/m^2 = \frac{100\times10^6}{(10^3)^2} N/mm^2$

at elastic limit $\sigma_t = 100 N/mm^2$

Poisson's Ratio ν or $1/m = 0.3$

To find:

Diameter of shaft 'd'

Soln

1. Maximum Principal stress Theory <7.1>

$$\sigma_t = \frac{\sigma_x+\sigma_y}{2} +$$

$$\sigma_{t_1} = \frac{1}{2}\left[(\sigma_x+\sigma_y) + \sqrt{(\sigma_x-\sigma_y)^2 + 4\tau_{xy}^2}\right] \langle 7.1 \rangle$$

An Axial tensile stress $\sigma_x = \sigma_1$

$$\sigma_x = \frac{P_t}{A} = \frac{10\times10^3}{\pi/4\times d^2}$$

$$\sigma_x = \frac{12.73\times10^3}{d^2}$$

$$\sigma_y = 0$$

Note

$\sigma_x = \sigma_1$

$\sigma_y = \sigma_2$

σ_z:

Soln

1. Maximum Principal Stress Theory <7.3>

$$\sigma_1 \text{ or } \sigma_2 \text{ or } \sigma_3 = \sigma_y$$

For Principal stress. Theory <7.2>

$$\sigma_1 = \frac{1}{2}\left[(\sigma_x+\sigma_y) + \sqrt{(\sigma_x-\sigma_y)^2 + 4\tau_{xy}^2}\right]$$

σ_x = Stress due to tensile Load

$$\sigma_x = \frac{P_t}{A} = \frac{10\times10^3}{\pi/4\times d^2}$$

$$\boxed{\sigma_x = \frac{12.73\times10^3}{d^2}}$$

τ = stress due to shear Load

$$\tau = \frac{P_s}{A} = \frac{5\times10^3}{\pi/4\times d^2}$$

$$\boxed{\tau = \tau_{xy} = \frac{6.365\times10^3}{d^2}}$$

Here $\sigma_y = 0$

$$\sigma_1 = \frac{1}{2}\left[\left(\frac{12.73\times10^3}{d^2}+0\right) + \sqrt{\left[\frac{12.73\times10^3}{d^2}-0\right]^2 + 4\times\left(\frac{6.365\times10^3}{d^2}\right)^2}\right]$$

$$\sigma_1 = \frac{12.73\times10^3}{2d^2} + \frac{1}{2}\sqrt{\left[\frac{12.73\times10^3}{d^2}\right]^2 + 4\left[\frac{6.365\times10^3}{d^2}\right]^2}$$

$$\sigma_1 = \frac{6.365\times10^3}{d^2} + \frac{1}{2}\times\frac{6.365\times10^3}{d^2}\sqrt{2^2+4}$$

$$\sigma_1 = \frac{6.365\times10^3}{d^2} + \frac{1}{2} \times \frac{6.365\times10^3}{d^2}\sqrt{4+4}$$

$$\sigma_1 = \frac{6.365\times10^3}{d^2}\left[1 + \left[\tfrac{1}{2}\times\sqrt{8}\right]\right]$$

$$\sigma_1 = \frac{6.365\times10^3}{d^2}\left[2.41\right]$$

$$\boxed{\sigma_1 = \frac{15.366\times10^3}{d^2}}$$

$$\sigma_2 = \frac{1}{2}\left[(\sigma_x+\sigma_y) - \sqrt{(\sigma_x-\sigma_y)^2 + 4\tau_{xy}^2}\right] \quad 7.27$$

$$\sigma_2 = \frac{1}{2}\left[\left(\frac{12.73\times10^3}{d^2}+0\right) - \sqrt{\left(\frac{12.73\times10^3-0}{d^2}\right)^2 + 4\times\left(\frac{6.365\times10^3}{d^2}\right)^2}\right]$$

$$\sigma_2 = \frac{12.73\times10^3}{2d^2} - \frac{1}{2}\sqrt{\left(\frac{12.73\times10^3}{d^2}\right)^2 + 4\left(\frac{6.365\times10^3}{d^2}\right)^2}$$

$$\sigma_2 = \frac{6.365\times10^3}{d^2} - \frac{1}{2}\times\frac{6.365\times10^3}{d^2}\sqrt{4+4}$$

$$\sigma_2 = \frac{6.365\times10^3}{d^2}\left[1 - \frac{1}{2}\times\sqrt{8}\right]$$

$$\sigma_2 = \frac{6.365\times10^3}{d^2}\times\left[-0.41\right]$$

$$\boxed{\sigma_2 = -\frac{2.609\times10^3}{d^2}}$$

σ_1 or σ_2 or $\sigma_3 = \sigma_y$

[which is Maximum]

$\sigma_t = \sigma_y$

$\sigma_1 = \sigma_y$

$\sigma_1 = \sigma_t$

$$\frac{15.366 \times 10^3}{d^2} = 100$$

$$\frac{15.366 \times 10^3}{100} = d^2 \qquad d^2 = 153.66$$

$$\boxed{d = 12.4\,mm}$$

(ii) Maximum shear ~~str~~ stress theory <7.3>

$(\sigma_1 - \sigma_2)$ (or) $(\sigma_2 - \sigma_3)$ (or) $(\sigma_3 - \sigma_1) = \sigma_y$

which is Maximum

$\sigma_1 - \sigma_2 = \sigma_y$ | $\sigma_t = \sigma_y$

$$\frac{15.366 \times 10^3}{d^2} - \left[-\frac{2.609 \times 10^3}{d^2}\right] = 100$$

$$\frac{17.975 \times 10^3}{d^2} = 100$$

$$\frac{17.975 \times 10^3}{100} = d^2 \qquad d^2 = 179.75$$

$$\boxed{d = 13.40\,mm}$$

3) Maximum Strain Theory <7.3>

$$\sigma_1 - \nu[\sigma_2 + \sigma_3] \text{ (or) } \sigma_2 - \nu(\sigma_3 + \sigma_1) \text{ (or)}$$

$$\sigma_3 - \nu[\sigma_1 + \sigma_2] = \sigma_y$$

which is Maximum

$$\sigma_1 - \nu\left[\sigma_2 + \sigma_3\right] = \sigma_t \qquad \sigma_3 = 0$$

$$\sigma_t = \sigma_y$$

$$\frac{15.366\times10^3}{d^2} - 0.3\left[\frac{-2.609\times10^3}{d^2} + 0\right] = 100$$

$$\frac{15.366\times10^3}{d^2} + \frac{782.7}{d^2} = 100$$

$$\frac{16.148\times10^3}{d^2} = 100$$

$$\frac{16.148\times10^3}{100} = d^2 \qquad d^2 = 161.48$$

$$\boxed{d = 12.70\text{ mm}}$$

IV Maximum Strain Energy theory <7.3>

$$\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu\left[\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1\right] = \sigma_y^2$$

$$\left(\frac{15.366\times10^3}{d^2}\right)^2 + \left[\frac{-2.609\times10^3}{d^2}\right]^2 + 0 - 2\times0.3\left[\left(\frac{15.366\times10^3}{d^2}\right)\left(\frac{-2.609\times10^3}{d^2}\right) + \left(\frac{-2.609\times10^3}{d^2}\right)\times0 + (0)\times\left(\frac{15.366\times10^3}{d^2}\right)\right] = 100^2$$

$$\sigma_3 = 0 \qquad \sigma_y = \sigma_t$$

$$\frac{236.11\times10^6}{d^4} + \frac{6.806\times10^6}{d^4} + -0.6\left(\frac{15.366\times10^3}{d^2}\right)\left(\frac{-2.609\times10^3}{d^2}\right) = 11$$

$$\frac{236.11\times10^6}{d^4} + \frac{6.806\times10^6}{d^4} + \frac{24.05\times10^6}{d^4} = 100^2$$

$$\frac{1}{d^4}\left[236.11\times10^6 + 6.806\times10^6 + 24.05\times10^6\right] = 100^2$$

$$\frac{1}{d^4} \times 266.96\times10^6 = 100^2$$

$$\frac{266.94\times10^6}{100^2} = d^4$$

$$\boxed{2.66\times10^4 = d^4}$$

$$d = (2.66\times10^4)^{1/4}$$

Ans $\boxed{\cancel{d = 40.42\, mm}}$

$$\boxed{d = 12.78\, mm}$$

II Distortion Energy Theory (von mises Hencky)

$$\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1\sigma_2 - \sigma_2\sigma_3 - \sigma_3\sigma_1 = \sigma_y^2$$

$$\left(\frac{15.366\times10^3}{d^2}\right)^2 + \left(\frac{-2.609\times10^3}{d^2}\right)^2 + 0 - \left(\frac{15.366\times10^3}{d^2}\right)\left(\frac{-2.609\times10^3}{d^2}\right) - 0 - 0 = 100^2$$

$$\frac{236.13\times10^6}{d^4} + \frac{6.806\times10^6}{d^4} + \frac{40.08\times10^6}{d^4} = 100^2$$

$$\frac{283.016 \times 10^{6}}{d^{4}} = 100^{2}$$

$$\frac{283.016 \times 10^{6}}{100^{2}} = d^{4}$$

$$d^{4} = 28301.6$$

$$d = [28301.6]^{1/4}$$

Ans $\boxed{d = 12.97 \text{ mm}}$

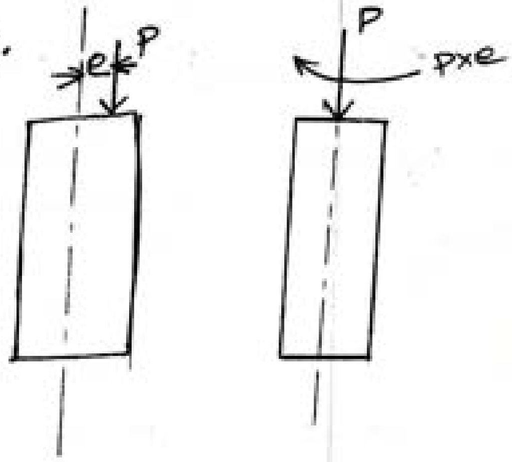
(7) A cylindrical shaft made of steel of yield strength 700 MPa is subjected to static Loads consisting of bending moment 10 kN.m and a torsional moment 30 kN.m Determine the diameter of the shaft using Two ~~different~~ different theories of failure, and assuming a factor of safety of 2. Take E = 210 GPa and Poisson's Ratio = 0.25

H.W

⇒ Eccentric Loading - Direct and Bending stress combined.

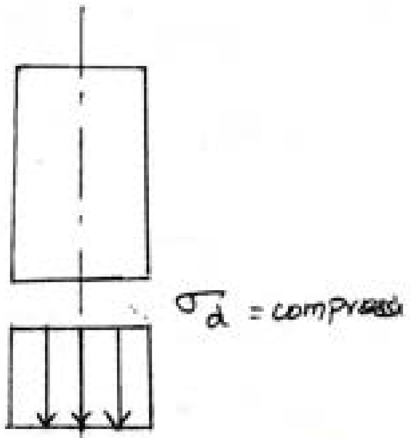
An external Load, whose Line of action is paralle but does not coincide with the centroidal Axis of the machine components is known as eccentric Load

The distance b/w the centroidal axis of the machine component and the eccentric Load is calle eccentricity.



1. Direct stress

In the case shown it is compressive Load



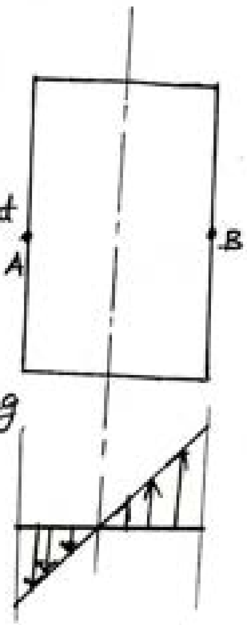
This is given by

$$\sigma_d = \frac{\text{Force}}{\text{Cross sectional Area}}$$

$$\boxed{\sigma_d = \frac{P}{A}}$$

2. Bending stress

This is due to bending moment (Pxe). This results in different types of stresses on either side of the natural axis of the ~~bending~~ section, compressive on one side and tensile on the other.



$$\sigma_b = \frac{M_b \times y}{I}$$

$$\frac{M_b}{I} = \frac{\sigma_b}{y}$$

$$\sigma_b = \frac{M_b \times y_c}{I} \quad \text{[compressive Load]}$$

$$\sigma_b = \frac{M_b \times y_t}{I} \quad \text{[tensile Load]}$$

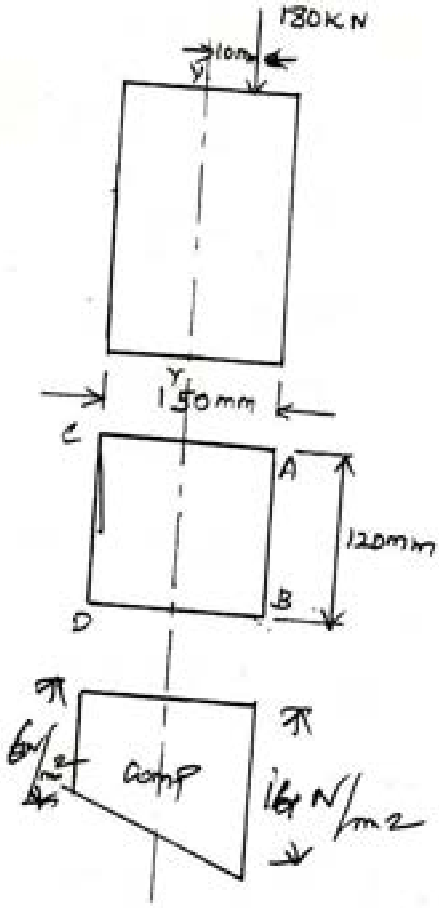
$$M_b = P \times e$$

Total stress (or) combined stress

$$\boxed{\sigma = \sigma_d + \sigma_b}$$

Problem

1. A rectangular strut is 150 mm wide and 120 mm thick. It carries a Load of 180 kN at an eccentricit of 10mm in a plane bisecting the thickness as shown in Fig. Find the maximum and minimum intensities of Stress in the Section.



Given

wide $b = 150mm$

thick $t = 120mm$

Load $P = 180\ kN = 180 \times 10^3 N$

Eccentricity $e = 10mm$

To find

Maximum & minimum Intensities of Stress $\therefore \sigma$ [combined stress]

Soln

Maximum Intensities Stress $\sigma_{Max} = \cancel{\sigma_b + \sigma_d}\ \sigma_d + \sigma_b$

Minimum Intensities Stress $\sigma_{min} = \cancel{\sigma_b - \sigma_d}\ \sigma_d - \sigma_b$

σ_d = direct stress due to compressive Load

$$\sigma_d = P/A = \frac{180 \times 10^3}{b \times t} = \frac{180 \times 10^3}{150 \times 120}$$

$$\boxed{\sigma_d = 10 N/mm^2}$$

σ_b: bending stress due to eccentricity

$$\sigma_b = \frac{M_b \times y}{I}$$

$$y = \frac{b}{2} = \cancel{t} \; b/2 = \frac{150}{2}$$

$$I_{yy} = \frac{\cancel{b} d\, b^3}{12}$$

$$\boxed{y = 75\,mm}$$

$\cancel{I=}$ d

$$I = \frac{t \times b^3}{12} = \frac{120 \times 150^3}{12} = 33.75 \times 10^6\ mm^4$$

$$M_b = P \times e = 180 \times 10^3 \times 10 = 1.8 \times 10^6\ N.mm$$

$$\sigma_b = \frac{1.8 \times \cancel{10^6} \times 75}{33.75 \times \cancel{10^6}}$$

$$\boxed{\sigma_b = 4\ N/mm^2}$$

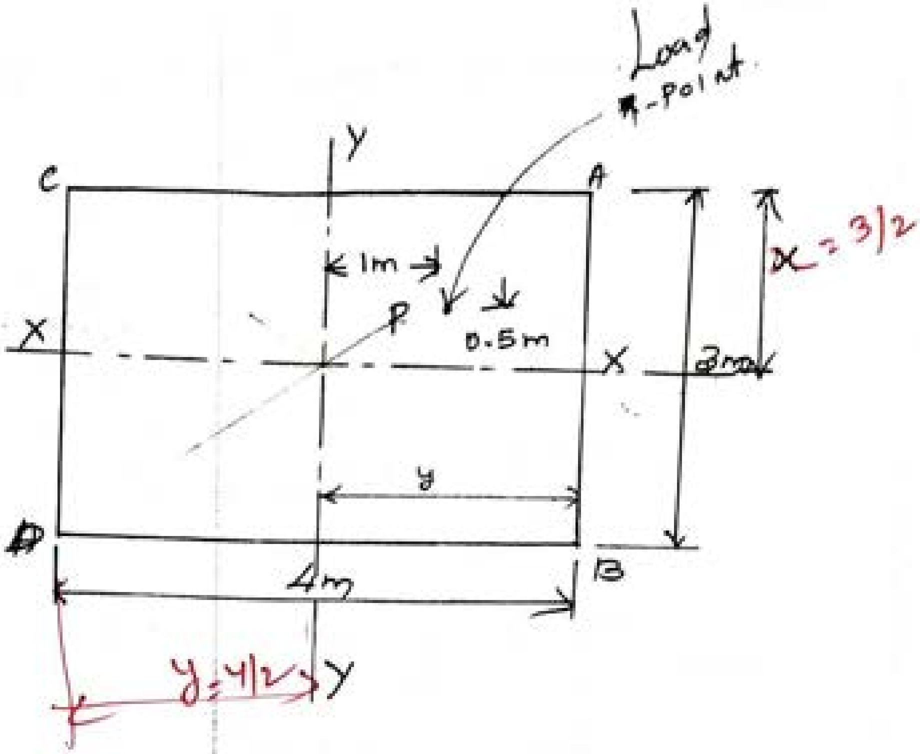
$$\sigma_{max} = \sigma_d + \sigma_b = 10 + 4$$

$$\boxed{\sigma_{max} = 14\ N/mm^2}$$

$$\sigma_{min} = \sigma_d - \sigma_b = 10 - 4$$

$$\boxed{\sigma_{min} = 6\ N/mm^2}$$

3) A masonry pier of width 4 m and thickness 3 m, sup (32)
a load of 30 kN as shown in Fig. Find the stresses developed at each corner of the pier.



Given

width $b = 4m$

thickness $t = 3m$

Load $P = 30kN = 30 \times 10^3 N$

eccentricity at x axis $e_x = 0.5m$

eccentricity at y axis $e_y = 1m$.

To Find

Stress at each corner (A B C D)

Soln

combined stress σ

$$\sigma = \sigma_d + \sigma_b$$

$$\sigma_d = P/A$$

$$\sigma_d = \frac{M_b \times y}{I}$$ General Relation

$$\sigma_b = \frac{M_b \times x}{I_{xx}} \ \& \ \frac{M_b \times y}{I_{yy}}$$

$M_b = P \times e$

ie $M_b = P \times e_x$ & $P \times e_y$

$A = b \times t = 4 \times 3 = 12 \, m^2$

$$I_{xx} = \frac{bt^3}{12} = \frac{4 \times 3^3}{12} = 9 \, m^4$$

$$I_{yy} = \frac{tb^3}{12} = \frac{3 \times 4^3}{12} = 16 \, m^4$$

$x = 3/2 = 1.5 \, m$ $y = 4/2 = 2 \, m$

stress at corner A

$$\sigma_A = \frac{P}{A} + \frac{P \times e_x \times x}{I_{xx}} + \frac{P \times e_y \times y}{I_{yy}}$$

$$\sigma_A = \frac{30}{12} + \frac{30 \times 0.5 \times 1.5}{9} + \frac{30 \times 1 \times 2}{16}$$

$$\sigma_A = 2.5 + 2.5 + 3.75$$

$$\boxed{\sigma_A = 8.75\, kN/m^2}$$

Stress at corner B

$$\sigma_B = \frac{P}{A} + \frac{P \times e_x \times X}{I_{xx}} - \frac{P \cdot e_y \times Y}{I_{yy}}$$

$$\sigma_B = \frac{30}{12} + \frac{30 \times 0.5 \times 1.5}{9} - \frac{30 \times 1 \times 2}{16}$$

$$\boxed{\sigma_B = 1.25\, kN/m^2}$$

Stress at corner C

$$\sigma_C = \frac{P}{A} - \frac{P\, e_x X}{I_{xx}} + \frac{P \times e_y \times Y}{I_{yy}}$$

$$\sigma_C = \frac{30}{12} - \frac{30 \times 0.5 \times 1.5}{9} + \frac{30 \times 1 \times 2}{16}$$

$$\boxed{\sigma_C = 3.75\, kN/m^2}$$

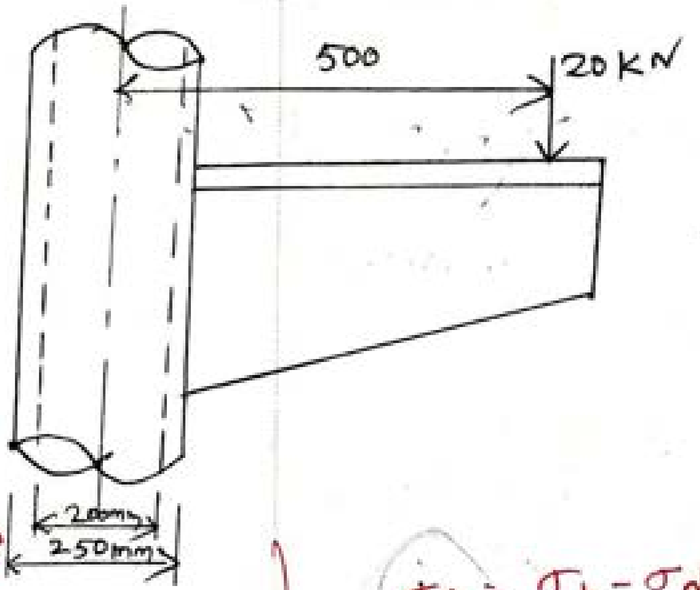
Stress at corner D

$$\sigma_D = \frac{P}{A} - \frac{P \cdot e_x X}{I_{xx}} - \frac{P \times e_y \times Y}{I_{yy}}$$

$$\sigma_D = \frac{30}{12} - \frac{30 \times 0.5 \times 1.5}{9} - \frac{30 \times 1 \times 2}{16}$$

$$\boxed{\sigma_D = -3.75\, kN/m^2}$$

3. A hollow circular column of external diameter 250mm and internal diameter 200mm, carries a projecting bracket on which a load of 20 kN rest, as shown in fig. The centre of the load from the centre of column is 500mm. Find the stresses at the side of the column.



$A = \pi/4\,(D^2 - d^2)$

$\sigma_d = P/A$

$= 1.13\,MPa$

$Z = \frac{I}{y} = \frac{\pi/64\,[D^4 - d^4]}{D/2}$

$Z = 905.8 \times 10^3\,mm^3$

B.M

$M = P.e$

$\sigma_b = M/Z$ $= 11.04\,MPa$

$\sigma_c = \sigma_b + \sigma_D$

$\sigma_t = \sigma_b - \sigma_d = 9.91\,MPa$

⇒ Bending stress in Curved Beams: <6.2>

curved beams, the natural axis of the cross section is shifted towards the centre of curvature of the beam causing a non-Linear (hyperbolic) distribution of stress, as shown in Fig.

It may be noted that the neutral axis lies b/w the Centroidal axis and the centre of curvature and always occurs within the curved beams.

Application: Crane hooks
chain Links
Frames of punches
Presses
Planers etc.

Bending stress <6.2>
$$\sigma_b = \frac{M_b}{a \times e}\left[\frac{y}{r_n - y}\right] \text{ (or) } \frac{M_b y}{ae[r_n - y]}$$

M: Bending moment

a: Area of cross section.

e: Distance from the centroidal axis to the Natural Axi

e: $r - r_n$

r_n = Radius of curvature in natural axis

y = Distance of fibre from NA.

outer bending stress in fibre

$$\sigma_{b\,max} = \frac{M_b h_o}{a e r_o}$$

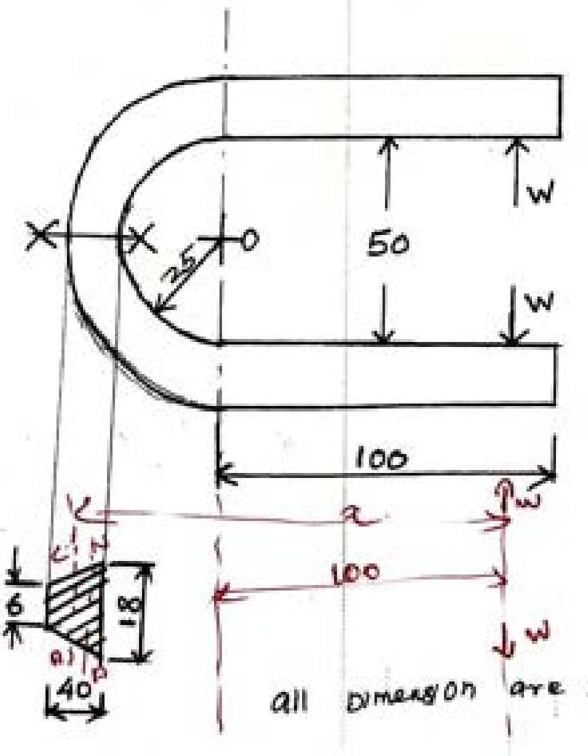
Inside bending stress in fibre

$$|\sigma_b|_{i\,max} = \frac{M_b h_i}{a e r_i}$$

Diagram Refer in Databook page NO: 6.2

Problem: 1

The Frame of a Punch Press is shown in Fig.
Find the stresses at the inner and outer surface at section X-X of the frame, if W = 5000N.



all dimension are in mm

Given

Load $W = 5000 N$

To Find

Stress in inner and outer surface ~~At~~ $\sigma_{bi\,Max}$ & σ_{bo}

Soln

Maximum bending stress at the inner surface

$$[\sigma_b]\,Max = \frac{M_b h_i}{a e r_i}$$

Maximum bending stress at the outer ~~inner~~ surface

$$\sigma_b\ Max = \frac{M_b h_o}{a e r_o}$$

M_b = bending Moment at the centroidal Axis

M_b = Load × Distance $= W \times x$ | $W = 5000 N$

x = Distance b/w the Load & centroidal axis

From Data book $x = 100 + R$
<6.3>

<6.3> $$R = r_i + \frac{h[b_i + 2b_o]}{3[b_i + b_o]}$$

$b_o = 6\,mm$
$b_i = 18\,mm$
$h = 40\,mm$
] From Data book 6.3

$r_i = 25mm$ Refer Fig <6-3>

$$R = 25 + \frac{40[18 + 2\times 6]}{3[18+6]}$$

$$R = 25 + \frac{40[30]}{3[24]}$$

$$R = 25 + \frac{1200}{72} = 25 + 16.66$$

$$\boxed{R = 41.66mm}$$

$$x = 100 + R = 100 + 41.66$$

$$\boxed{x = 141.66 \text{ mm}}$$

$$M_b = W \times x = 5000 \times 141.66$$

$$M_b = 708,300 \text{ N.mm}$$

$$\boxed{M_b = 708.3\times10^3 \text{ N.mm}}$$

The Section XX is subjected to Tensile Load of W=5000N and a Bending moment of M=708.35N-mm

WKT,

$$\sigma_t = \frac{W}{A} = \frac{5000}{480} = 10.42 \text{ MPa.}$$

$$A = \tfrac{1}{2}(18+6)\,40$$

$$= 480mm^2$$

e = Distance b/w centroidal axis and natural axis

$$e = R - r_n$$

$$r_n = \frac{\frac{1}{2}[b_i + b_o]h}{\left(\frac{b_i r_o - b_o r_i}{h}\right)\ln\left(\frac{r_o}{r_i}\right) - (b_i - b_o)} \quad \angle 6.3 \rangle$$

From Fig $b_i = 18mm$

$b_o = 6mm$

$h = 40mm$

$r_o = h + r_i$

$r_i = 25mm$

$r_o = 40 + 25$

$r_o = 65mm$

$$r_n = \frac{\frac{1}{2}[18+6]40}{\left[\frac{18\times 65 - 6\times 25}{40}\right] \ln\left[\frac{65}{25}\right] - [18-6]}$$

$$r_n = \frac{480}{25.5 \times 0.955 - 12}$$

$$r_n = \frac{480}{24.35 - 12} = \frac{480}{12.352}$$

$$\boxed{r_n = 38.85mm}$$

$e = R - r_n = 41.66 - 38.85$

$$\boxed{e = 2.81\ mm}$$

a = Area of cross section.

$a = \frac{1}{2}[b_i + b_o]h = \frac{1}{2}[18 + 6]40$

$$\boxed{a = 480\ mm^2}$$

h_i = Distance from the neutral Axis to the inner surface

$h_i = r_n - r_i = 38.85 - 25$

$\boxed{h_i = 13.85 \text{ mm}}$

h_o = Distance from the ~~neta~~ neutral Axis to the outer surface

$h_o = r_o - r_n = 65 - 38.85$

$\boxed{h_o = 26.15 \text{ mm}}$

Stress in inner surface

$$|\sigma_b|_{max} = \frac{M_b h_i}{a e r_i} = \frac{708.3 \times 10^3 \times 13.85}{480 \times 2.81 \times 25}$$

$$\boxed{|\sigma_b|_{max} = 290.92 \text{ N/mm}^2}$$

Stress in outer surface

$$\sigma_{b\,max} = \frac{M_b h_o}{a e r_o} = \frac{708.3 \times 10^3 \times 26.15}{480 \times 2.81 \times 65}$$

$$\boxed{\sigma_{b\,max} = 211.26 \text{ N/mm}^2}$$

Direct Tensile Stress $\sigma_t = \frac{\text{Load}}{\text{Area}} = \frac{5000}{480}$

$\sigma_t = 10.42$ N/mm^2

Resultant stress on the inner surface σ_{Ri}

$$\sigma_{Ri} = \sigma_t + |\sigma_b|_{Max} = 10.42 + 290.92$$

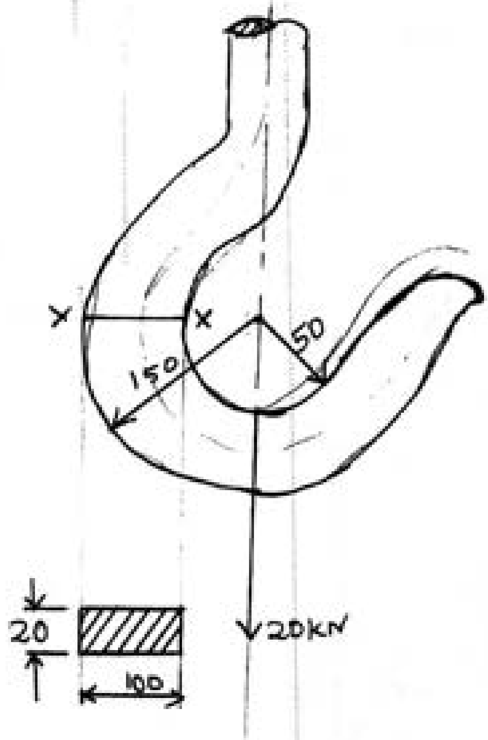
$$\boxed{\sigma_{Ri} = 301.94\ N/mm^2}$$

Resultant stress on the outer surface σ_{Ro}

$$\sigma_{Ro} = \sigma_t - \sigma_{bMax} = 10.42 - 211.26$$

$$\sigma_{Ro} = -200.84\ N/mm^2$$

2. The crane hook carries a load of 20kN as shown in Fig. The section at X-X is rectangular whose horizontal side is 100mm. Find the stresses in the inner and outer fibres at the given section.



Given:

Load $W = 20KN = 20\times10^3 N$

outer radius $r_o = 150mm$

Inner radius $r_i = 50mm$

To Find

Stresses in Inner & outer Surface $|\sigma_b|_{Max}$ &

$\sigma_{b\,Max}$.

Soln

Stresses in Inner Surface $|\sigma_b|_{Max}$

$$|\sigma_b|_{Max} = \frac{M_b h_i}{a e r_i}$$

Stresses in outer Surface $\sigma_{b\,Max}$

$$\sigma_{b\,Max} = \frac{M_b h_o}{a e r_i}$$

M_b = bending Moment

M_b = Load x Distance $= W \times x$

Load $W = 20\times10^3 N$

Distance $x = R = r_1 + \frac{h}{2}$

$$\left.\begin{aligned} r_1 &= 50mm \\ h &= 100mm \end{aligned}\right\} \text{From <DB 6.3>}$$

$R = r_i + h/2$

$R = 50 + \frac{100}{2}$

$\boxed{R = 100\,mm}$

$M_b = W \times R = 20 \times 10^3 \times 100$

$\boxed{M_b = 2 \times 10^6\ N.mm}$

e = Distance b/w centroidal axis and neutral axis.

$e = R - r_n$

r_n: Radius of curvature of the neutral Axis

$r_n = \frac{h}{\ln\left(\frac{r_o}{r_i}\right)}$

$r_n = \frac{100}{\ln\left(\frac{150}{50}\right)}$

$h = 100\,mm$

$r_i = 50\,mm$

From DB6:

$r_o = 150\,mm$ (or) $r_i + h$

$= 50 + 100 = 150m$

$\boxed{r_n = 91.02\ mm}$

$e = R - r_n = 100 - 91.02$

$\boxed{e = 8.92\ mm}$

a = Area of Cross Section

$a = 100 \times 20$

$\boxed{a = 2000\,mm^2}$

h_i = Distance from the neutral axis to the inside fibre

$h_i = r_n - r_i = 91.02 - 50$

$\boxed{h_i = 41.02\ mm}$

h_o : Distance from the neutral axis to outside fibre

$h_o = r_o - r_i = 150 - 50$

$\boxed{h_o = 100\ mm}$

$h_o = 58.93\ mm$

Maximum bending stress at the inside fibre

$$|\sigma_b|_{Max} = \frac{M_b h_i}{a e r_i} = \frac{2\times10^6 \times 41.02}{2000 \times 8.92 \times 50}$$

$$\boxed{|\sigma_b|_{Max} = 91.97\ N/mm^2}$$

Maximum bending stress at the outside fibre

$$\sigma_{b\,Max} = \frac{M_b h_o}{a e r_o} = \frac{2\times10^6 \times 100}{2000 \times 8.92 \times 150}$$

$$\boxed{\sigma_{b\,Max} = 74.73\ N/mm^2}$$

Direct tensile stress σ_t

$$\sigma_t = \frac{Load}{Area} = \frac{20\times10^3}{2000}$$

$$\boxed{\sigma_t = 10\ N/mm^2}$$

Resultant stress in inside surface σ_{Ri}

$$\sigma_{Ri} = \sigma_t + (\sigma_b)_{max} = 10 + 91.97 .$$

$$\boxed{\sigma_{Ri} = 101.97 \ N/mm^2}$$

Resultant stress in outside surface σ_{Ro}

$$\sigma_{Ro} = \sigma_t - \sigma_{bMax} = 10 - 74.73$$

$$\boxed{\sigma_{Ro} = -64.73 \ N/mm^2}$$

⇒ Variable stresses in Machine parts:

The previous chapter, the stresses due to static Loading only.

But only a few machine parts are subjected to static Loading.

Ex, such as axles, shafts, crankshafts, connecting rods, spring, pinion teeth etc)

Types of varying stress

(i) completely reversed (or) cyclic stresses

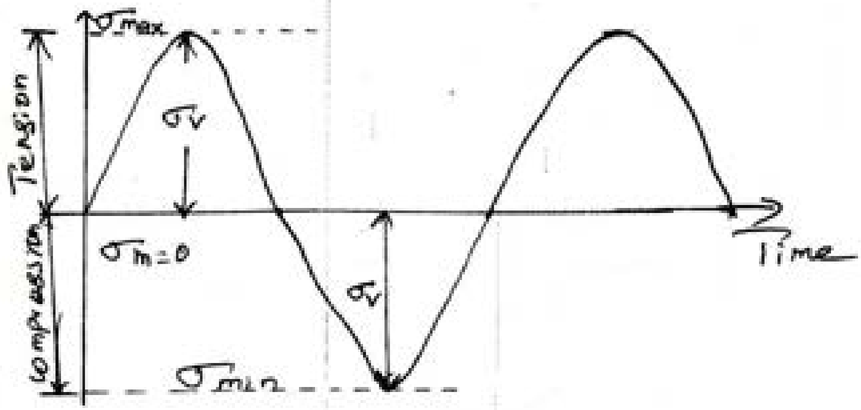
(ii) Fluctating stresses.

(iii) Repeated stresses

(iv) Alternating stresses

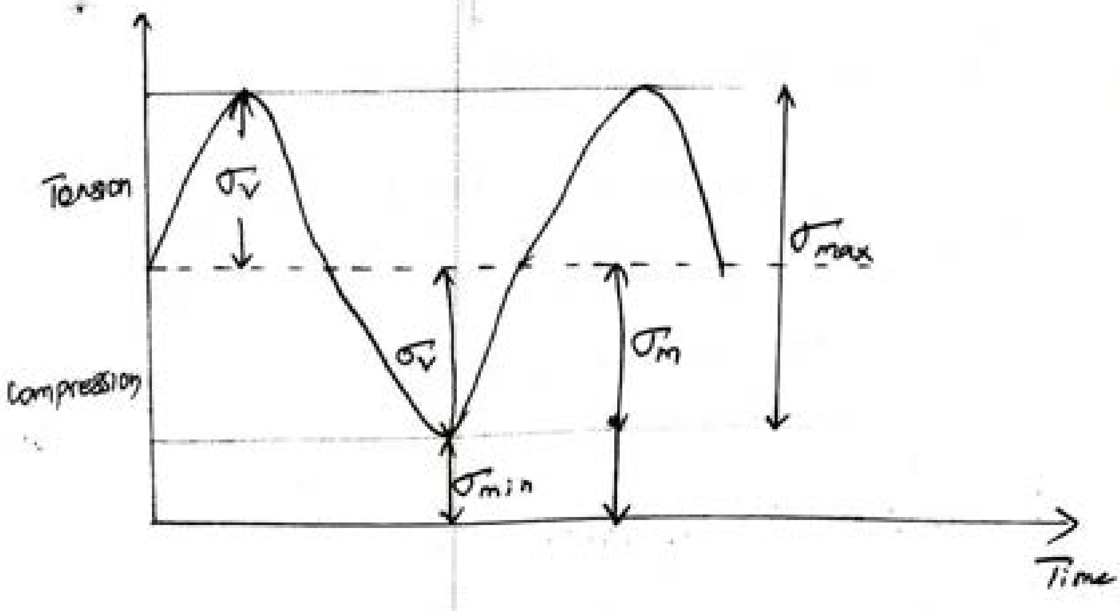
(I) completely reversed (or) cyclic stresses:

Stresses which change from one value of tension to the same value of compression is known as completely reversed as cyclic stresses (σ_v)



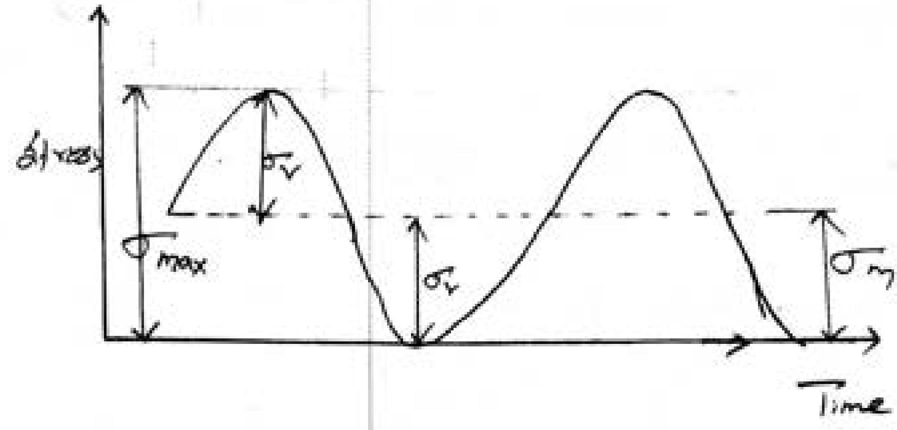
(II) Fluctuating stresses

Stresses which vary from a minimum value to a maximum of same nature [compressive or tensile] are called as fluctuating stresses.



(iii) Repeated Stress

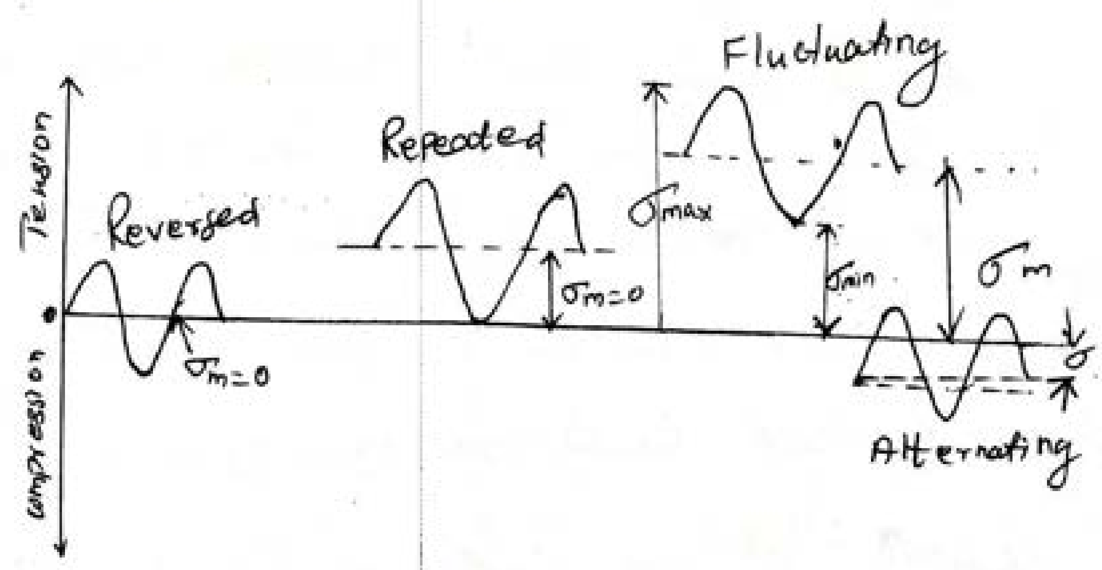
This refers to a stress, which various from zero to a maximum value of a same nature.



IV Alternative Stress

Stress varying from a minimum value to a maximum value of the opposite neture [from a minimum compressive to maximum tensile] is known as alternating st.

On common axis, these stresses may be plotted as given below.



PSG.DB

Terms used in variable stress conditions. <7.6>

1. Mean or average stress $\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$

2. Variable stress (or) amplitude stress $\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$

3. Stress Ratio R

$$R = \frac{\sigma_{min}}{\sigma_{max}}$$

It is completed reversed stress $\sigma_{min} = -\sigma_{max}$

$$R = \frac{\sigma_{min}}{\sigma_{max}} = \frac{-\sigma_{max}}{\sigma_{max}} = -1$$

4. Endurance Limit (σ_{-1})

The endurence limit (σ_{-1}) of a material as determined by the rotating beam method is for reversed bending load. There are many machine members which are subjected to loads other than reversed bending loads.

Thus the endurance limit will also be different for different types of loading

<1.42> $\sigma_{-1} = 0.5\sigma_u$ for steel [for bending]

$\sigma_{-1} = 0.45\sigma_u$ [for tension / compression]

$= 0.4\sigma_u$ for CI

$= 0.3\sigma_u$ for non ferrous and alloys

> Factors affecting endurance strength:

1. Load Factor (K_L)

This factors varies with the loading type. reversed axial load, reversed bending etc.

$K_L = \cancel{at}\ 1$ Reversed bending

$= 0.1$ Reversed axial bending

$= 0.6$ reversed torsion

2. Surface finish factor (K_{sf})

This factors arises due to the surface conditio of the material. If the ~~mirror~~ surface is mirror li this factor equals unity. If poor surface is there, it is less than unity. ~~The value~~ of

$K_{sf} = 0.9$ for ground or cold rolled surface

$= 0.7$ to 0.85 machined surface

$= 0.3$ to 0.7 hot rolled surface.

3. Size factor (K_{sz})

A Large size speciman will have more defects than a small one. So as size increase, this factor reduce.

$K_{sz} = 1$ $d < 7.5mm$

$= 0.85$ $d < 50mm$ $d > 7.5mm$

$= 0.75$ $d > 50mm$

4. Reliability factor (k_R)

Considerable scatter is found in fatigue ~~steel~~ tests. A normal distribution gives a good agreement with a standard deviation of 8% (0.08). This standard deviation can be subtracted from the 50% mean strength to obtain desired reliability.

$$k_R = 1 - 0.08 D_f$$

Reliability Factor

Survival rate	Deviation Multiplier, D_f	Reliability factor k_R
50	0	1
90	1.4	0.89
95	1.6	0.87
98	2.0	0.84
99	2.4	0.8
99.9	3.4	0.75

5. Miscellaneous factor k

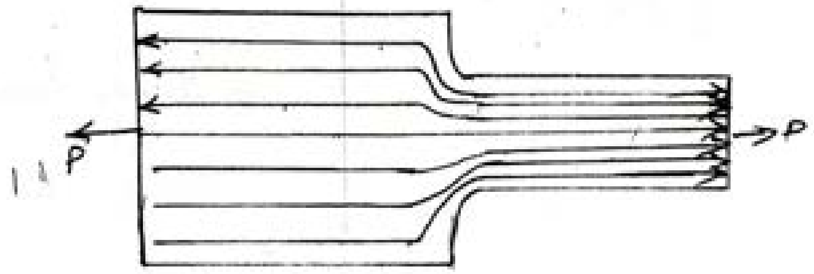
Other than the above, factors like temperature, impact factor may also be considered in determining the endurance limit

The actual or modified or component endurance strength

$$\sigma_{-1m} = \frac{\sigma_{-1}\, k_L\, k_{SZ}\, k_{SF}\, k_R\, k}{\cancel{k_t}\; K_f}$$

→ Stress Concentration

Whenever a machine component change the shape of its cross section, the simple stress distribution no longer holds good and the neighbourhood of the discontinuity is differed. This irregularity in the stress distribution caused by abrupt changes of forr is called stress concentration.



→ Stress concentration factor k_t <7.8>

Stress concentration factor k_t is defined as the ratio of the maximum stress at the change of cross section to the normal stress.

$$k_t = \frac{\sigma_{max}}{\sigma_0}$$ → Nominal stress.

Fatigue stress concentration factor k_f <7.6>

$$k_f = 1 + q\,(k_t - 1)$$

↓

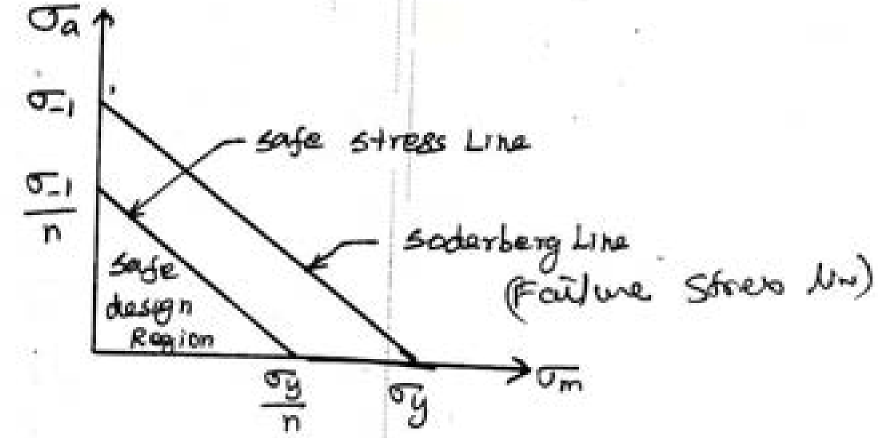
notch sensitivity factor.

$$q = \frac{k_f - 1}{k_t - 1}$$

⇒ Soderberg and Goodman Diagrams

(i) Soderberg Diagrams: <8.7-4> <7.6>

Soderberg method is particular used in Ductile material.



In this Diagram, amplitude stress and mean stress are plotted on Y and X axes respectively. Soderberg Line joins the endurance limit on Y axis and yield stress on the X axis. This diagram may be used for ductile Material.

~~(ii) Goo~~

$$\frac{1}{n} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_a}{\sigma_{-1}} \quad , \quad \frac{1}{n} = \frac{\tau_m}{\tau_y} + \frac{\tau_a}{\tau_{-1}} \quad \langle 7.4 \rangle$$

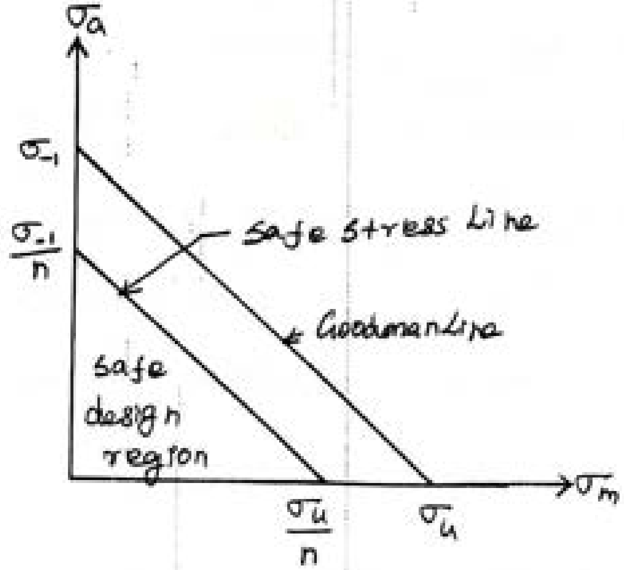
$$\sigma_{-1m} = k_f \times \frac{1}{(\sigma_{-1})\ k_L \cdot k_{SF} \times k_{SZ} \cdot k_R \cdot k}$$

~~Modified Eqn~~

$$\frac{1}{n} = \frac{\sigma_m}{\sigma_y} + k_f \frac{\sigma_a}{(\sigma_{-1})\ k_L\ k_{SF}\ k_{SZ}\ k_R\ k}$$

$$\frac{1}{n} = \frac{\tau_m}{\tau_y} + k_f \frac{\tau_a}{(\tau_{-1})\ k_L\ k_{SF}\ k_{SZ}\ k_R\ k}$$

(ii) Goodman Diagram:



If endurance strength (y axis) is Jointed with ultimate stress on the x axis, it is called Goodman Line.

This diagram may be used for

$$\frac{1}{n} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_a}{\sigma_{-1}} \quad , \quad \frac{1}{n} = \frac{\tau_m}{\tau_u} + \frac{\tau_a}{\tau_{-1}} \qquad \langle 7.4 \rangle$$

~~Modified~~ $$\frac{1}{n} = k_f \left[\frac{\sigma_m}{\sigma_u} + \frac{\sigma_a}{\sigma_{-1} \times \text{all factor}} \right] , \quad \frac{1}{n} = k_f \left[\frac{\tau_m}{\tau_u} + \frac{\tau_a}{\tau_{-1} \times \text{all fact}} \right]$$

<u>combined</u> $\langle 7.6 \rangle$

$$\sigma_{eq} = \frac{\sigma_y}{n} = \sigma_m + k_f \times \frac{\sigma_a \sigma_y}{\sigma_{-1} \, k_L \, k_{SF} \, k_{SZ} \, k_R \, K}$$

$$\tau_{eqn} = \frac{\tau_y}{n} = \tau_m + k_f \frac{\tau_a \tau_y}{\tau_{-1} \, k_L \, k_{SF} \, k_{SZ} \, k_R \, K}$$

$$\frac{1}{n} = \left[\left(\frac{\sigma_{eq}}{\sigma_y} \right)^2 + \left(\frac{\tau_{eq}}{\tau_y} \right)^2 \right]^{1/2}$$

Problem: based on varying stresses

1. A machine component is subjected to a flexural stress which fluctuates b/w $+300\ MN/m^2$ and $-150\ MN/m^2$. Determine the value of minimum ultimate strength according to 1. Gerber relation 2. Modified Goodman relation, and 3. Soderberg relation

Take yield strength = 0.55 Ultimate strength:

Endurance strength = 0.5 ultimate strength: and

Factor of safety = 2.

Given:

Maximum stress $\sigma_{max} = 300\ MN/m^2 = 300 \times 10^6\ N/m^2$

$\sigma_{max} = 300\ N/mm^2$

Minimum stress $\sigma_{min} = -150\ MN/m^2 = -150\ N/mm^2$

Yield strength $\sigma_y = 0.55\,\sigma_u$

endurance strength $\sigma_{-1} = 0.5\,\sigma_u$

Factor of safety $n = 2$

To Find

The value of ultimate strength σ_u

by using 1) Gerber Relation

2) Modified ~~Soderberg~~ Goodman relations

3) Soderberg relation

Soln

1. Gerber Eqn.

$$\frac{1}{n} = \left(\frac{\sigma_m}{\sigma_u}\right)^2 \times n + \frac{\sigma_a}{\sigma_{-1}}$$

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = \frac{300 - 150}{2} \qquad \langle 7.6 \rangle$$

$$\sigma_m = 75 \, N/mm^2$$

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{300 - (-150)}{2}$$

$$\sigma_a = 225 \, N/mm^2$$

$\sigma_{-1} = 0.5 \, \sigma_u \rightarrow$ given.

$n = 2 \rightarrow$ given

$$\frac{1}{2} = \frac{75^2}{\sigma_u^2} \times 2 + \frac{225}{0.5 \sigma_u}$$

$$\frac{1}{2} = \frac{75^2 \times 2}{\sigma_u^2} + \frac{450}{\sigma_u}$$

$$\frac{1}{2} = \frac{11250}{\sigma_u^2} + \frac{450}{\sigma_u}$$

$$\frac{1}{2} = \frac{11250 + 450 \sigma_u}{\sigma_u^2}$$

$$\frac{\sigma_u^2}{2} = 11250 + 450 \sigma_u$$

$$\sigma_u^2 = 2[11250 + 450 \sigma_u]$$

$$\sigma_u^2 = 22500 + 900\sigma_u$$

$$\sigma_u^2 - 900\sigma_u - 22500 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad a = 1, \quad b = -900, \quad c = -22500$$

$$\sigma_u = \frac{-(-900) \pm \sqrt{(-900)^2 - 4 \times 1 \times (-22500)}}{2 \times 1}$$

$$\sigma_u = \frac{900 \pm 948.7}{2}$$

Ans $\boxed{\sigma_u = 924.35 \ N/mm^2}$

2. Modified Goodman relation: <7.6>

$$\frac{1}{n} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_a}{\sigma_{-1}}$$

$$\frac{1}{2} = \frac{75}{\sigma_u} + \frac{225}{0.5\sigma_u}$$

$$\frac{1}{2} = \frac{1}{\sigma_u}\left[\frac{75}{1} + \frac{225}{0.5}\right]$$

$$\frac{1}{2} = \frac{1}{\sigma_u}\left[75 + 450\right]$$

$$\frac{1}{2} = \frac{1}{\sigma_u} \times 525$$

$\sigma_u = \cancel{525}$

$$\frac{1}{2} = \frac{525}{\sigma_u}$$

$$\sigma_u = 2 \times 525$$

Ans $\boxed{\sigma_u = 1050 \ N/mm^2}$

3. soderberg Relation:

$$\frac{1}{n} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_a}{\sigma_{-1}}$$

$$\frac{1}{2} = \frac{75}{0.55\sigma_u} + \frac{225}{0.5\sigma_u}$$

$$\frac{1}{2} = \frac{1}{\sigma_u}\left[\frac{75}{0.55} + \frac{225}{0.5}\right]$$

$$\frac{1}{2} = \frac{1}{\sigma_u}\left[136.36 + 450\right]$$

$$\frac{1}{2} = \frac{586.36}{\sigma_u}$$

$$\sigma_u = 586.36 \times 2$$

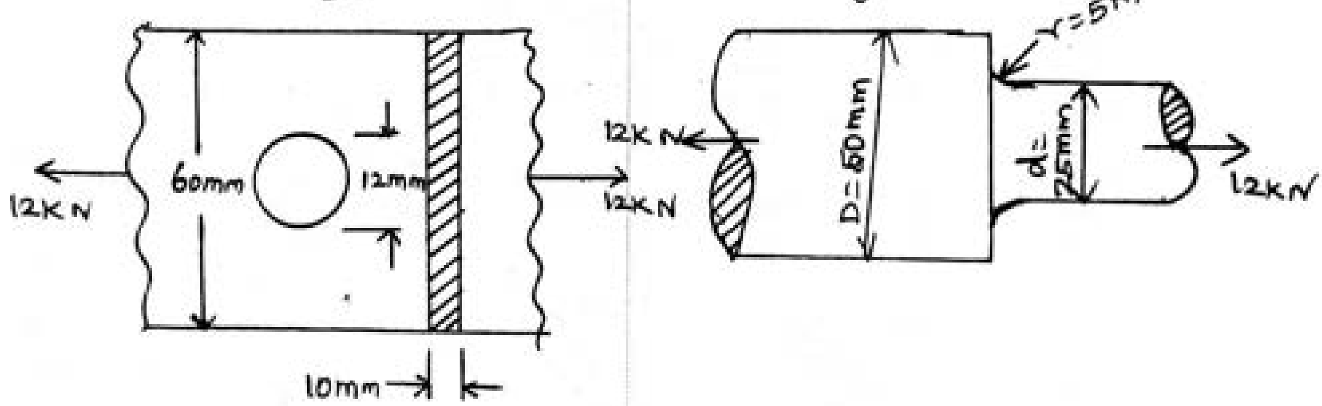
Ans $\boxed{\sigma_u = 1172.72 \ N/mm^2}$

2. Find the Maximum stress induced in the following Causes taking stress concentraction into account.

1. A rectangular plate 60mm x 10mm with a hole 12 mm diameter as shown in Fig (a) and subjected to a tensile Load of 12KN.

2. A stepped shaft as shown in Fig (b) and carrying a tensile load of 12KN.

Fig (a) Fig (b)



Given

Rectangle plate: width ~~b = 60~~ b (or) w = 60 mm

thickness h or t = 10 mm

diameter of hole (a ϕ) = 12 mm

Load P (or) W = 12KN = 12×10^3 N

Stapped bar:

D ⇒ Dia of Large end D = 50mm

Dia of Small end d = 25mm

Load P = 12KN = 12×10^3 N

Radius of fillet ⇒ r = 5mm

To find:

Maximum stress σ_{max}.

Soln

1. Rectangle plate:

From data book P.No <7.10>

$$k_t = \frac{\sigma_{max}}{\sigma_{nom}}$$

Refer Diagram

$$\sigma_{nom} = \frac{\text{Load}}{\text{Area}}$$

$P = 12 \times 10^3\,N$

$h = 10\,mm$

$W = 60\,mm$

~~$a = \pi/4 \times d^2$~~

~~a = Area of hole~~

~~$a = \pi/4 \times 12^2 = 113$~~

a = dia of hole

$a = 12\,mm$

$$\sigma_{max} = k_t \times \sigma_{nom}.$$

$$\sigma_{nom} = \frac{P}{(W-a)h}$$

$$\sigma_{nom} = \frac{12 \times 10^3}{[60-12]10}$$

$$\boxed{\sigma_{nom} = 25\,N/mm^2}$$

k_t Refer Diagram <DB 7.10>

$$a/w = \frac{12}{60} = 0.2$$

$$\boxed{k_t = 2.5}$$

$$\sigma_{max} = 2.5 \times 25$$

$$\boxed{\sigma_{max} = 62.5\,N/mm^2}$$

2. stepped bar.

$$k_t = \frac{\sigma_{max}}{\sigma_{non.}} \Rightarrow \sigma_{max} = k_t \times \sigma_{nom}$$

$$\sigma_{nom} = \frac{Load}{Area} = \frac{Load}{\text{Area of stepped bar}}$$

$$\sigma_{nom} = \frac{12 \times 10^3}{\pi/4 \times 25^2}$$

$$\boxed{\sigma_{nom} = 24.4 \ N/mm^2}$$

k_t Refer DB Pg no <7.11>

x axis $r/d = \frac{5}{25} = 0.2$

y axis k_t

$$D/d = \frac{50}{25} = 2$$

$k_t = 1.5$

$$\sigma_{max} = 1.5 \times 24.5$$

$$\boxed{\sigma_{max} = 36.75 \ N/mm^2}$$

3. A bar of circular cross section is subjected to alternating ~~to~~ tensile force varying from a minimum of 200KN to a maximum of 500 KN. It is to be manufactured of a material with an ultimate tensile strength of 900MPa and a endurance limit of 700MPa. Determine the diameter of bar using ~~of~~ safety factors of 3.5 related to ultimate tensile strength and 4 related to endurance limit and a stress concentration of 1.65 for fatigue load. Use Goodman stright Line as basis for design.

Given data:

minimum Tensile Load $W_{min} = 200KN = 200\times10^3 N$

maximum Tensile Load $W_{max} = 500KN = 500\times10^3 N$

Ultimate tensile strength $\sigma_u = 900MPa = 900 N/mm$

endurance Limit $\sigma_{-1} = 700 MPa = 700 N/mm^2$

Factor of safety for ultimate strength $n_u = 3.5$

FOS for endurance limit $n_{\sigma_{-1}} = 4$

Fatigue Stress concentration Factor $K_f = 1.65$

To Find:

Diameter of circular section 'd'

Soln

~~Good~~

Goodman Eqn <7.67

$$\frac{1}{n} = K_t \left[\frac{\sigma_m}{\sigma_u} + \frac{\sigma_a}{\sigma_{-1}} \right] \qquad <7.67>$$

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

$$\sigma_{max} = \frac{Load}{Area} = \frac{W_{max}}{A} = \frac{500 \times 10^3}{\pi/4 \times d^2}$$

$$\sigma_{max} = \frac{500 \times 10^3}{0.785 \times d^2}$$

$$\sigma_{min} = \frac{W_{min}}{Area} = \frac{200 \times 10^3}{\pi/4 \times d^2} = \frac{200 \times 10^3}{0.785 \times d^2}$$

$$\sigma_m = \frac{\frac{500 \times 10^3}{0.785 d^2} + \frac{200 \times 10^3}{0.785 d^2}}{2}$$

$$\sigma_m = \frac{1}{0.785 d^2} \times \frac{500 \times 10^3 + 200 \times 10^3}{2 \cancel{d^2}}$$

$$\sigma_m = \frac{350 \times 10^3}{0.785 d^2}$$

$$\boxed{\sigma_m = \frac{445.85 \times 10^3}{d^2}}$$

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{\frac{500\times10^3}{0.785d^2} - \frac{200\times10^3}{0.785d^2}}{2}$$

$$\sigma_a = \frac{1}{0.785d^2} \times \frac{500\times10^3 - 200\times10^3}{2}$$

$$\sigma_a = \frac{1}{0.785d^2} \times \frac{300\times10^3}{2}$$

$$\sigma_a = \frac{150\times10^3}{0.785d^2}$$

$$\boxed{\sigma_a = \frac{191.08\times10^3}{d^2}}$$

$k_f = 1.6$ → fatigue stress concentration factor.

To find k_t — Theoretical stress conc. factor

$$k_f = 1 + q[k_t - 1] \quad \langle 7.6 \rangle$$

notch sensitivity factor. $q = 1$ [assumed]

$$1.6 = 1 + 1[k_t - 1]$$

$$1.6 - 1 = 1[k_t - 1]$$

$$0.6 = k_t - 1 \Rightarrow k_t = 1 + 0.6$$

$$\boxed{k_t = 1.6}$$

$$\sigma_u = \frac{\cancel{900}}{\cancel{3}}$$

$$\sigma_u = \frac{900}{FS_u} = \frac{900}{3.5}$$

$$\sigma_u = 257.14 \ N/mm^2$$

$$\sigma_{-1} = \frac{700}{4} = 175 \ N/mm^2$$

$$\frac{1}{n_u} = k_t \left[\frac{\sigma_m}{\sigma_u} + \frac{\sigma_a}{\sigma_{-1}} \right]$$

$$\frac{1}{3.5} = 1.6 \left[\frac{\frac{445.85 \times 10^3}{d^2}}{257.14} + \frac{\frac{191.08 \times 10^3}{d^2}}{175} \right]$$

$$\frac{1}{3.5} = 1.6 \left[\frac{445.85 \times 10^3}{257.14\, d^2} + \frac{191.08 \times 10^3}{175\, d^2} \right]$$

$$\frac{1}{3.5} = 1.6 \left[\frac{1733.88}{d^2} + \frac{1091.88}{d^2} \right]$$

$$\frac{1}{3.5} = \frac{1.6}{d^2} \times \left[1733.88 + 1091.88 \right]$$

$$\frac{1}{3.5} = \frac{1.6}{d^2} \times 2825.76$$

$$\frac{1}{3.5} = \frac{4521.216}{d^2}$$

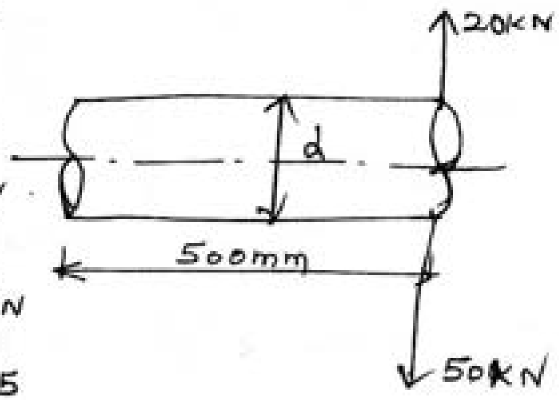
$$d^2 = \frac{4521.216}{3.5} = 1291.77$$

$$d = \sqrt{1291.77} = 35.94$$

$$\boxed{d = 36 \, mm}$$

4) A circular bar of 500 mm length is supported freely at its two ends. It is acted upon by a central concentrated cyclic Load having a minimu value of 20 kN and a maximum value of 50 kN. Determine the diameter of bar by taking a factor of safety of 1.5, size effect of 0.85, surface finis factor of 0.9. The material properties of bar are given by: ultimate strength of 650 MPa, yield streng of 500 MPa and endurence strength of 350 MPa.

Given



Length $l = 500mm$

minimum Load $W_{min} = 20 \times 10^3 N$

maximum Load $W_{max} = 50 \times 10^3 N$

Factor of safety $n = 1.5$

Size effect factor $K_{sz} = 0.85$

Surface finish factor $K_{sf} = 0.9$

Ultimate strength $\sigma_u = 650\ MPa = 650\ N/mm^2$

Yield strength $\sigma_y = 500\ MPa = 500\ N/mm^2$

Endurance strength $\sigma_{-1} = 350\ MPa = 350\ N/mm^2$

To Find:

Diameter of shaft d.

soln

By soderberg Eqn <7.6>

$$\frac{1}{n} = \frac{\sigma_m}{\sigma_y} + K_f \times \frac{\sigma_a}{\sigma_{-1} \times K_{sz} \times K_{sf}}$$

$$\left.\begin{aligned} n &= 1.5 \\ \sigma_y &= 500\ N/mm^2 \\ \sigma_{-1} &= 350\ N/mm^2 \\ K_{sz} &= 0.85 \\ K_{sf} &= 0.9 \end{aligned}\right\} \text{given}$$

$\sigma_m =$ mean stress <7.6>

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

σ_{max} By ~~beging~~ bending Relation

$$\frac{M_b}{I} = \frac{\sigma}{y} \quad \langle 7.1 \rangle$$

$$\sigma = \frac{M_b \times y}{I}$$

$$\sigma_{max} = \frac{M_{bmax} \times y}{I}$$

M_b = bending Moment = Load x Dis

$$M_{bmax} = \frac{\text{Maximum Load} \times \text{Distance}}{4}$$

$$M_{bmax} = \frac{50 \times 10^3 \times 500}{4}$$

$$M_{bmax} = 6.25 \times 10^6\ N.mm$$

$$M_{bmin} = \frac{20 \times 10^3 \times 500}{4}$$

$$M_{bmin} = 2.5 \times 10^6\ N.mm$$

$$Y = d/2$$

$$I = \pi/64 \times d^4$$

$$I/y = z$$

$$\sigma_{max} = \frac{6.25\times10^6 \times d/2}{\pi/64 \times d^4}$$

$$\sigma_{max} = \frac{6.25\times10^6 \times \cancel{d}}{\pi/\cancel{64}_{32} \times d^{\cancel{4}3} \times \cancel{2}}$$

$$\sigma_{max} = \frac{6.25\times10^6}{\pi/32 \times d^3}$$

$$\boxed{\sigma_{max} = \frac{63.66\times10^6}{d^3}}$$

$$\sigma_{min} = \frac{M_{bmin} \times \cancel{y}}{\cancel{I}} \quad \langle 7.1 \rangle$$

$$\sigma_{min} = \frac{2.5\times10^6 \times \cancel{d}}{\pi/\cancel{64}_{32} \times \cancel{2} \times d^{\cancel{4}3}}$$

$$\boxed{\sigma_{min} = \frac{25.46\times10^6}{d^3}}$$

$$\sigma_m = \frac{\frac{63.66\times10^6}{d^3} + \frac{25.46\times10^6}{d^3}}{2}$$

$$\boxed{\sigma_m = \frac{44.56\times10^6}{d^3}}$$

σ_a: Amplitude stress

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$$

$$\sigma_a = \frac{\frac{63.66\times10^6}{d^3} - \frac{25.46\times10^6}{d^3}}{2}$$

$$\boxed{\sigma_a = \frac{19.1\times10^6}{d^3}}$$

Fatigue ~~stress~~ concentration stress factor.

$$\frac{1}{n} = \frac{\sigma_m}{\sigma_y} + K_f \frac{\sigma_a}{\sigma_{-1}\times K_{sz}\times K_{sf}}$$

$K_f = 1$ assumed.

$$\frac{1}{1.5} = \frac{\frac{44.56\times10^6}{d^3}}{500} + 1\times\frac{\frac{19.1\times10^6}{d^3}}{350\times0.85\times0.89}$$

$$\frac{1}{1.5} = \frac{44.56\times10^6}{500\, d^3} + \frac{19.1\times10^6}{350\times0.85\times0.89\times d^3}$$

$$\frac{1}{1.5} = \frac{89.12\times10^3}{d^3} + \frac{72.13\times10^3}{d^3}$$

$$\frac{1}{1.5} = \frac{1}{d^3}\left[89.12\times10^3 + 72.13\times10^3\right]$$

$$\frac{1}{1.5} = \frac{1}{d^3}\times161.25\times10^3$$

$$d^3 = 161.25\times10^3\times1.5 = 241.88\times10^3$$

$$d^3 = 241.88 \times 10^3$$

$$d = [241.88 \times 10^3]^{1/3}$$

$$\boxed{d = 62.3 \text{ mm}}$$

Say

~~d=~~

(ii) Goodman Eqn

$$\frac{1}{n} = k_t \left[\frac{\sigma_m}{\sigma_u} + \frac{\sigma_a}{\sigma_{-1} \times k_{sz} \times k_{sf}} \right]$$

$$\frac{1}{1.5} = 1 \times \left[\frac{\frac{44.56 \times 10^6}{d^3}}{650} + \frac{\frac{19.1 \times 10^6}{d^3}}{350 \times 0.85 \times 0.89} \right] \quad k_t = 1$$

$$\frac{1}{1.5} = \frac{68.55 \times 10^3}{d^3} + \cancel{19.1 \times 10^6} \frac{72.13 \times 10^3}{d^3}$$

$$\frac{1}{1.5} = \frac{140.68 \times 10^3}{d^3}$$

$$d^3 = \cancel{140} \; 140.68 \times 10^3 \times 1.5 = 211.025 \times 10^3$$

$$d = [211.025 \times 10^3]^{1/3}$$

Ans $\boxed{d = 59.53 \text{ mm}}$

5. A 500mm diameter shaft is made from carbon steel having ultimate tensile strength of 630 MPa. It is subjected to a torque which fluctuates b/w 2000 N.m to − 800 N.m. Using Soderberg method, Calculate the factor of safety. Assume suitable value for any other data needed.

Given:

Material: carbon steel

Ultimate tensile strength $\sigma_u = 630$ MPa

Maximum Torque $T_{max} = 2000$ N.m $= 2000 \times 10^3$ N.mm

minimum Torque $T_{min} = -800$ N.m $= -800 \times 10^3$ N.mm

Diameter of shaft $d = 500$ mm

To Find

Factor of safety (n)

soln

Soderberg Eqn <7.1>

$$\boxed{\frac{1}{n} = \frac{\tau_m}{\tau_y} + k_f \frac{\tau_a}{\tau_{-1} \times}} \; k_{sz} \times k_{sf}$$

$$\tau_m = \frac{\tau_{max} + \tau_{min}}{2}$$

~~τ_{max}~~ :

By using Torsion Eqn <7.1>

$$\tau = \frac{M_t \times r}{J} \qquad M_t = T$$

r = radius of shaft

$r = d/2$

$J = \pi/32 \times d^4$

$$\tau_{max} = \frac{T_{max} \times r}{J}$$

$$\tau_{max} = \frac{200 \times 10^3 \times d/2}{\pi/32 \times d^4}$$

$$\tau_{max} = \frac{200 \times 10^3 \times \cancel{d}}{\cancel{\pi/32}_{16} \times d^{\cancel{4}\,3} \times \cancel{2}}$$

$$\tau_{max} = \frac{200 \times 10^3}{0.196 \times \cancel{d^3}\, 500^3}$$

$$\cancel{\boxed{\tau_{max} = \frac{1.018 \times 10^6}{d^3}\ N/mm^2}}$$

$$\boxed{\tau_{max} = 0.0186\ N/mm^2}$$

$$\tau_{min} = \frac{T_{min} \times r}{J} = \frac{-800 \times 10^3 \times \cancel{d/2}}{\cancel{\pi/32}_{16} \times d^{\cancel{4}\,3}} = \frac{-800 \times 10^3}{\pi/16 \times 500^3}$$

$$\cancel{\boxed{\tau_{min} = \frac{-4.07 \times 10^6}{d^3}\ N/mm^2}}$$

$$\boxed{\tau_{min} = -0.0325\ N/mm^2}$$

$k_f = 1$ (assume).

DB

Endurance limit stress $\tau_{-1} = 0.22\,\sigma_u$ <1.42>

$$\tau_{-1} = 0.22 \times 200$$

$$\boxed{\tau_{-1} = 44\ N/mm^2}$$

$\tau_y = \frac{\sigma_y}{2}$ <7.67>

σ_y = yield strength

For carbon steel material <DB 1.9> For C_{45} s

$\sigma_y = 36 \ kgf/mm^2$

$\sigma_y = 36 \times 10 \ N/mm^2$

$\sigma_y = 360 \ N/mm^2$

$\tau_y = \frac{360}{2} \Rightarrow \boxed{\tau_y = 180 \ N/mm^2}$

$K_{SF} = 0.87$
$K_{SZ} = 0.85$ } assume.

d $\tau_{max} = \frac{1.018 \times 10^6}{d^3} = \frac{1.018 \times 10^6}{500^3}$

~~$\tau_{max} = 8.14 \times 10^{-3} \ N/mm^2$~~

~~$\tau_{min} = \frac{-4.07 \times 10^6}{d^3} = \frac{-4.07 \times 10^6}{500^3}$~~

~~$\tau_{min} = -0.0325 \ N/mm^2$~~

$\tau_m = \frac{\tau_{max} + \tau_{min}}{2} = \frac{0.0186 + (-0.0325)}{2}$

$\boxed{\tau_m = -6.95 \times 10^{-3} N/mm^2}$

$\tau_a = \frac{\tau_{max} - \tau_{min}}{2} = \frac{0.0186 - (-0.0325)}{2}$

$\boxed{\tau_a = 0.0255 \ N/mm^2}$

$$\frac{1}{n} = \frac{\tau_m}{\tau_y} + k_f \frac{\tau_a}{\tau_{-1} \, k_{sz} \times k_{sf}} \qquad \langle 7.6 \rangle$$

$$\frac{1}{n} = \frac{\overset{-6.95\times10^{-3}}{\cancel{-0.0121}}}{180} + 1 \times \frac{0.0255}{44 \times 0.85 \times 0.87}$$

$$\cancel{\frac{1}{n} = -6.72\times10^{-5} + 6.23\times10^{-4}}$$

$$\cancel{\frac{1}{n} = 5.56\times10^{-4}}$$

$$\frac{1}{n} = -3.86\times10^{-5} + 7.83\times10^{-4}$$

$$\frac{1}{n} = 7.44\times10^{-4}$$

$$\frac{1}{7.44\times10^{-4}} = n$$

$$\boxed{n = 1343}$$

6) A cantilever beam made of cold drawn carbon steel of circular cross - section as shown in Fig. is subjected to a load which varies from -F to 3F. Determine the maximum load that this member can withstand for an indefinite life using factor of safety as 2. The theoretical stress concentration factor is 1.42 and the notch sensitivity is 0.9. Assume the following values

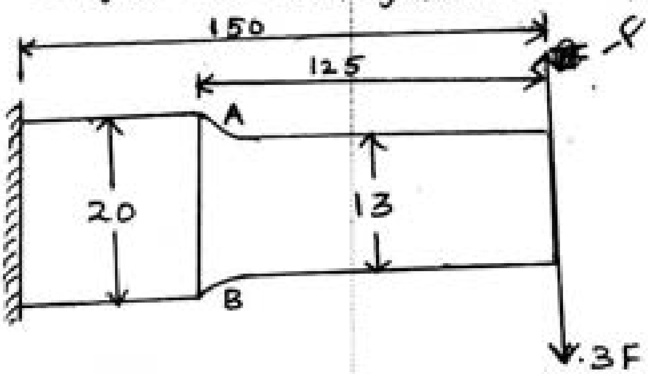
Ultimate stress = 550 Mpa

Yield stress = 470 MPa

Endurence Limit = 275 MPa

size factor = 0.85

Surface finish factor = 0.89



Given:

Minimum Load $W_{min} = -F$

Maximum Load $W_{max} = 3F$

FOS $n = 2$

Stress Concentration factor $K_t = 1.42$

not sensitivity $q = 0.9$

$\sigma_u = 550 \ N/mm^2$

yield stress $\sigma_y = 470 \ N/mm^2$

Endurance stress $\sigma_{-1} = 275 \ N/mm^2$

size factor $K_{sz} = 0.85$

Surface finish factor $K_{sf} = 0.89$

dia of smallend $d = 13 mm$

dia of large end $D = 20 mm$

Length of beam $L = 150 mm$.

Length of small dia $l = 125 mm$

To Find:

Maximum Load 'F'

Soln

~~Soode~~r

Soderberg Eqn

$$\frac{1}{n} = \frac{\sigma_m}{\sigma_y} + k_f \frac{\sigma_a}{\sigma_{-1} \times k_{sz} \times k_{sF}} \quad \langle 7.6 \rangle$$

σ_m: mean stress

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} \quad \langle 7.6 \rangle$$

$$\sigma_{max} = \frac{M_{b} \times y}{I} \quad \langle 7.1 \rangle$$

M_{b} = Bending Moment at Point A = Load $\times$ Distance.

$M_{b\,max}$ = W_{max} $\times$ Length of small shaft

$$M_{bmax} = 3F \times 125 \quad \Rightarrow \text{Load at end.}$$

$$\boxed{M_{bmax} = 375\,F}$$

$$y = d/2 = \frac{13}{2} =$$

$$\boxed{y = 6.5\,mm}$$

I = Moment of inertia

$$I = \pi/64 \times d^4 = \pi/64 \times 13^4$$

$$\boxed{I = 1401.98\,mm^4}$$

$$\sigma_{max} = \frac{M_{bmax} \times y}{I} = \frac{375F \times 6.5}{1401.98}$$

$$\sigma_{max} = \frac{2437.5\,F}{1401.98}$$

$$\boxed{\sigma_{max} = 1.73\,F}$$

$$\sigma_{min} = \frac{M_{bmin} \times y}{I}$$

$$M_{b\,min} = W_{min} \times Dis = -F \times 125$$

$$\boxed{M_{bmin} = -125\,F}$$

$$\sigma_{min} = \frac{-125F \times 6.5}{1401.98}$$

$$\boxed{\sigma_{min} = -0.58\,F}$$

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = \frac{1.73F + (-0.58F)}{2}$$

$$\boxed{\sigma_m = 0.58F} \ N/mm^2$$

σ_a = amplitude stress

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{1.73F - (-0.58F)}{2}$$

$$\boxed{\sigma_a = 1.15F \ N/mm^2}$$

σ_y = yield stress

$\boxed{\sigma_y = 470 \ N/mm^2}$ → given

$\sigma_{-1} = 275 \ N/mm^2$

$k_{sf} = 0.89$

$k_{sz} = 0.85$

$k_t = 1.42$

$k_f = 1 + q(k_t - 1)$ → <7.6>

$= 1 + 0.9[1.42 - 1]$

$\boxed{k_f = 1.378}$

FOS $n = 2$

$$\frac{1}{n} = \frac{\sigma_m}{\sigma_y} + k_f \frac{\sigma_a}{\sigma_{-1} \times k_{s2} \times k_{sf}}$$

$$\frac{1}{2} = \frac{0.58F}{470} + 1.378 \frac{1.15F}{275 \times 0.85 \times 0.89}$$

$$\frac{1}{2} = 1.23 \times 10^{-3} F + 7.61 \times 10^{-3} F$$

$$\frac{1}{2} = 8.84 \times 10^{-3} F$$

$$F = \frac{1}{2 \times 8.84 \times 10^{-3}}$$

$$\boxed{F = 56.51 \text{ N}}$$

Goodman Eqn

$$\frac{1}{n} = k_t \left[\frac{\sigma_m}{\sigma_u} + \frac{\sigma_a}{\sigma_{-1} \times k_{s2} \times k_{sf}} \right]$$

$$\frac{1}{2} = 1.42 \left[\frac{0.58F}{550} + \frac{1.15F}{275 \times 0.85 \times 0.89} \right]$$

$$\frac{1}{2} = 1.497 \times 10^{-3} F + 7.84 \times 10^{-3} F$$

$$\frac{1}{2} = 9.34 \times 10^{-3} F$$

$$F = \frac{1}{2 \times 9.34 \times 10^{-3}}$$

$$\boxed{F = 53.49 \text{ N}}$$

7. A steel cantilever is 200mm long. It is subjected to an axial load which varies from 150N (compress) to 450N (tension) and also a transverse load at its free end which varies from 80N ~~and~~ up to 120N down. The cantilever is of circular cross section. It ~~is~~ ~~is~~ is of diameter 2d for the first 50mm and of diameter d for remaining length. Determine its diameter taking a factor of safety of 2. Assume the following Data

Yield stress = 330MPa

Endurance limit in reversed loading = 300 MPa

Correction factor = 0.7 in reversed axial load

= 1.0 in reversed bending

Stress Concentration Factor = 1.44 for bending

= 1.64 for axial loading

Size effect Factor = 0.85

Surface effect factor = 0.90

Notch Sensitivity Factor = 0.90

Given Data:

Length of cantilever l = 200mm

minimum Axial Load W_{min} = -150 N

maximum Axial Load W_{max} = 450N

Maximum transverse Load $W_{t\,max} = 120N$

Minimum transverse Load $W_{t\,max} = -80N$

Fos $n = 2$

Yield stress $\sigma_y = 330 N/mm^2$

Endurance strength $\sigma_{-1} = 300 N/mm^2$

Correction factor $k_a = 0.7$

$k_b = 1$

Stress concentration factor $k_{ta} = 1.64$

$k_{tb} = 1.44$

Size effect factor $k_{sz} = 0.85$

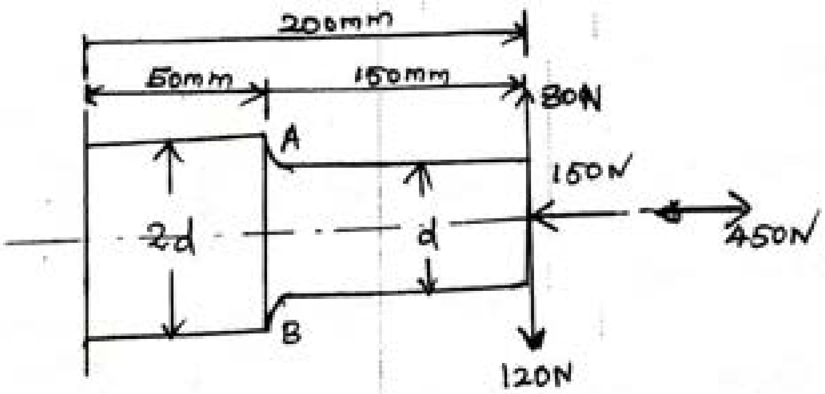
Surface effect factor $k_{SF} = 0.9$

Notch sensitivity index $= q = 0.9$

<u>To Find:</u>

Diameter of Shaft:

<u>Soln</u>



$$\cancel{\frac{1}{n} = \left[\left(\frac{\sigma_{eq}}{\sigma_y}\right)^2 + \left(\frac{\tau_{eq}}{\tau_y}\right)^2\right]^{1/2}} \quad \cancel{\angle 7.67}$$

$$\sigma_{Total} = \sigma_{eq\,a} + \sigma_{eq\,trans}$$

σ_{eq_a} = Equivalent stress in axial Load at point A

$$\sigma_{eq_a} = \sigma_m + k_f \frac{\sigma_a \sigma_y}{\sigma_{-1} \times K_a \times \text{[illegible]} \times K_{sz} \times K_{sf}} \quad \angle 7.67$$

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} \quad \angle 7.67$$

$$\sigma_{max} = \frac{Load}{Area} = \frac{W_{max}}{Area} = \frac{450}{\pi/4 \times d^2}$$

$$\boxed{\sigma_{max} = \frac{572.95}{d^2}} \quad N/mm^2$$

$$\sigma_{min} = \frac{W_{min}}{Area} = \frac{-150}{\pi/4 \times d^2}$$

$$\boxed{\sigma_{min} = -\frac{190.98}{d^2}}$$

σ_{max}

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = \frac{\frac{572.95}{d^2} + \left(-\frac{190.98}{d^2}\right)}{2}$$

$$\boxed{\sigma_{max} = \frac{381.96}{\cancel{d^2}}} \qquad \boxed{\sigma_m = \frac{190.98}{d^2}} \quad N/mm^2$$

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{572.95/d^2 - (-190.98/d^2)}{2}$$

$$\boxed{\sigma_a = \frac{381.96}{d^2}} \; N/mm^2$$

$$k_f = 1 + q\,[k_{ta} - 1] \quad \text{—— <7.1>}$$

$$k_f = 1 + 0.9\,[1.64 - 1]$$

$$\boxed{k_f = 1.576}$$

$$\sigma_{eq} = \frac{190.98}{d^2} + \frac{1.576 \times \frac{381.96}{d^2} \times 330}{300 \times 0.7 \times [illegible] \times 0.89 \times 0.9}$$

$$\sigma_{eq} = \frac{190.98}{d^2} + \cancel{\frac{720.04}{d^2}} \; \frac{1180.96}{d^2}$$

$$\cancel{\boxed{\sigma_{eq_a} = \frac{911.07}{d^2}}} \; \cancel{N/mm^2} \qquad \boxed{\sigma_{eqa} = \frac{1371.94}{d^2} \; N/mm^2}$$

$$\cancel{\tau_{eq} = \tau_m + k_f \frac{\tau_a \tau_y}{\tau_{-1} \times k_1 \times k_{bb} \times k_{sz} \times k_{sf}}}$$

$$\cancel{\tau_m = \frac{\tau_{max} + \tau_{min}}{2} \quad <7.1>}$$

$$\cancel{\tau_{max} =}$$

~~1180~~

~~1371~~

$\sigma_{eq_{b}}$ = Equivalent stress in ~~axial~~ transverse Load at point A

So $\sigma_{eq_b} = \sigma_m + k_y \dfrac{\sigma_a \sigma_y}{\sigma_{-1} \times K_b \times [redacted] \times K_{sz} \times K_{sf}}$

$\sigma_m = \dfrac{\sigma_{max} + \sigma_{min}}{2}$ $\qquad y = d/2$

$\sigma_{max} = \dfrac{M_b \times y}{I}$ $\qquad I = \pi/64 \times d^4$

M_b = Load x Distance [End Load]

$M_{b\,max} = W_{t\,max} \times L = 120 \times 150$

$M_{b\,max} = 18 \times 10^3$ N.mm

Reversed Bending so -ive

$M_{b\,min} = W_{t\,min} \times L = -80 \times 150$

$M_{b\,min} = -12 \times 10^3$ N.mm

$\dfrac{M_b}{I} = \dfrac{\sigma_{max}}{y}$

$\sigma_{max} = \dfrac{M_{b\,max} \times y}{I}$

$= \dfrac{M_{b\,max} \times d}{2 \times \pi/64 \times d^4}$ (32, d^3)

$\sigma_{max} = \dfrac{M_{b\,max} \times y}{I} = \dfrac{18 \times 10^3 \times \cancel{d}}{2 \times \pi/64 \times d^{\cancel{4}\,3}}$ (32)

$\sigma_{max} = \dfrac{183.34 \times 10^3}{d^3}$ N/mm²

$\sigma_{min} = \dfrac{M_{b\,min} \times y}{I} = \dfrac{-12 \times 10^3 \times \cancel{d}}{2 \times \pi/64 \times d^{\cancel{4}\,3}}$ (32)

$\sigma_{min} = \dfrac{-122.23 \times 10^3}{d^3}$ N/mm²

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = \frac{183.34 \times 10^3/d^3 + (122.23 \times 10^3)/d^3}{2}$$

$$\sigma_m = \frac{61.109 \times 10^3}{d^3 \times 2}$$

$$\boxed{\sigma_m = \frac{30.55 \times 10^3}{d^3}} \; N/mm^2$$

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{\frac{183.34 \times 10^3}{d^3} - \left(-\frac{122.23 \times 10^3}{d^3}\right)}{2}$$

$$\boxed{\sigma_a = \frac{152.78 \times 10^3}{d^3}} \; N/mm^2$$

$$K_f = 1 + q\left[K_{t_b} - 1\right] \quad \langle 7.8 \rangle$$

$$= 1 + 0.9\left[1.44 - 1\right]$$

$$\boxed{K_f = 1.396}$$

$$\sigma_{eq_b} = \frac{30.55 \times 10^3}{d^3} + 1.396 \times \frac{\frac{152.78 \times 10^3}{d^3} \times 330}{300 \times 1 \times \text{[redacted]} \times 0.85 \times 0.9}$$

$$\sigma_{eq_b} = \frac{30.55 \times 10^3}{d^3} + \cancel{\frac{212.97 \times 10^3}{d^3}} \; \frac{292.89 \times 10^3}{d^3}$$

$$\cancel{\boxed{\sigma_{eq_b} = \frac{243.52 \times 10^3}{d^3}}} \qquad \boxed{\sigma_{eq_b} = \frac{323.44 \times 10^3}{d^3}}$$

Total equivalent stress at point A

$$\sigma_{Total\ eq} = \sigma_{eq_b} + \sigma_{eq_a}$$

$$\sigma_{tot} = \frac{\overset{1371.94}{\cancel{911.07}}}{d^2} + \frac{\overset{323.44}{\cancel{243.52}} \times 10^3}{d^3}$$

$$\sigma_{total} = \frac{\sigma_y}{Fos} = \frac{330}{2} = 165$$

$$165 = \frac{\overset{1371.94}{\cancel{911.07}}}{d^2} + \frac{\overset{323.44}{\cancel{243.52}} \times 10^3}{d^3}$$

$$165 = \frac{\overset{1371.94d + 323.44 \times 10^3}{\cancel{911.07d + 243.52 \times 10^3}}}{d^3}$$

$$165 d^3 = \overset{1371.94d + 323.44 \times 10^3}{\cancel{911.07d + 243.52 \times 10^3}}$$

By trial and error Method

$\cancel{\boxed{d = 11.54\ mm}}$

$\boxed{d = 12.73\ mm}$

$\cancel{2100.45}$

$165d^3 - 1371.94d - 323.44 \times 10^3 = 0$

8. A hot rolled steel shaft is subjected to a torsional moment that various from 330 N.m clockwise to 110 N.m counter clockwise and an applied bending ~~Momment~~ Moment at a critical section varies from 440 N.m to −220 N.m. The shaft is of uniform cross section and no key way is pressed at critical section. Determine the required shaft diameter. The material has an ultimate strength of 550 ~~N/mm~~ MN/m^2 and a yield strength of 410 MN/m^2. Take the endurance limit as half the ultimate strength, ~~load~~ factor of safety of 2, size factor of 0.85 and a surface finish factor of 0.62.

Given:

$$T_{max} = 330\ N.m\ (cw)$$
$$= 330 \times 10^3\ N.mm$$
$$T_{min} = -110\ N.m\ (ccw) = -110 \times 10^3\ N.mm$$

$$M_{b\,max} = 440\ N.m = 440 \times 10^3\ N.mm$$

$$M_{b\,min} = -220\ N.m = -220 \times 10^3\ N.mm$$

$$\sigma_u = 550\ MN/m^2 = 550\ N/mm^2$$

$$\sigma_y = 410\ MN/m^2 = 410\ N/mm^2$$

$$\sigma_{-1} = \frac{\sigma_u}{2} = \frac{550}{2}$$

$$\sigma_{-1} = 275\ N/mm^2$$

Fos $n = 2$ $\quad K_{sz} = 0.85 \quad K_{sf} = 0.62$

<u>To Find</u>

Diameter of shaft. d

<u>Soln</u>

Combined stress

$$\frac{1}{n} = \left[\left(\frac{\sigma_{eq}}{\sigma_y}\right)^2 + \left(\frac{\tau_{eq}}{\tau_y}\right)^2\right]^{1/2} \quad \angle 7.6\rangle$$

$$\sigma_{eq} = \sigma_m + k_f \frac{\sigma_a \sigma_y}{\sigma_{-1}} \quad \angle 7.6\rangle$$

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

$$\sigma_{max} = \frac{M_{bmax} \times y}{I}$$

$$\sigma_{max} = \frac{440 \times 10^3 \times \cancel{d}}{\pi/\cancel{64}_{32} \times d^{\cancel{4}3} \times \cancel{2}}$$

$$\sigma_{max} = \frac{4.481 \times 10^6}{d^3} \ N/mm^2$$

$$\sigma_{min} = \frac{M_{bmin} \times Y}{I} = \frac{-220 \times 10^3 \times \cancel{d}}{\pi/\cancel{64}_{32} \times d^{\cancel{4}3} \times \cancel{2}}$$

$$\boxed{\sigma_{min} = \frac{-2.24 \times 10^6}{d^3}}$$

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = \frac{\frac{4.481 \times 10^6}{d^3} + \frac{(-2.24 \times 10^6)}{d^3}}{2}$$

$$\boxed{\sigma_m = \frac{1.12 \times 10^6}{d^3}}$$

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{\frac{4.481 \times 10^6}{d^3} - \frac{(-2.24 \times 10^6)}{d^3}}{2}$$

$$\boxed{\sigma_a = \frac{3.36 \times 10^6}{d^3}}$$

$k_f = 1$ (assume)

$\sigma_y = 410\ N/mm^2$

$\sigma_{-1} = 275\ N/mm^2$

$$\sigma_{eq} = \frac{1.12 \times 10^6}{d^3} + 1 \times \frac{\frac{3.36 \times 10^6}{d^3} \times 410}{275 \times 0.85 \times 0.62}$$

$$\sigma_{eq} = \frac{1.12 \times 10^6}{d^3} + \frac{9.50 \times 10^6}{d^3}$$

$$\boxed{\sigma_{eq} = \frac{10.62 \times 10^6}{d^3}}\ N/mm^2$$

$$\tau_{eq} = \tau_m + k_f \frac{\tau_a \tau_y}{\tau_{-1} \times k_{sz} \times k_{sf}}$$

$$\tau_m = \frac{\tau_{max} + \tau_{min}}{2}$$

$$\tau_{max} = \frac{T_{max} \times r}{J} = \frac{330 \times 10^3 \times \cancel{d}}{\frac{\pi}{\cancel{32}_{16}} \times d^{\cancel{4}3} \times \cancel{2}}$$

$$\boxed{\tau_{max} = \frac{1.68 \times 10^6}{d^3}} \ N/mm^2$$

$$\tau_{min} = \frac{T_{min} \times r}{J} = \frac{-110 \times 10^3 \times \cancel{d}}{\frac{\pi}{\cancel{32}_{16}} \times d^{\cancel{4}3} \times \cancel{2}}$$

$$\boxed{\tau_{min} = \frac{-560 \times 10^3}{d^3}}$$

$$\tau_m = \frac{\frac{1.68 \times 10^6}{d^3} + \frac{-560 \times 10^3}{d^3}}{2}$$

$$\boxed{\tau_m = \frac{560 \times 10^3}{d^3}} \ N/m^2$$

$$\tau_a = \frac{\tau_{max} - \tau_{min}}{2} = \frac{\frac{1.68 \times 10^6}{d^3} - \frac{-560 \times 10^3}{d^3}}{2}$$

$$\boxed{\tau_a = \frac{1.12 \times 10^6}{d^3}}$$

$k_f = 1$ assume

$$\tau_y = \frac{\sigma_y}{2} \quad \langle 7.1 \rangle$$

$$\tau_y = \frac{410}{2}$$

$$\tau_y = 205 \ N/mm^2$$

$$\tau_{-1} = 0.22 \ \sigma_u \quad \langle 1.42 \rangle$$

$$\tau_{-1} = 0.22 \times 550$$

$$\tau_{-1} = 121 \ N/mm^2$$

$$\tau_{eq} = \frac{560 \times 10^3}{d^3} + 1 \times \frac{\frac{1.12 \times 10^6}{d^3} \times 205}{121 \times 0.85 \times 0.62}$$

$$\tau_{eq} = \frac{560 \times 10^3}{d^3} + \frac{3.6 \times 10^6}{d^3}$$

$$\boxed{\tau_{eq} = \frac{4.16 \times 10^6}{d^3}} \ N/mm^2$$

$$\frac{1}{n} = \left[\left(\frac{\sigma_{eq}}{\sigma_y} \right)^2 + \left(\frac{\tau_{eq}}{\tau_y} \right)^2 \right]^{1/2}$$

$$\frac{1}{2} = \left[\left(\frac{\frac{10.62 \times 10^6}{d^3}}{410} \right)^2 + \left(\frac{\frac{4.16 \times 10^6}{d^3}}{205} \right)^2 \right]^{1/2}$$

Both side squaring

$$\left[\frac{1}{2}\right]^2 = \left(\frac{10.62\times10^6}{410\times d^3}\right)^2 + \left(\frac{4.16\times10^6}{205\times d^3}\right)^2$$

$$\frac{1}{4} = \left(\frac{25.90\times10^3}{d^3}\right)^2 + \left(\frac{20.29\times10^3}{d^3}\right)^2$$

$$\frac{1}{4} = \frac{670.81\times10^6}{d^6} + \frac{411.79\times10^6}{d^6}$$

$$\frac{1}{4} = \frac{1.09\times10^9}{d^6}$$

$$d^6 = 1.09\times10^9\times4$$

$$d^6 = 4.33\times10^9$$

$$d = [4.33\times10^9]^{1/6}$$

Ans $\boxed{d = 40\,mm}$

9. A pulley is keyed to a shaft ~~to a~~ midway b/w Two bearings. The shaft is made of cold draw steel for which the Ultimate strength is 550MPa and yield strength is 440 MPa. The ~~bed~~ bending moment at the pulley varies from -150 N.m to $+400$ N.m as the torque on the shaft various from

-50 N.m to $+150$ N.m. Obtain the diameter of the shaft for an indefinite life. The stress concentration factor for ~~at~~ the key ~~way~~ way at the pulley in bending and in torsion are 1.6 and 1.3 respectively. Take the following values.

Factor of safety $= 1.5$

Load correction factor $= 1$ in bending,
0.6 in torsion.

Size effect factor $= 0.85$

Surface effect factor $= 0.88$.

H.W Refer Previous Problem.

$\boxed{\text{Ans } d = 35\text{mm}}$

A. C- Clamp is subjected to a max. load of 'W' as shown in Fig. If the max. tensile stress in the clamp is limited to 140 MPa. Find the value of load W.

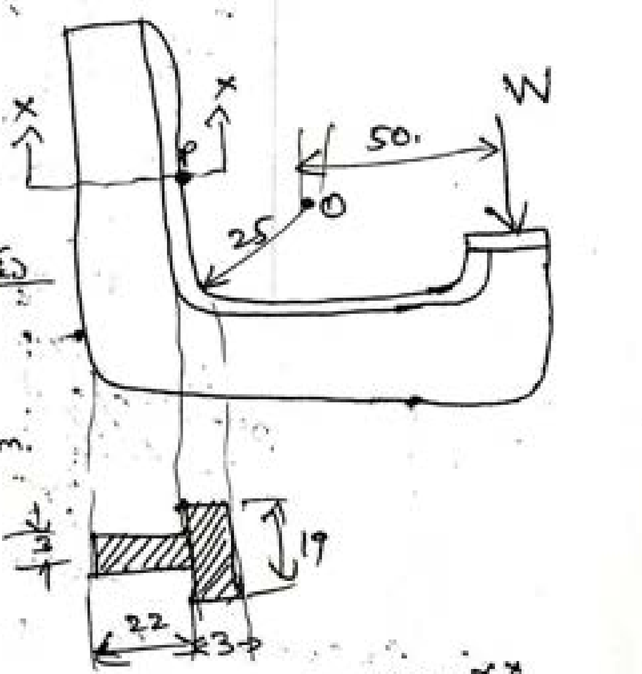
Given:

$\sigma_t (max) = 140\ N/mm^2$

$R_i = 25 mm$; $R_o = 25 + 25 = 50\ mm$.

$b_i = 19\ mm$, $t_i = 3 mm$; $t = 3mm$; $h = 25m$.

Area. $A = (3 \times 22) + (3 \times 19) = 123\ mm^2$



Section - XX

$R_n = 31.64\ mm$.

$R = 33.2\ mm$.

Calculation of eccentricity:

$e = R - R_n = 1.56\ mm$

$x = 50 + R$

$= 50 + 33.2 = 83.2 mm$.

$M = W \times x$.

$= W \times 83.2 mm$

$\boxed{M = 83.2 W \quad N\text{-}mm}$

Calculation of Direct tensile stress:

Direct stress at section XX

$\sigma_t = \frac{W}{A}$.

$= \frac{W}{123} = \frac{1}{123} \times W$

$\sigma_t = 0.008 W\ N/mm^2$

$h_i = 6.64\ mm$.

max. Bending stress at point P,

Calculation of Bending Stress:

$\sigma_{bi} = \frac{M\ h_i}{A e r_i} = 0.115 W\ N/mm^2$.

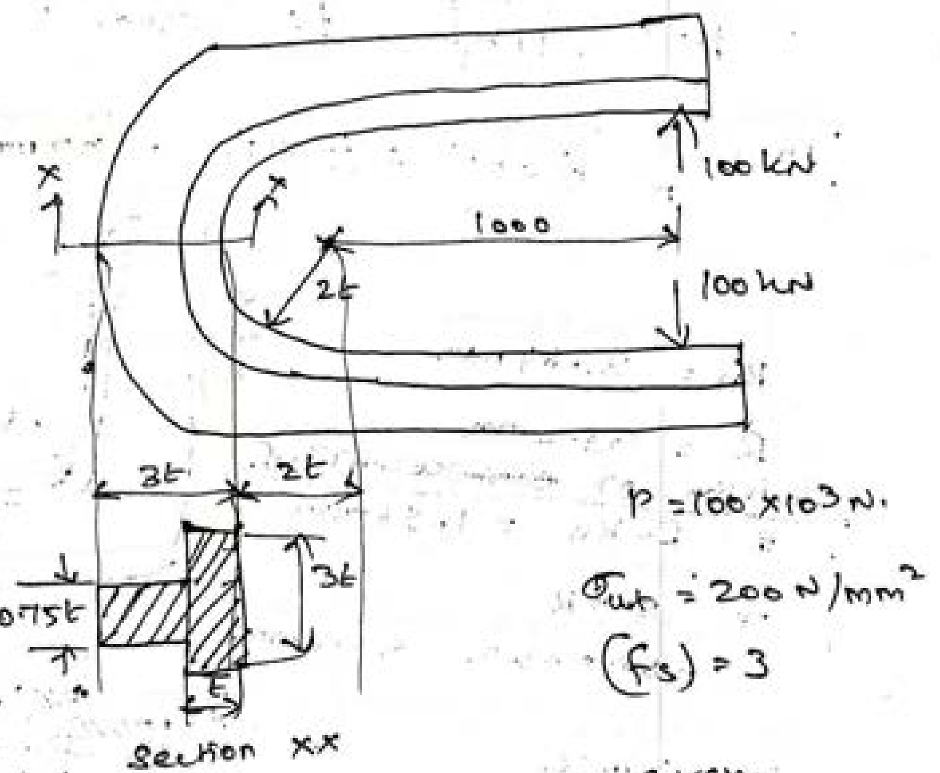
WKT, Calculation of load

$140 = \sigma_t + \sigma_{bi}$.

$140 = 0.008 W + 0.115 W$

$\boxed{W = 1138 N}$

The C- frame of a ~~[illegible]~~ 100 kN Capacity press as shown in fig. The material of the frame is grey CI FG 200 and the factor of safety is 3. Determine the Dimensions of the frame.



$P = 100 \times 10^3$ N.

$\sigma_{ut} = 200\ N/mm^2$

$(f_s) = 3$

(i). Calculation of permissible Tensile Stress.

$$\sigma_{max} = \frac{\sigma_{at}}{(f_s)} = \frac{200}{3} = 66.67\ N/mm^2.$$

(ii) Calculation of eccentricity (e)

$\boxed{e = R - R_N}$

$b_i = 3t, \quad h = 3t, \quad R_i = 2t$

$R_o = 5t, \quad t_i = t \quad t = 0.75b.$

$R_N = 2.8134t$

$R = 3t.$

$\boxed{e = 0.1866t}$

UNIT-2 II PART

DESIGN OF COUPLING

Introduction:

Shaft are usually available up to 7 metres Length due to inconvenience in transport. In order to have a greater length, it becomes necessary to Join two or more pieces of shaft by means of a coupling.

Purpose:

→ To provide for the connection of shaft of unit that are manufactured separately such as a motor and generator and to provide for disconnection for repairs or alternations.

→ To provide for the ~~misalinment~~ misallignment of the shaft or introduce mechanical flexibility.

→ To reduce the Transmission of shock loads from one shaft to another.

→ To introduce protection against overloads.

→ It should have no projecting parts.

⇒ Requirements of a Good Shaft coupling:

* It should be easy to connect or disconnect.
* It should transmit the full power from one shaft to the other shaft with out Losses.
* It should hold the shafts in perfect alignment.
* It should reduce the transmission of shock loads from one shaft to another shaft.
* It should have no Projecting Parts.

⇒ ~~Th~~ Types of coupling:

~~1. Flexibl~~

1. Rigid coupling

It is used to connect two shafts which are perfectly aligned. types of Rigid coupling

a. sleeve (or) muff coupling.

b. clamp (or) split-muff or compression coupling.

c. Flange coupling.

2. Flexible coupling.

It is used to connect two shafts having both lateral and angular misallignment. types

a. Bushed pin type coupling.

b. Universal coupling

c. Old ham coupling.

Design of Muff Coupling:

It is simplest type of rigid coupling, made of cast iron. It consists of a hollow cylinder whose inner diameter is the same as that of the shaft. It is fitted over the ends of the two shaft by means of a gib head key.

The power is transmitted from one shaft to another ~~a~~ shaft by means of key and a sleeve.

Diagram Reference <D.B XEROX 14>

~~Design Procedure for the muff coup~~

Components of coupling [Muff]

1. shaft
2. sleeve or muff
3. key.

Material..

1. Shaft - castiron
2. Sleeve - castiron
3. key - castiron

Design Procedure for sleeve or muff coupling

Step I Design of shaft (d)

$$T = \frac{\pi}{16} \times [\tau_s] \times d^3$$

$T = T_{max}$

Power $P = \frac{2\pi NT}{60}$. T in N.m.

Step-II

Dimension of muff coupling. < D.B XEROX]

D = outside dia of sleeve = 2d + 13

L = Length of sleeve L = 3.5d

Step-III Design of sleeve. <D.B XEROX a>

$$T = \frac{\pi}{16} \times \tau_{cm} \left[\frac{D^4 - d^4}{D}\right]$$

$\tau_c = ?$

$\tau_{cm} \leq [\tau_{cm}]$

Design is safe

Step-IV Design of key <D.B XEROX a>

Dimension of key <D.B 5.19>

w = width of key.
t = thickness of key
l = length of key = $\frac{L}{2}$

a. failure of induced shear.

$$T = l \times w \ \tau_{sk} \ d/2$$

$$\tau_s = ?$$

$$\tau_s < [\tau_{sk}]$$

Design is safe.

b. failure of crushing stresses

$$T = l \times [t/2] \ \sigma_{ck} \ [d/2]$$

$$\sigma_c = ?$$

$$\sigma_c < [\sigma_{ck}]$$

Design is safe.

Problems:

1. Design and make a neat dimensioned sketch of a muff coupling which is used to connect two steel shaft transmitting 40kW at 350 rpm. The material for the shaft and key is plain carbon steel for which allowable shear and crushing stresses may be taken as 40 MPa and 80MPa respectively. The material for the muff is castiron for which the allowable may be assumed as 15 MPa.

Given data:

Power $P = 40\,kW = 40 \times 10^3\,W$

Speed $N = 350$ rpm

allowable shear stress for shaft & key.

$$[\tau_s] = [\tau_{sk}] = 40\,MPa = 40\,N/mm^2$$

Crushing stress in key $[\sigma_{ck}] = 80\,MPa = 80\,N/mm^2$

allowable shear stress for muff $[\tau_{cm}] = 15$ MPa =

$[\tau_{cm}] = 15$

To find

Design the muff coupling.

Soln.

Step-I Design of shaft diameter 'd'

$$T = \pi/16 \times [\tau_s]\, d^3$$

Power $P = \frac{2\pi N T}{60}$

$$T = \frac{P \times 60}{2 \times \pi \times N} = \frac{40 \times 10^3 \times 60}{2 \times \pi \times 350}$$

$T = 1100$ N.m

$T = 1100 \times 10^3$ N.mm

$$1100 \times 10^3 = \pi/16 \times 40 \times d^3$$

$\boxed{d = 52\text{ mm}}$ say

$\boxed{d = 55\text{ mm}}$

Step-II Dimension of sleeve coupling <D.B xeror>

D: outside dia of sleeve $= 2d + 13 = 2 \times 52 + 13$

$\boxed{D = 117\text{ mm}}$

L: Length of sleeve $= 3.5\,d = 3.5 \times 52$

$\boxed{L = 182\text{ mm}}$

5 N/mm

Step-III Design of sleeve <DB xerox 9>

$$T = \pi/16 \; \tau_{cm} \left[\frac{D^4 - d^4}{D} \right]$$

$$1100\times10^3 = \frac{\pi}{16} \times \tau_{cm} \left[\frac{117^4 - 52^4}{117} \right]$$

$$1100\times10^3 = \pi/16 \times \tau_{cm} \left[1.53\times10^6 \right]$$

$$1100\times10^3 = \pi/16 \times \tau_{cm} \times 1.53\times10^6$$

$$1100\times10^3 = 302.2\times10^3 \; \tau_{cm}$$

$$\tau_{cm} = \frac{1100\times10^3}{302.2\times10^3}$$

$$\boxed{\tau_{cm} = 3.63 \; N/mm^2}$$

$$[\tau_{cm}] = 15 \; N/mm^2$$

$$\tau_{cm} \leq [\tau_{cm}]$$

Design is safe.

Step-IV Design of key <D.B xerox 9>

Dimension of key for <5.19>

For shaft diameter 52 mm

Width $b = 16$ mm

thickness $t = 10$ mm

Length of key $= \frac{L}{2} = \frac{182}{2} = 91$ mm

1. Check for Induced shear

$$T = l \times w \times \tau_{sk} \times d/2$$

$$1100 \times 10^3 = 91 \times 16 \times \tau_{sk} \times \frac{52}{2}$$

$$1100 \times 10^3 = 37856 \times \tau_{sk}$$

$$\boxed{\tau_{sk} = 29.05 \ N/mm^2}$$

$$[\tau_{sk}] = 40 \ N/mm^2$$

$$\tau_{sk} < [\tau_{sk}]$$

Design is safe.

2. Check for crushing stress

$$T = l \times [t/2] \times \sigma_{ck} \times \frac{d}{2}$$

$$1100 \times 10^3 = 91 \times \frac{10}{2} \times \sigma_{ck} \times \frac{52}{2}$$

$$1100 \times 10^3 = 11.83 \times 10^3 \ \sigma_{ck}$$

$$\sigma_{ck} = 92.98 \ N/mm^2$$

$$[\sigma_{ck}] = 80 \ N/mm^2$$

Design is not safe

∴ safer value increase the length of key

$$l = 110 mm$$

$$1100 \times 10^3 = 110 \times \frac{10}{2} \times \sigma_{ck} \times 52/2$$

$$\sigma_{ck} = 76.92 \ N/mm^2$$

Design is safe.

2. Design a muff coupling to connect two shaft transmitting 40kW at 120rpm. The permissible shear and crushing stress for the shaft and key material are 30 MPa and 80 MPa respectively. The material for muff is cast iron with permissible shear stress. of 15 MPa. Assume that the maximum Torque is 25% greater than the mean Torque.

τ_s $\sigma_{cs} = \sigma_{cr}$

<u>Given Data:</u>

$$\text{Power} \quad P = 40\,kW = 40 \times 10^3\,W$$

$$\text{Speed} \quad N = 120 \text{ rpm}$$

Permissible shear stress for shaft & key

$$[\tau_s] = [\tau_{sk}] = 30\text{ MPa}$$
$$= 30\text{ N/mm}^2$$

Permissible crushing stress for key

$$[\sigma_c k] = 80\text{ MPa} = 30\text{ N/mm}^2$$

Permissible shear stress for muff $[\tau_{cm}] = 15$ MPa

$$[\tau_{cm}] = 15\text{ N/mm}^2$$

$$T = 1.25\, T_{min}$$

<u>To Find:</u>

Design a muff coupling

$\frac{125}{100} = 1.25$

$\frac{25}{100} = 0.25$ $1.25 = 1.25$

Soln

Step-I Design of shaft diameter 'd'

$$T = \pi/16 \times [\tau_s] \times d^3$$

$$T = 1.25\ T_{min}$$

$$\text{Power } P = \frac{2\pi N T_{min}}{60}$$

$$40 \times 10^3 = \frac{2 \times \pi \times 120 \times T_{min}}{60}$$

$$T_{min} = 3.183 \times 10^3 \text{ N.m}$$

$$T_{min} = 3.183 \times 10^6 \text{ N.mm}$$

$$T = 1.25 \times T_{min} = 1.25 \times 3.183 \times 10^6$$

$$\boxed{T = 3.97 \times 10^6 \text{ N.mm}}$$

$$3.97 \times 10^6 = \pi/16 \times 30 \times d^3$$

$$d = 87.6 \text{ mm} \quad \text{say}$$

$$\boxed{d = 88 \text{ mm}}$$

Step-II Dimension of ~~coupling~~ sleeve coupling

$$D = \text{Diameter of sleeve} = 2d + 13 = 2 \times 88 + 13$$

$$\boxed{D = 189 \text{ mm}}$$

$$L = \text{Length of sleeve} = 3.5 \times d = 3.5 \times 88$$

$$\boxed{L = 308 \text{ mm}}$$

Step-III Design of sleeve <D.B XEROX 9>

$$T = \pi/16 \times \tau_{cm} \left[\frac{D^4 - d^4}{D}\right]$$

$$3.97 \times 10^6 = \pi/16 \times \tau_{cm} \left[\frac{189^4 - 88^4}{189}\right]$$

$$\tau_{cm} = 3.15 \text{ N/mm}^2$$

$$[\tau_{cm}] = \cancel{30} \; 15 \text{ N/mm}^2$$

$$\tau_{cm} < [\tau_{cm}]$$

Design is safe.

$\tau_{cm} \Rightarrow$ shear stress on muff

Step-IV Design of key

Dimension of key <D.B 5.19>

For shaft diameter 88 mm

width of key - 25 mm

thickness of key - 14 mm

Length of key $l = \frac{L}{2} = \frac{308}{2} = 154\text{mm}$

(i) check for Induced shear

$$T = l \times w \times \tau_{sk} \times d/2$$

$$3.97 \times 10^6 = 154 \times 25 \times \tau_{sk} \times \frac{88}{2}$$

$$\boxed{\tau_{sk} = 23.43 \text{ N/mm}^2}$$

$$[\tau_{sk}] = 50 \text{ N/mm}^2$$

$\tau_{sk} < [\tau_{sk}]$ Design is safe.

(ii) Check for ~~indirect~~ crushing stress

$$T = l \times t/2 \times \sigma_{ck} \times d/2$$

$$3.97 \times 10^6 = 154 \times \frac{14}{2} \times \sigma_{ck} \times 88/2$$

$$\sigma_{ck} = 83.69 \ N/mm^2$$

$$[\sigma_{ck}] = 80 \ N/mm^2$$

Design is not safe

Safer value increase in length of key

$$l = 170 \ mm$$

$$3.97 \times 10^6 = 170 \times \frac{14}{7} \times \sigma_{ck} \times 88/2$$

$$\sigma_{ck} = 75.82 \ N/mm^2$$

Design is safe.

3) Design a muff coupling to connect two steel shafts transmitting 25kW power at 360 rpm. The shaft and key are made of plain carbon steel 30C8 [$S_{yt} = S_{yc} =$ 400 N/mm²]. The sleeve is made made of grey cast iron FG200 ($S_{ut} = 200$ N/mm²). The factor of safety for shaft and key is 4. For sleeve, the factor of safety is 6 based on ultimate strength.

Given

Power = 25 kW N = 360 rpm $\tau_{cM} = \frac{200}{6} N/mm^2$

$$[\tau_s] = [\tau_{sk}] = \frac{400}{4} N/mm^2 \quad \sigma_{ck} = \frac{400}{4} N/mm^2$$

⇒ Design of Flange coupling

A Flange Coupling usually applies to a coupling have two separate castiron flange.

Each Flange is mounted on the shaft and keyed to it. The faces are turned up at right angle to the Axis of the shaft.

The Two flanges are coupled together by means of bolt and Nuts.

Major Parts of Flange coupling

1. Flange
2. Hub
3. shaft
4. bolt
5. Key

Material for Flange coupling

Shaft, Bolt, Key } steel.

Flange, hub } castiron.

→ Design Procedure for Unprotected type Flange coupling or Rigid type Flange coupling:

Step I Design of shaft diameter 'd'

$$T = \pi/16 \times [\tau_s]_{all} \times d^3$$

$$\text{Power } P = \frac{2\pi N T_{mean}}{60 \times 1000}$$

$$T_{mean} = ?$$

$$T_{mean} = T.$$

If service factor is given $T = S.F \times T_{mean}$

(or)

T or T_{max} = over Torque × T_{mean}.

$[\tau_s]_{all}$ = allowable shear stress for shaft material.

Step-2 Dimension: of Coupling (xerox pg no: 9)

D = Diameter of hub (or) outside dia = 2d

D_1 = Dia of bolt circle = 3d

D_2 = outside dia of flange = 4d

L = Length of hub = 1.5d

t_f = thickness of flange

~~dr~~ =

n = No of bolt.

Step-3: Design of bolt

a. Dia of bolt:

a. $T = n \times \pi/4 \times d_1^2 \times [\tau_b]_{all} \times \left[\frac{D_1}{2}\right]$

$d_1 =$ diameter of bolt find.

b: Check for crushing stress

$T = n \times d_1 \times t_f \times \sigma_{cb} [D_1/2]$

$\sigma_{cb} = ?$

$\sigma_{cb} < [\sigma_{cb}]$

Design is safe.

Step-4 Design of key.

For key Dimension <PSG D.B 5.16>

$l =$ Length of key $= l = L$

$w =$ width of key

$t =$ thickness of key

a. Check for Induced shear stress

$T = l \times w \times \tau_k \times d/2$

$\cancel{t_k = ?}$ $\tau_k = ?$

$\tau_k < [\tau_k]_{all}$ Design is safe.

τ_k is Not safe

safer value Increase the length of key

b. Check for crushing stress.

$$T = l \times (t/2) \times \sigma_c \times (d/2)$$

$\sigma_c = ?$

$\sigma_c < [\sigma_c]_{all}$

Design is safe.

σ_c is Not safe

safer value increase the Length of key

Step-5 Design of hub.

$$T = \pi/16 \times \tau_c \times \left[\frac{D^4 - d^4}{D}\right]$$

$\tau_c = ?$

$\tau_c < [\tau_c]_{all}$

Design is safe.

Step 6 Design of flange

$$T = \pi \times \left[\frac{D^2}{2}\right] \times \tau_c \times t_f$$

$\tau_c = ?$

$\tau_c < [\tau_c]_{all}$ Design is safe.

Value collection for $[\tau_s]_{all}$, $[\tau_b]_{all}$ and $[\tau_k]_{all}$, $[\sigma_s]$ $[\sigma_{ck}]_{all}$ $[\sigma_{cb}]_{all}$

$[\tau_s]_{all} = \frac{[\sigma_s]_{all}}{n}$ n=2 for steel.

$[\sigma_s]_{all} = \frac{\sigma_y}{n} = \frac{\sigma_y}{2}$

σ_y = yield stress <PSG DB 1.9>

$\sigma_y = 36$ kgf/mm² = 360 N/mm²

Value collection for $[\tau_c]_{all}$

$[\tau_c]_{all} = \frac{\sigma_u}{n} = \frac{\sigma_u}{9}$

n = 9 for CI

σ_u = <PSG.DB. 1.4>

$\sigma_u = 220$ N/mm²

Problem based on Rigid ~~com~~ coupling (or) UnProtective type.

1. Design a rigid type of flange coupling to connect two shafts. The Input shaft transmits 37.5 kW power at 180 rpm. to the output shaft through the coupling. The service factor for the application is 1.5. select suitable material for various parts of the coupling.

Given

Power $P = 37.5\ kW = 37.5 \times 10^3\ W$

Speed $N = 180\ rpm$

Service factor $= 1.5$

Find:

Design the Rigid type coupling

Soln

Step-I Diameter of shaft 'd'

$$T = \pi/16 \times [\tau_s]_{all} \times d^3$$

$$P = \frac{2\pi N T}{60 \times 1000} \Rightarrow 37.5 \times 10^3 = \frac{2 \times \pi \times 180 \times T}{60 \times 1000}$$

$T = 1.989 \times 10^6\ N.mm$

$T = SF \times 1.989 \times 10^6 = 1.5 \times 1.989 \times 10^6$

$$\boxed{T = 2.98 \times 10^6\ N.mm}$$

$[\tau_s]_{all} = ?$

Soln

Assume shaft, key, bolt material steel.

From PSG DB

$C_{40} \rightarrow$ carbon steel $\angle 1.9 \rangle$

$\sigma_u = 63 - 70 \text{ kgf/mm}^2$

$\sigma_u = 70 \text{ kgf/mm}^2 = 70 \times 10$

$\sigma_u = 700 \text{ N/mm}^2$

$\sigma_y = 36 \text{ kgf/mm}^2 = 36 \times 10$

$\sigma_y = 360 \text{ N/mm}^2$

$\tau = \frac{\sigma_y}{2} = \frac{360}{2}$

$n = $

$[\tau_s]_{all} = \frac{[\sigma_s]_{all}}{n}$

$[\tau_s]_{all} = \frac{[\sigma_s]_{all}}{2}$ $\quad n = 2$ For steel.

$[\sigma_s]_{all} = \frac{\sigma_y}{n} = \frac{\sigma_y}{2}$

$[\sigma_s]_{all} = \frac{360}{2} = 180 \text{ N/mm}^2$

$[\tau_s]_{all} = \frac{180}{2} = 90 \text{ N/mm}^2$

$[\tau_s]_{all} = [\tau_b]_{all} = [\tau_k]_{all} = 90 \text{ N/mm}^2$

$[\sigma_s]_{all} = [\sigma_{cb}]_{all} = [\sigma_{ck}]_{all} = 180 \text{ N/mm}^2$

$$2.98\times10^6 = \pi/16 \times 90 \times d^3$$

$$d^3 = \frac{16\times 2.98\times10^6}{\pi\times 90}$$

$$\boxed{d = 55.24\ mm}$$

standard diameter $\boxed{d = 55\ mm}$ D.B <7.20>

Step-2 Dimension of coupling.

d = Dia of shaft $= 55mm$

D = out side dia of hub $= 2d = 2\times55 = 110\ mm$

D_1 = Dia of bolt circle $= 3d = 3\times55 = 165\ mm$

D_2 = Diameter of outer flange $= 4d = 4\times55 = 220mm$

t_f = thickness of flange $= d/2 = \frac{55}{2} = 27.5mm$

L = Length of flange $= 1.5d = 1.5\times55 = 82.5\ mm$

n = No of bolts $= 4$ Nos.

Step-3 Design of bolt

$$T = n\times \pi/4 \times d_1^2 \times [\tau_b]_{all} \times \frac{D_1}{2}$$

$$2.98\times10^6 = 4\times \pi/4 \times d_1^2 \times 90 \times \frac{165}{2}$$

$$\boxed{d_1 = 11.3\ mm}$$

check for crushing stress.

$$T = n \times d_1 \times t_f \times \sigma_{cb} \times \frac{D_1}{2}$$

$$2.98 \times 10^6 = 4 \times 11.3 \times 27.5 \times \sigma_{cb} \times \frac{165}{2}$$

$$\boxed{\sigma_{cb} = 29.05\ N/mm^2}$$

$$[\sigma_{cb}]_{all} = 180\ N/mm^2$$

$$\sigma_{cb} < [\sigma_{cb}]_{all}$$

Design is safe.

Step-4 Design of key:

For key Dimension <PSG D.B 5.16>

For shaft dia $d = 55$ mm

W = width of Key = 16 mm

t = Thickness of key = 10 mm

l = Length of key = $l = L = 82.5$ mm.

a. Check for Induced shear stress

$$T = l \times w \times \tau_k \times d/2$$

$$2.98 \times 10^6 = 82.5 \times 16 \times \tau_k \times \frac{55}{2}$$

$$\tau_k = 82 \ N/mm^2$$

$$[\tau_k]_{all} = 90 \ N/mm^2$$

$$\tau_k < [\tau_k]_{all}$$

Design is safe.

b. Check for crushing stress

$$T = l \times t/2 \times \sigma_{ck} \times \frac{d}{2}$$

$$2.98 \times 10^6 = 82.5 \times \frac{10}{2} \times \sigma_{ck} \times \frac{55}{2}$$

$$\sigma_{ck} = 262.69 \ N/mm^2$$

$$[\sigma_{ck}]_{all} = 180 \ N/mm^2$$

$$\sigma_{ck} > [\sigma_{ck}]_{all}$$

Design is not safe

So safer value increase the length of key

$l = 125mm$

$$2.98 \times 10^6 = 115 \times \frac{10}{2} \times \sigma_{ck} \times \frac{55}{2}$$

$$\sigma_{ck} = 173.32 \ N/mm^2$$

$$\sigma_{ck} < [\sigma_{ck}]_{all}$$

Design is safe

Step-5 Design of hub

$$T = \pi/16 \times \tau_c \times \left[\frac{D^4 - d^4}{D}\right]$$

$$2.98 \times 10^6 = \pi/16 \times \tau_c \left[\frac{110^4 - 55^4}{110}\right]$$

$$\boxed{\tau_c = 12.16 \ N/mm^2}$$

$$[\tau_c]_{all} = ?$$

For hub & Flange Material is castiron

$$[\tau_c]_{all} = \frac{\sigma_u}{n} = \frac{\sigma_u}{9}$$

<PSGDB . 1.4>

$n = FOS$

$n = 9$ [Force]

CI 220 SAE $\sigma_u = 220 \ N/mm^2$

$$[\tau_c]_{all} = \frac{220}{9}$$

$$[\tau_c]_{all} = 24.4\ N/mm^2$$

$$\tau_c < [\tau_c]_{all}$$

Design is safe.

Step-6 Design of Flange

$$T = \pi \times \frac{D^2}{2} \times \tau_c \times t_f$$

$$2.98 \times 10^6 = \pi \times \frac{110^2}{2} \times \tau_c \times 27.5$$

$$\tau_c = 5.70\ N/mm^2$$

$$\tau_c < [\tau_c]_{all}$$

Design is safe.

2. Design a cast iron flange coupling for a mild steel shaft transmitting 90KW at 250rpm, the allowable shear stress in the shaft ~~Material as a solid~~ is 40 MPa and Angle of twist is not to exceed 1° in a length of 20meters The allowable shear stress in the coupling bolt is 30 MPa. Take $G = 84 \, KN/mm^2$.

soln

Given:

Power $P = 90KW = 90 \times 10^3 W$

Speed $N = 250rpm$

allowable Shear stress for shaft $[\tau_s]_{all} = 40MPa = 40 N/mm^2$

Angle of twist $\theta = 1^\circ = \pi/180 = 0.0175$ radian

Allowable shear stress for bolt $[\tau_b]_{all} = 30MPa = 30N/mm^2$

Rigidity Modulus $G = 84 \, KN/mm^2 = 84 \times 10^3 \, N/mm^2$

To Find:

Design a CI Flange coupling.

Soln

step-I Design of shaft diameter 'd'

$$\boxed{T = \pi/16 \times d^3 \times [\tau_s]_{all}}$$

Power $P = \dfrac{2\pi N T_{mean}}{60 \times 1000}$

$$90 \times 10^3 = \frac{2 \times \pi \times 250 \times T_{mean}}{60 \times 1000}$$

$$\boxed{T_{mean} = 3440 \times 10^3 \text{ N.mm}}$$

$$T_{mean} = T = 3440 \times 10^3 \text{ N.mm}$$

$$T = 3.44 \times 10^6 \text{ N.mm}$$

$$3.44 \times 10^6 = \frac{\pi}{16} \times d^3 \times 40$$

$$d^3 = \frac{16 \times 3.44 \times 10^6}{\pi \times 40}$$

$$\boxed{d = 75.94 \text{ mm}}$$

Standard dia $\boxed{d = 80 \text{ mm}}$ <PSG DB 7.20>

Considering Rigidity of shaft

$$\frac{T}{J} = \frac{G\theta}{L}$$

$$\frac{3.44 \times 10^6}{\pi/32 \times d^4} = \frac{84 \times 10^3 \times 0.0175}{20}$$

15

$$\frac{3.44\times10^6}{0.0981 d^4} = 73.5$$

$$d^4 = \frac{3.44\times10^6}{0.0981\times73.5}$$

$$d^4 = 4.77\times10^5$$

$$d = \left[4.77\times10^5\right]^{1/4}$$

$$\boxed{d = 26.28 mm}$$

Fa Standard dia d = <PSG D.B7.20>

Taking Largest of the two values.

$$\boxed{d = 80 mm}$$

Step-2 Dimension of coupling

D = outside dia of hub = 2d = 2×80 = 160mm

D_1 = Dia of bolt circle = 3d = 3×$\cancel{160}$ 80 = 240mm

D_2 = Dia of outer flange = 4d = 4×80 = 320mm

L = Length of flange = 1.5×d = 1.5×80 = 120mm

t_f = thickness of flange = $d/2$ = $\frac{80}{2}$ = 40mm

n = No of bolt = 4 Nos.

Step-

Step-3 Design of bolt

$$T = n \times \pi/4 \times d_1^2 \times [\tau_b]_{all} \times \frac{D_1}{2}$$

$$3.44 \times 10^6 = 4 \times \pi/4 \times d_1^2 \times 30 \times \frac{240}{2}$$

$$\boxed{d_1 = 18 \text{ NOS}}$$

$$: [\tau_b]_{all} = 30 N/m$$

check for crushing stress.

$$T = n \times d_1 \times t_f \times \sigma_{cb} \times \frac{D_1}{2}$$

$$3.44 \times 10^6 = 4 \times 18 \times 40 \times \sigma_{cb} \times \frac{240}{2}$$

$$\boxed{\sigma_{cb} = 9.95 \ N/mm^2}$$

$$[\tau_s]_{all} = \frac{[\sigma_s]_{all}}{2}$$

$$[\sigma_s]_{all} = [\tau_s]_{all} \times 2 = 40 \times 2 = 80 \ N/mm^2$$

$$[\sigma_s]_{all} = [\sigma_{cb}]_{all} = [\sigma_{ck}]_{all} = 80 \ N/mm^2$$

$$[\tau_s]_{all} = [\tau_k]_{all} = 40 \ N/mm^2$$

$$\sigma_{cb} < [\sigma_{cb}]_{all}$$

Design is safe.

Step-4 Design of key

key Dimension <PSG 0.B.5.16>

For shaft dia $d = 80mm$

width of key $w = 22mm$

thickness of key $t = 14mm$

length of key = Length of flange = 120mm

$l = 120mm$

a. check for Induced shear stress

$$T = l \times w \times \tau_k \times d/2$$

$$3.44 \times 10^6 = 120 \times 22 \times \tau_k \times \frac{80}{2}$$

$$\tau_k = 32.57 \ N/mm^2$$

$$[\tau_k]_{all} = 40 \ N/mm^2$$

Design is safe.

b. check for crushing stress

$$T = l \times \frac{t}{2} \times \sigma_{ck} \times \frac{d}{2}$$

$$3.44 \times 10^6 = 120 \times \frac{14}{2} \times \sigma_{ck} \times \frac{80}{2}$$

$$\sigma_{ck}] = 102.38 \ N/mm^2$$

$\sigma_{ck} > [\sigma_{ck}]_{all}$ Design is Not safe.

safer value Increase the length of key

$$l = 160\,mm$$

$$3.44 \times 10^6 = 160 \times \frac{14}{2} \times \sigma_{ck} \times \frac{80}{2}$$

$$\sigma_{ck} = 76.78\ N/mm^2$$

$$\sigma_{ck} < [\sigma_{ck}]_{all}$$

Design is safe.

Step-5 Design of hub

$$T = \pi/16 \times \tau_c \times \left[\frac{D^4 - d^4}{D}\right]$$

$$3.44 \times 10^6 = \pi/16 \times \tau_c \times \left[\frac{160^4 - 80^4}{160}\right]$$

$$\boxed{\tau_c = 4.56\ N/mm^2}$$

$$[\tau_c]_{all}$$

For hub & flange material is cast iron.

$$[\tau_c]_{all} = \frac{\sigma_u}{n} = \frac{\sigma_u}{9}$$ | $n = FOS = 9$ for CI

<PSG D.B 1.4>

CI ~~F~~20 SAE $\sigma_u = 220\ N/mm^2$

$$[\tau_c]_{all} = \frac{220}{9}$$

$$[\tau_c]_{all} = 24.4\ N/mm^2$$

$$\tau_c < [\tau_c]_{all}$$

Design is safe

Step 6 Design of Flange

$$T = \pi \times \frac{D^2}{2} \times \tau_c \times t_f$$

$$3.44 \times 10^6 = \pi \times \frac{160^2}{2} \times \tau_s \times 40$$

$$\boxed{\tau_s = 2.13 \ N/mm^2}$$

$$\tau_s < [\tau_s]_{all}$$

Design is safe.

3. Design a rigid flange coupling to transmit a torque of 250N.m b/w two coaxial shafts. The shaft is made of alloy steel, flanges out of cast iron and bolts out of steel. Four bolt are used to couple the flanges. The shafts are keyed to the flange hub. The permissible stresses are given below

Shear stress on shaft = 100Mpa

Bearing (or) crushing stress on shaft = 250Mpa shear stress on keys = 100 Mpa.

Bearing stress on keys = 250 Mpa.

shearing stress on CI = 200 MPa

Shearing stress on bolts = 100 MPa

After designing the various Elements, make a neat sketch of the assembly indicating the important dimensions. The stresses developed in various members may be checked if thumb rules are using for fixing the dimensions.

Given data:

Torque $T_{mean} = T = 250\ N.m = 250 \times 10^3\ N.mm$

No of bolt $n = 4$

Permissible stresses on shaft $[\tau_s]_{all} = 100\ MPa = 100\ N/mm^2$

crushing stress on shaft $[\sigma_s]_{all} = 250\ MPa = 250\ N/mm^2$

shear stress on key $[\tau_K]_{all} = 100\ MPa = 100\ N/mm^2$

Bearing stress on Key $[\sigma_{cK}]_{all} = 250\ N/mm^2$

shearing stress on CI $[\tau_c]_{all} = 200\ MPa = 200\ N/mm^2$

Shearing stress on bolt $[\tau_b]_{all} = 100\ MPa = 100\ N/mm^2$

To Find:

Design of Rigid Flange coupling.

Soln

StepI Design of shaft diameter 'd'

$$T = \pi/16 \times d^3 \times [\tau_s]_{all}$$

$$250\times10^3 = \frac{\pi}{16} \times d^3 \times 100$$

$$d^3 = 12.73\times10^3$$

$$d = 23.35\ mm$$

say

Standard dia <PSG D.B.7.20>

$$\boxed{d = 25mm}$$

Step-2 Dimension of coupling..

D = out side dia of hub = 2d = 2×25 = 50mm

D_1 = Dia of bolt circle = 3d = 3×25 = 75 mm

D_2 = Dia of outer flange = 4d = 4×25 = 100mm

L = Length of flange = 1.5d = 1.5×25 = 37.5 mm

t_f = thickness of flange = $\frac{d}{2} = \frac{25}{2}$ = 12.5mm

n = No of bolt = 4 Nos given.

Step-3 Design of bolt

$$T = n \times \pi/4 \times d_1^2 \times [\tau_b]_{all} \times \frac{D_1}{2}$$

$$250 \times 10^3 = 4 \times \frac{\pi}{4} \times d_1^2 \times 100 \times \frac{75}{2}$$

$$\boxed{d_1 = 4.6\,mm}$$

Check for crushing stress

$$T = n \times d_1 \times t_f \times \sigma_{cb} \times \frac{D_1}{2}$$

$$250 \times 10^3 = 4 \times 4.6 \times 12.5 \times \sigma_{cb} \times \frac{75}{2}$$

$$\boxed{\sigma_{cb} = 28.98\ N/mm^2}$$

$$[\tau_b]_{all} = \frac{[\sigma_{cb}]_{all}}{2} \Rightarrow 100 = \frac{[\sigma_{cb}]_{all}}{2}$$

$$[\sigma_{cb}]_{all} = 200\ N/mm^2$$

$\sigma_{cb} < [\sigma_b]_{all}$ Design is safe.

Step-4 Design of key

Key dimension <PSG D.B 5.16>

Width of key $w = 8mm$

thickness of key $t = 7mm$

Length of key $l = L = 37.5mm$

a. Check for Induced shear stress

$$T = l \times w \times \tau_k \times \frac{d}{2}$$

$$250 \times 10^3 = 37.5 \times 8 \times \tau_k \times \frac{25}{2}$$

$$\boxed{\tau_k = 66.66 \ N/mm^2}$$

$$[\tau_k]_{all} = 100 \ N/mm^2$$

$$\tau_k < [\tau_k]_{all}$$

Design is safe.

b. Check for crushing stress.

$$T = l \times \frac{t}{2} \times \sigma_{ck} \times \frac{d}{2}$$

$$250 \times 10^3 = 37.5 \times \frac{7}{2} \times \sigma_{ck} \times \frac{25}{2}$$

$$\boxed{\sigma_{ck} = 152.38 \ N/mm^2}$$

$$[\sigma_{ck}]_{all} = 250 \ N/mm^2$$

$$\sigma_{ck} < [\sigma_{ck}]_{all}$$

Design is safe.

Step-5 Design of hub

$$T = \frac{\pi}{16} \times \tau_c \times \left[\frac{D^4 - d^4}{D}\right]$$

$$250 \times 10^3 = \frac{\pi}{16} \times \tau_c \times \left[\frac{50^4 - 25^4}{50}\right]$$

$$\boxed{\tau_c = 10.86 \ N/mm^2}$$

$$[\tau_c]_{all} = 200 \ N/mm^2$$

$$\tau_c < [\tau_c]_{all}$$

Design is safe.

Step-6 Design of Flange

$$T = \pi \times \frac{D^2}{2} \times \tau_c \times t_f$$

$$250 \times 10^3 = \pi \times \frac{50^2}{2} \times \tau_c \times 12.5$$

$$\boxed{\tau_c^f = 5.09 \ N/m^2}$$

$$[\tau_c]_{all} = 200 \ N/mm^2$$

$$\therefore \tau_c < [\tau_c]_{all}$$

Design is safe.

→ Protective type Flange coupling.

The small ~~change~~ change the ~~F~~ Top of Flange part will be extent.

The find the thickness of protecting Flange t_p in step-2.

Problem:

Design and draw a protective type of cast iron flange coupling for a steel shaft transmitting 15KW at 200rpm. and having an allowable shear stress of 40 Mpa. The working ~~string~~ Stress in the bolts should not exceed 30MPa. Assume that the same material is used for shaft and key and that the crushing stress is Twice the value of shear stress. The Maximum Torque is 25% greater then the full load Torque. The shear stress for castiron is 14 MPa.

Given

Power $P = 15 KW = 15 \times 10^3 W$

Speed $N = 200$ rpm.

allowable shear stress for shaft $[\tau_s]_{all} = 40$ MPa.

$= 40 N/mm^2$

Working stress in bolt $= 30\,MPa = 30\,N/mm^2$ soln

(or)

Shear stress in bolt $[\tau_b]_{all} = 30\,N/mm^2$

Same material for shaft & key

$$[\tau_s]_{all} = [\tau_k]_{all} = 40\,N/mm^2$$

Crushing stress = Twice the shear stress

$$[\sigma_{ck}]_{all} = [\sigma_s]_{all} = 2 \times [\tau_s]_{all} = 2 \times 40$$

$$[\sigma_{ck}]_{all} = 80\,N/mm^2$$

$T = 1.25 \times T_{mean}$.

Shear stress for CI $= 14\,MPa = 14\,N/mm^2$

$$[\tau_c] = 14\,N/mm^2$$

To Find:

Design & Protective type coupling Drawing

<u>Soln</u> StepI Design of shaft diameter 'd'

$$T = \frac{\pi}{16} \times [\tau_s]_{all} \times d^3$$

Power $P = \dfrac{2\pi N T_{mean}}{60 \times 1000}$

$$15 \times 10^3 = \frac{2 \times \pi \times 200 \times T_{mean}}{60 \times 1000}$$

$T_{mean} = 7.16 \times 10^5$ N.mm

$T = 1.25 \times T_{mean}$

$T = 1.25 \times 7.16 \times 10^5$

$$\boxed{T = 8.95 \times 10^5 \text{ N.mm}}$$

$$8.95 \times 10^5 = \frac{\pi}{16} \times 40 \times d^3$$

$d = 48.48$ mm

say

Standard Diameter $\boxed{d = 50\text{mm}}$ <PSG D.B 7.20>

<u>Step-2</u> Dimension of coupling.

D = outside dia of hub $= 2d = 2 \times 50 = 100$mm

D_1 = Dia of bolt circle $= 3d = 3 \times 50 = 150$mm

D_2 = Dia of outer flange $= 4d = 50 \times 4 = 200$mm

L = Length of flange $= 1.5d = 1.5 \times 50 = 75$mm

t_f = thickness of flange $= d/2 = \frac{50}{2} = 25$mm

t_p = thickness of protecting flange $= d/4 = \frac{50}{4} = 12.5$

$n =$ No of bolt $= 4$ Nos

Step 3 Design of bolts

$$T = n \times \frac{\pi}{4} \times d_1^2 \times [\tau_b]_{all} \times \frac{D_1}{2}$$

$$8.95 \times 10^5 = 4 \times \frac{\pi}{4} \times d_1^2 \times 30 \times \frac{150}{2}$$

$$\boxed{d_1 = 11.25\,mm}$$

check for crushing stress

$$T = n \times d_1 \times t_f \times \sigma_{cb} \times \frac{D_1}{2}$$

$$8.95 \times 10^5 = 4 \times 11.25 \times 25 \times \sigma_{cb} \times \frac{150}{2}$$

$$\boxed{\sigma_{cb} = 10.6\,N/mm^2}$$

$$[\tau_b]_{all} = \frac{[\sigma_b]_{all}}{2} \qquad [\sigma_b]_{all} = [\sigma_{cb}]_{all}$$

$$[\sigma_{cb}]_{all} = 2 \times [\tau_b]_{all} = 2 \times 30$$

$$[\sigma_{cb}]_{all} = 60\,N/mm^2$$

$$\sigma_{cb} < [\sigma_{cb}]_{all}$$

Design is safe.

Step-4 Design of key

Key Dimension <PSG D.B 5.16>

width of key $w = 16\,mm$

thickness of key $t = 10\,mm$

Length of key $l = L = 75\,mm$

a. Check for induced shear stress

$$T = l \times w \times \tau_K \times \frac{d}{2}$$

$$8.95 \times 10^5 = 75 \times 16 \times \tau_K \times \frac{50}{2}$$

$$\boxed{\tau_K = 29.83\ N/mm^2}$$

$$[\tau_K]_{all} = 40\ N/mm^2$$

$\tau_K < [\tau_K]_{all}$ Design is safe.

b. Check for Induce crushing stress

$$T = l \times \frac{t}{2} \times \sigma_{cK} \times \frac{d}{2}$$

$$8.95 \times 10^5 = 75 \times \frac{10}{2} \times \sigma_{cK} \times \frac{50}{2}$$

$$\boxed{\sigma_{cK} = 95.46\ N/mm^2}$$

$$[\sigma_{cK}]_{all} = 80\ N/mm^2$$

$$\sigma_{cK} > [\sigma_{cK}]_{all}$$

Design is not safe.

Safer value Increase the Length of Key
$l = 100\,mm$

$$8.95\times10^5 = 100\times\frac{10}{2}\times\sigma_{ck}\times\frac{50}{2}$$

$$\boxed{\sigma_{ck} = 71.6\ N/mm^2}$$

$$\sigma_{ck} < [\sigma_{ck}]_{all}$$

Design is safe.

Step-5. Design of hub.

$$T = \pi/16\times\tau_c\times\left[\frac{D^4 - d^4}{D}\right]$$

$$8.95\times10^5 = \frac{\pi}{16}\times\tau_c\left[\frac{100^4 - 50^4}{100}\right]$$

$$\boxed{\tau_c = 4.86\ N/mm^2}$$

$$[\tau_c]_{all} = 14\ N/mm^2$$

$\tau_c < [\tau_c]_{all}$ Design is safe.

Step-6 Design of Flange

$$T = \pi\times\frac{D^2}{2}\times\tau_c\times t_f$$

$$8.95\times10^5 = \pi\times\frac{100^2}{2}\times\tau_c\times25$$

$$\boxed{\tau_c = 2.27\ N/mm^2}$$

$$[\tau_c]_{all} = 14\ N/mm^2$$

$\tau_c < [\tau_c]_{all}$ Design is safe.

⇒ Design of Flexible coupling:

A Flexible coupling used ~~to~~ so as ~~the~~ to permit misalignment of the shaft without undue absorption of the power which the shaft are transmitting.

→ Design Procedure For Flexible coupling (or) bushed pin coupling.

Step I Design of shaft diameter 'd'

$$T = \frac{\pi}{16} \times d^3 \times [\tau_s]_{all}$$

$$d = ?$$

$$\text{Power } P = \frac{2\pi N T_{mean}}{60 \times 1000}$$

$$T_{mean} = \text{in N.mm}$$

$$T = T_{mean}$$

If service Factor is given

$$T = S.F \times T_{mean}.$$

Step-2 Dimension of coupling

d = dia of shaft

D = outer diameter of hub = 2d

D_1 = Pcd of bolt = 3d

D_2 = outer diameter of flange = 4d

~~d~~ n = No of bolt

d_1 = Dia of bolt $= \frac{0.5d}{\sqrt{n}}$

L = Length of flange = 1.5d

t = thickness of flange $t_f = \frac{d}{2}$.

Step-3 ~~Dim~~ Design of key

Dimension of key <P.54 . D.B 5.19>

~~Inter~~ width of key w

thickness of key t

Length of key $l = L$

a. Check for Induced shear

$$T = l \times w \times \tau_k \times \frac{d}{2}$$

$\tau_k = ?$

$\tau_k < [\tau_k]_{all}$

Design is safe.

b. Check for Crushing stress

$$T = l \times \frac{t}{2} \times \sigma_{ck} \times \frac{d}{2}$$

$\sigma_{ck} = ?$

$\sigma_{ck} < [\sigma_{ck}]$

Design is safe.

Step-4 Design of bollt

a. check for Induced shear

$$T = n \times \frac{\pi}{4} \times d_1^2 \times \tau_b \times \frac{D_1}{2}$$

$\tau_b = ?$

$\tau_b < [\tau_b]_{all}$

Design is safe.

b. Maximum bending stress & shear stress

$$\sigma_{b\,max} = \frac{\sigma_b}{2} + \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

<7.2 Modified>

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

$$\sigma_b = \frac{M}{Z}$$ <D.B x10>

$$Z = \frac{\pi}{32} \times d^3 \quad \langle D.B \times 10 \rangle$$

$$M = W\left[\frac{l}{2} + 5\right] \quad \langle D.B \times 9 \rangle$$

$$T = l \times w \left[\tau_b\right]_{all} \times \frac{d}{2} \quad \langle D.B \times 10 \rangle$$

$$W = ?$$

$$\tau = \frac{\text{Load}}{\text{Area}} = \frac{W}{\frac{\pi}{4} \times d_1^2}$$

Step 5   Design of hub

$$T = \frac{\pi}{16} \times \tau_c \times \left[\frac{D^4 - d^4}{D}\right]$$

$$\tau_c = ?$$

$$\tau_c < \left[\tau_c\right]_{all}$$

Design is safe.

Step-6 Design of Flange

$$T = \pi \times \frac{D^2}{2} \times \tau_c \times t_f$$

$\tau_c = ?$

$\tau_c < [\tau_c]_{all}$

Design is safe.

Step-7 Design of bush.

$$W = P_b \times d_2 \times l$$

$$d_2 = \frac{W}{P_b \times l}$$

$d_2 = ?$

$$\sigma_{bush} = \frac{W}{L \times d_2}$$

$\sigma_{bush} = ?$

$$\tau_{bush} = \frac{[\sigma_{bush}]_{all}}{2}$$

$[\sigma_{bush}]_{all} = ?$

τ_{bush} is not given assume 2.

Problem based on Flexible coupling:

1. Design a bushed-pin type of flexible coupling to connect a pump shaft to a motor shaft transmitting 32 kW at 960 rpm. The overall torque is 20 percent more than the mean Torque. The material properties are as follows.

   (i) The allowable shear and crushing stress for shaft material is 40 MPa and 80 MPa respectively

   (ii) The allowable shear stress for cast iron is 15 MPa.

   (iii) The allowable bearing pressure for rubber bush is $0.8\ N/mm^2$

   (iv) The material of the pin is same as that of shaft and key.

Draw neat sketch of the coupling.

Given data:

Bushed-Pin type

Power $P = 32\ kW = 32 \times 10^3\ W$

Speed $N = 960\ rpm$

$T = 20\%\ T_{mean} = 1.2\ T_{mean}$

allowable shear stress for shaft, key, bolt (or) pins

$$[\tau_s]_{all} = [\tau_k]_{all} = [\tau_b]_{all} = 40 N/mm^2$$

Crushing stress for shaft, key, bolt or pins

$$[\sigma_s]_{all} = [\sigma_k]_{all} = [\sigma_b]_{all} = 80 N/mm^2$$

The allowable shear stress on CI =

$$[\tau_c]_{all} = 15 MPa$$

Bearing Pressure $P_b = 0.8 N/mm^2$

To ~~F~~ Find

Design the Bush pin coupling.

Soln.

<u>Step-I</u> Design of shaft diameter 'd'

$$T = \frac{\pi}{16} \times d^3 \times [\tau_s]_{all}$$

$$\text{Power } P = \frac{2\pi N T_{mean}}{60 \times 1000}$$

$$32 \times 10^3 = \frac{2 \times \pi \times 960 \times T_{mean}}{60 \times 1000}$$

$$T_{mean} = 3.18 \times 10^5 N.mm$$

$$T = 1.2 \times T_{mean} = 1.2 \times 3.18 \times 10^5$$

$$\boxed{T = 3.81 \times 10^5 N.mm}$$

$$3.81 \times 10^5 = \frac{\pi}{16} \times d^3 \times 40$$

$$\boxed{d = 36.5\,mm}$$

Standard diameter $\boxed{d = 40mm}$ <PSG D.B 7.20>

Step-2 Dimension of coupling

d= Dia of shaft = 40mm

D= outer Diameter of hub = 2d = 2x40 = 80mm.

D_1= P.Cd of bolt = 3d = 3x40 = 120mm

D_2= outer diameter of flange = 4d = 4x40 = 160mm

d_1= Dia of bolt = $\frac{0.5\,d}{\sqrt{n}} = \frac{0.5 \times 40}{\sqrt{3}}$ = 11.54mm

n= No of bolts

n= 3 Nos

L= Length of flange = 1.5d = 1.5 x 40 = 60mm

t= thickness of flange $t_f = \frac{d}{2} = \frac{40}{2}$ = 20mm.

Step-3

Design of key <PSG D.B 5.19>

Dimension of key

width of key W= 12mm

thickness of key t= 8mm

Length of key l= 1.5d = 1.5x40 = 60mm.

a. Check For Induced Shear

$$T = l \times w \times \tau_K \times \frac{d}{2}$$

$$3.81 \times 10^5 = 60 \times 12 \times \tau_K \times \frac{40}{2}$$

$$\boxed{\tau_K = 26.45 \ N/mm^2}$$

$$[\tau_K]_{all} = 40 \ N/mm^2$$

$\tau_K < [\tau_K]_{all}$ Design is safe.

b. Check For crushing stress

$$T = l \times \frac{t}{2} \times \sigma_{ck} \times d/2$$

$$3.81 \times 10^5 = 60 \times \frac{8}{2} \times \sigma_{ck} \times \frac{40}{2}$$

$$\boxed{\sigma_{ck} = 79.37 \ N/mm^2}$$

$$[\sigma_{ck}]_{all} = 80 \ N/mm^2$$

$$\sigma_{ck} < [\sigma_{ck}]_{all}$$

Design is safe.

Step-4

Design of bolt

a. Check For induced shear

$$T = n \times \frac{\pi}{4} \times d_1^2 \times \tau_b \times \frac{D_1}{2}$$

$$3.81 \times 10^5 = 3 \times \frac{\pi}{4} \times 11.54^2 \times \cancel{d_1} \ \tau_b \times \frac{120}{2}$$

$$\boxed{\tau_b = 20.23 N/mm^2}$$

$$\tau_b < [\tau_b]_{all}$$

Design is safe.

b) Maximum bending stress & shear stress

$$\sigma_{b\,max} = \frac{\sigma_b}{2} + \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$ <7.2 Modified>

~~σ_b = bendi~~

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

σ_b = bending stress

$$\frac{M_b}{I} = \frac{\sigma_b}{Y}$$ <7.1>

$$\sigma_b = \frac{M_b \times Y}{I} \text{ (or) } \frac{M}{Z}$$ <DB × 10>

(or)

<D.B × 10> $Z = \frac{\pi}{32} \times d_1^3 = \pi/_{32} \times (11.54)^3$

$Z = \cancel{13.07}$ 149. mm^3

<D.B × 9> $M = W\left[(l/_2) + 5\right]$

~~<D.B × 9> W = Pd × d₂ × l~~

~~<D.B × 9> $T = Pd \times d_2 \times \ln\left(\frac{D_1}{2}\right)$~~

~~$3.81 \times 10^5 / \cancel{80} = 0.8 \times d_2 \times \ln\left(\frac{120}{2}\right)$~~

~~$d_2 =$~~

$\langle D.3 \times 10 \rangle \quad T = l \times w \times [\tau_b]_{all} \times d/2$

$$3.81 \times 10^5 = 60 \times w \times 40 \times \frac{40}{2}$$

$$\boxed{W = 7.93\ N}$$

$$\cancel{w = P_b \times d_2 \times l}$$

$$\cancel{7.93 = 0.8 \times d_2 \times 60}$$

$$\boxed{d_2 = 0.165\ mm}$$

$\langle D.3 \times 9 \rangle \quad M = W\left[l/2 + 5\right]$

$$M = 7.93 \times \left[\frac{60}{2} + 5\right]$$

$$\boxed{M = 277.55\ N.mm}$$

$$\sigma_b = \frac{M}{Z} = \frac{277.55}{150.87}$$

$$\boxed{\cancel{\sigma_b = 21.23\ N/mm^2}} \quad \sigma_b = 1.84082\ N/mm^2.$$

τ = shear stress

$$\tau = \frac{Load}{Area} = \frac{W}{\pi/4 \times d_1^2}$$

$$\tau = \frac{7.93}{\pi/4 \times (11.54)^2}$$

$$\boxed{\tau = 0.089\ N/mm^2}$$

$$\sigma_{b\,Max} = \frac{\sigma_b}{2} + \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

$$= \frac{\overset{1.8407}{\cancel{21.23}}}{2} + \frac{1}{2}\sqrt{(\overset{1.8407}{\cancel{21.23}})^2 + 4\times(0.089)^2}$$

$$\boxed{\sigma_{bMax} = \cancel{21.23}\ N/mm^2}$$ $\sigma_{bmax} = 1.8431\ N/mm^2$.

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4(\tau)^2}$$

$$= \frac{1}{2}\sqrt{(\overset{1.8407}{\cancel{21.23}})^2 + 4\times(0.089)^2}$$

$$\boxed{\tau_{max} = \cancel{10.61}\ N/mm^2}$$ $\tau_{max} = 0.9235\ N/mm^2$.

Step- 5 Design of Flange

$$T = \pi \times \frac{D^2}{2} \times \tau_c \times t_f$$

$$3.81\times10^5 = \pi \times \frac{80^2}{2} \times \tau_c \times 20$$

$$\boxed{\tau_c = 1.89\ N/mm^2}$$

$$[\tau_c]_{all} = 15\ N/mm^2$$

$$\tau_c < [\tau_c]_{all}$$

Design is safe.

Step 6 Design of hub

$$T = \frac{\pi}{16} \times \tau_c \times \frac{D^4 - d^4}{D}$$

$$3.81 \times 10^5 = \frac{\pi}{16} \times \tau_c \times \left[\frac{80^4 - 40^4}{80}\right]$$

$$\tau_c = 4.04 \ N/mm^2$$

$$[\tau_c]_{all} = 15 \ N/mm^2$$

$$\tau_c < [\tau_c]_{all}$$

Design is safe.

Step 7 Design of bush <D.B×9>

$$W = P_b \times d_2 \times l$$

$$7.93 = 0.8 \times d_2 \times 60$$

$$\boxed{d_2 = 0.165 \ mm}$$

$$\sigma_{bush} = \frac{W}{L \times d_2}$$

$$\sigma_{bush} = \frac{7.93}{60 \times 0.165}$$

$$\boxed{\sigma_{bush} = 0.8 \ N/mm^2}$$

$$\tau_{bush} = \frac{[\sigma_{bush}]_{all}}{2}$$

$$2 = \frac{[\sigma_{bush}]_{all}}{2}$$

$$[\sigma_{bush}]_{all} = 4 \ N/mm^2$$

$$\sigma_{bush} < [\sigma_{bush}]_{all}$$

Design is safe.

Note:

If Notgiven

$\tau_{bush} = 2 \ N/mm^2$ (assume)

2) Design a bushed pin flexible coupling for connecting a motor shaft to a pump shaft for the following service conditions.

Power to be transmitted = 40 kW

speed of motor shaft = 1000 r.p.m

diameter of motor shaft = 50 mm

diameter of the pump shaft = 45 mm

The bearing pressure in the rubber bush and ~~the pins~~ allowable stress in the pins are to be ~~limited to~~ 0.45 N/mm² and 25 MPa respectively.

H.W

3. Design a flange coupling [bush type] to transmit 5 kW at 750 rpm with a service factor of 1.2 for shaft, key, bolt with permissible stress 50 N/mm² for castiron shear stress 15 N/mm² for shear stress for bush is 2 N/mm² for key crushing stress of 100 N/mm².

UNIT - III

TEMPORARY AND PERMANENT JOINT

Threaded fasters - Bolted Joints - including eccentric loading - knuckle Joints - cotter Joints - welded Joints, riveted Joints for structures - theory of bonded joints.

⇒ Design of welded Joints <PSG DB 11.1 - 11.6>

Introduction:

A welded Joint is a permanent Joint which is obtained by fusion of the edge of the two parts to be Jointed together, with (or) without the application of pressure and a fillar material.

Advantage:

* The welded structures are usually lighter ~~tam~~ than riveted structures.
* The welded Joints provide maximum efficiency.
* Alterations and additions can be easily made in the existing structure.

* The welded provides very rigid Joints.
* The process of welding takes Less time than the riveting.

Disadvantage

* It requires highly skilled Labour and Supervision.
* The inspection of welding work is more difficult than riveting work.
* Since there is an uneven heating and cooling during fabrication.

→ Welding Process.

* welding Processes that use heat along.

  eg: Fusion welding

* welding Processes that use a combination of heat and Pressure

  eg. Forge welding.

  Re plastic defor

  solid stag

Types of welded Joints:

1. Lap Joint or fillet Joint

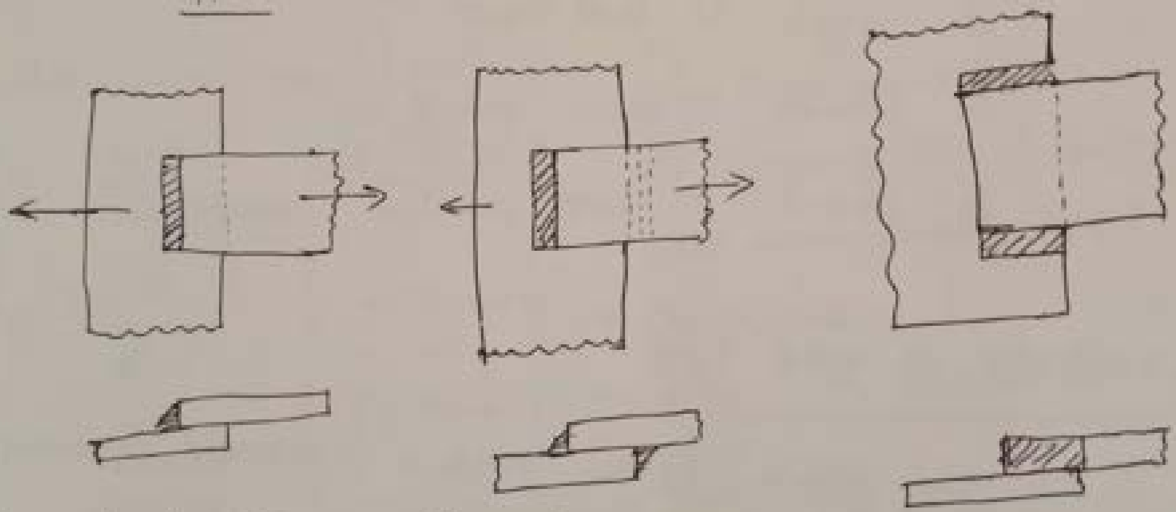
2 Butt Joint

1. Lap Joint

The lap Joint or the fillet Joint is obtained by overlapping the plates and then welding the edges of the plate.

The cross - section of the fillet is approximately triangular.

Types

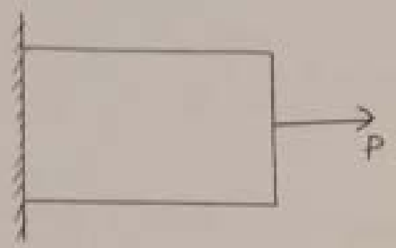


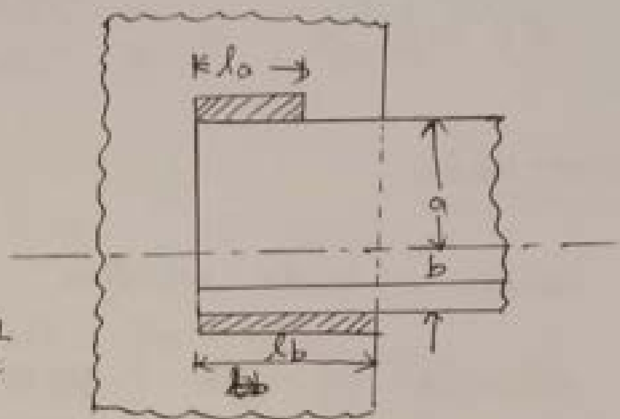
a) single transverse b) Double Transverse c) Parallel fillet

Type I

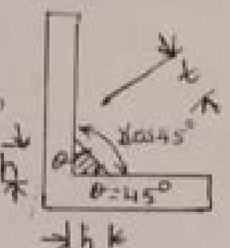
* Design of welded Joints based on strength of welding in axial Load.

The butt weld joint is subjected to tensile load 'P' the average tensile stress in the weld is given by





$\sin\theta = \frac{h}{t}$



Shear stress

$\tau = \frac{\text{Load}}{\text{Area of weld}}$

$\tau = \frac{P}{l \times t}$

$\tau = \frac{P}{\cos 45 \times h}$

$\boxed{\tau = \frac{P}{0.707 h}}$

h - size of weld

τ : P : Load.

$t = h \times \sin 45$

or

$t = h \times \cos 45$

l_a Length of weld in Top

l_b = length of weld in bottom

l = total weld length = $l_a + l_b$

P = Axial Load

a = Distance of topweld from gravity axis

b = Distance of bottom weld from gravity Axis

Moment of topweld = $l_a \times f \times a$ | M = force × dis

Moment of bottom weld = $l_b \times f \times b$

Sum of moment of top & bottom

$l_a \times f \times a = l_b \times f \times b$

$l_a \times a = l_b \times b$

W.k.t $l = l_a + l_b$

Butt Weld

The butt joint is obtained by placing the plates edge to edge as shown in fig.

square butt Joint   single V butt Joint

→ Types of butt Joint

1. square butt Joint
2. single ~~bevel~~ V butt Joint
3. single U butt Joint
4. Double V butt Joint
5. Double U butt Joint

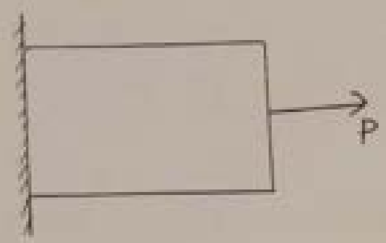
→ selection of weld type

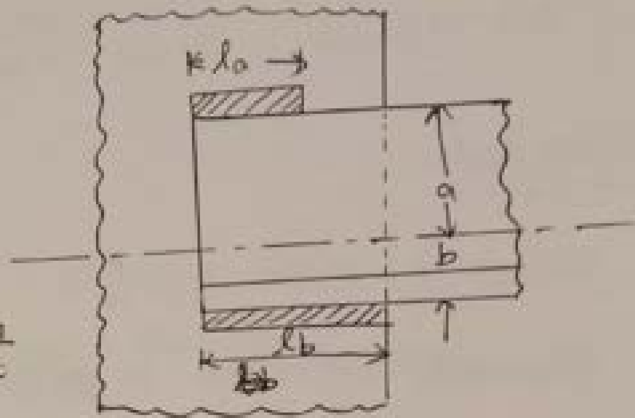
* The shape of the welded component required
* The thickness of the plates to be welded.
* The direction of the forces applied.

Type I

* Design of Welded Joints based on strength of welding in axial Load.

The butt weld joint is subjected to tensile load 'P' the average tensile stress in the weld is given by





~~same~~ $= \frac{h}{t}$

Shear stress ~~eqn~~

$$\tau = \frac{\text{Load}}{\text{Area of weld}}$$

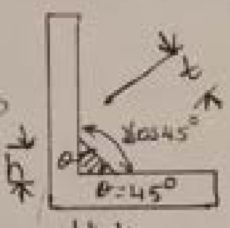
$$\tau = \frac{P}{L \times t}$$

$$\tau = \frac{P}{\cos 45 \times h}$$

$$\boxed{\tau = \frac{P}{0.707 h}}$$

h - size of weld

τ. P: Load.



$t = h \times \sin 45$

or

$t = h \times \cos 45$

l_a Length of weld in Top

l_b = Length of weld in bottom

l = total weld Length = $l_a + l_b$

P: Axial Load

a = Distance of topweld from gravity axis

b: Distance of bottom weld from gravity Axis

Moment of topweld $= l_a \times f \times a$

Moment of bottom weld $= l_b \times f \times b$

M: force axis

Sum of moment of top & botom

$$l_a \times f \times a = l_b \times f \times b$$

$$l_a \times a = l_b \times b$$

W.k.t $l = l_a + l_b$

Problems

1. A plate of 100mm wide and ~~plate~~ 12.5 mm thick is to be welded by means of another plate two parallel fillet weld. The Plate was subjected to axial Load of ~~15~~ 80 KN. The Maximum shear stress is not to exceed 56 N/mm². Find the length of weld.

<u>Given data:</u>

width of plate $w = 100mm$

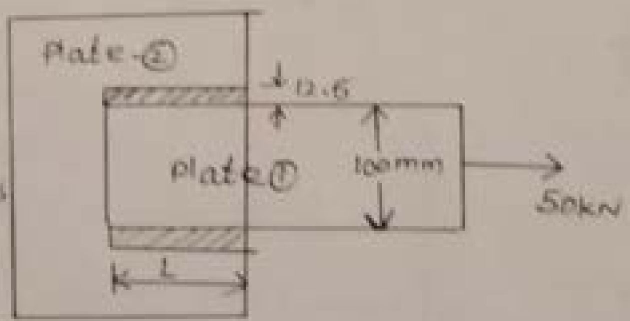
thickness of weld h (or) $t = 12.5mm$

Maximum shear stress $\tau_{max} = 56 N/mm^2$

Axial Load $P = 50kN = 50 \times 10^3 N$

<u>To Find:</u>

Length of weld (L)



<u>Soln</u>

Maximum shear stress

$$\tau_{max} = \frac{\text{Load}}{\text{Area}}$$

$$\tau_{max} = \frac{P}{2 \times t \times L} \qquad t = h \times \cos 45$$

$$\tau_{max} = \frac{P}{2 \times 0.707 h \times L}$$

$$56 = \frac{50\times10^3}{2\times0.707\times12.5\times L}$$

$$\boxed{L = 50.51\,mm}$$

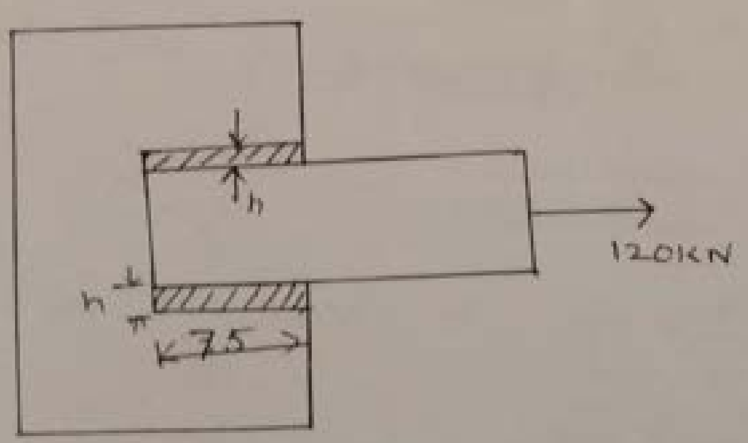
Method - 2 < PSG 0.13 11.3 >

$$\sigma = \frac{0.707\,P}{h\times L}$$

$$56 = \frac{50\times10^3\times0.707}{12.5\times L}$$

$$\boxed{l = 50.51\,mm}$$

2) Find the size of weld for connecting as shown in fig. The ~~D~~ Tensile ~~force~~ force is acting on 120KN. assume design shear stress of material 75 MPa.



Given:

Load $P = 120\,KN = 120\times10^3\,N$

Shear stress $\tau = 75\,MPa = 75\,N/mm^2$

Length of weld $L = 75\,mm$

To find

Size of weld 'h'

Soln

$$\text{Shear stress} = \frac{\text{Load}}{\text{Area of weld}}$$

$$\tau_{max} = \frac{P}{A} = \frac{P}{2\times t\times l}$$

$$75 = \frac{120\times10^3}{2\times t\times 75}$$

$$t = h\times\cos45 = 0.707h.$$

$$75 = \frac{120\times10^3}{2\times0.707h\times75}$$

$$\boxed{h = 15.08\,mm}$$

method -2 $\boxed{\text{PSG D.B } 11.3}$

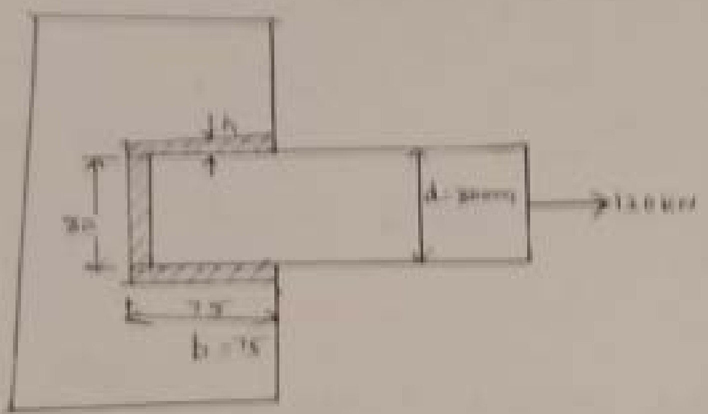
$$\sigma = \frac{0.707\,P}{h\times L}$$

$$75 = \frac{0.707\times120\times10^3}{h\times75}$$

$$\boxed{h = 15.08\,mm}$$

3. Find the size of weld for the connection as shown in fig. If the tensile load acting is 120 kN. Permissible shear stress on the welding is 75 MPa.

Given



wide (or) d $w = 80mm$

Shear stress $\tau_{max} =$ ~~120N/mm²~~ $= 75\ N/mm^2$

thickness of plate ~~of bar~~ $t = 12.5mm$

Axial Load $P =$ ~~80kN = 80×10³N~~ 120×10^3

To find

Size of weld 'h'

Soln

Maximum shear stress $= \dfrac{\text{Load}}{\text{Area of weld}} = \dfrac{P}{0.707h\,(2b+d)}$

$\boxed{t = 0.707h}$

Area of weld = $[2b + d]\, h \cdots$

$$\tau_{max} = \frac{P}{[2b+d]\frac{h}{\sqrt{2}}}$$

$$75 = \frac{120 \times 10^3}{[(2 \times 75) + 80] \times 0.707 \times h}$$

$$\boxed{h = 9.84\,mm}$$

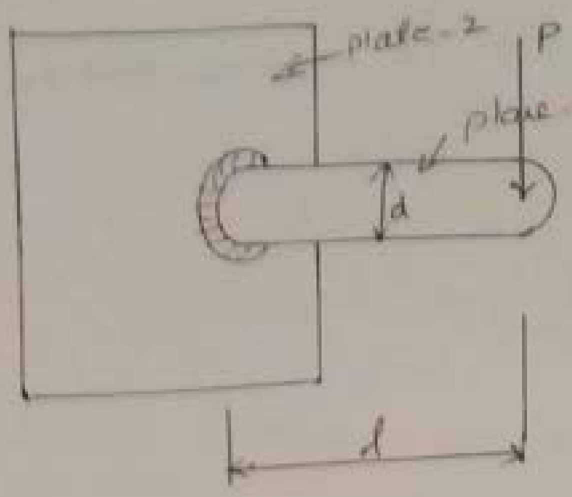
⇒ Type - 2

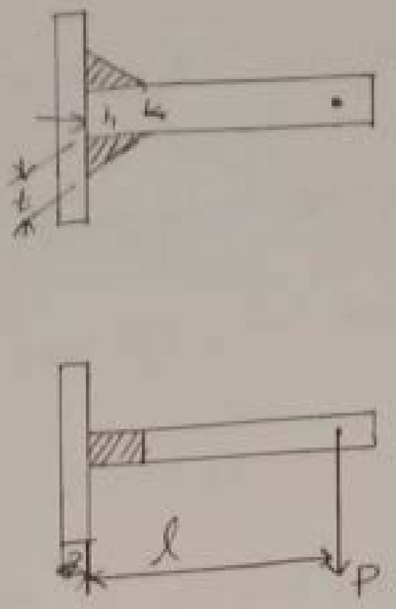
Eccentrically Loaded welded Joints.

An eccentric Load may be imposed on welded Joints in many ways. The stress induced on the Joint may be of different nature or of the same nature.

The induced stresses are combined depending upon the nature of stresses.

When the shear and bending stresses are simultaneously present in a joint, then maximum stresses are as follows





Step-1

Direct Stress $\tau = \dfrac{\text{Load}}{\text{Area of weld}}$ <7.17>

Step-2

Bending Stress $\sigma_b = \dfrac{M_b}{I/y}$ | $\dfrac{M_b}{I} = \dfrac{\sigma_b}{y}$

$\sigma_b = \dfrac{M_b}{Z}$ <7.17> × Tough

M_b: ~~bending of~~ Moment = Load × Distance

Z = section Modulus of weld &

$Z = Z_w \times t$ <11.5> PSG D.B

Z_w: section Modulus of weld shape

Z_w in PSG D.B <11.6>

Step-3

Maximum Shear Stress

$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$ PSG D.B <7.2> × Tough

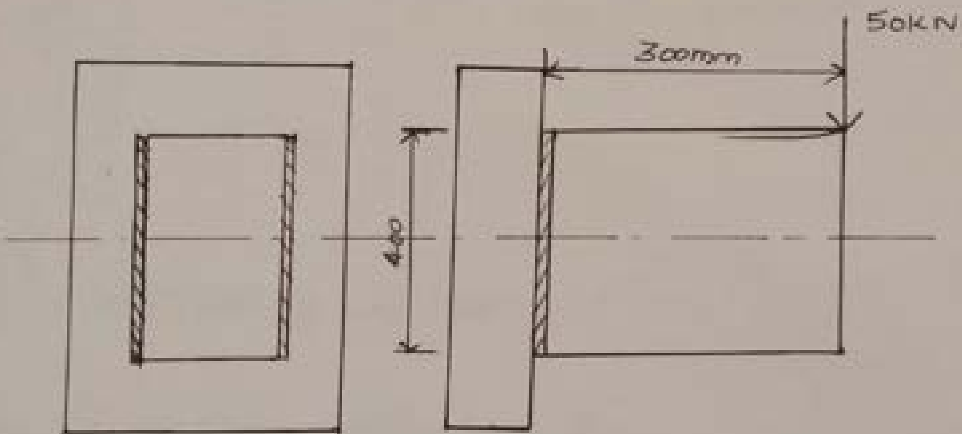
Formula for Area of weld in Diff Cross Section

Area of Rectangular plate $= (2b+2d)t$

Area of cylinder $= \pi d t$

Problems:

1. A bracket is welded to the vertical plate by means of two fillet welds as shown in Fig. Determine the size of ~~weld~~ the welds, if the permissible shear stress is limited to 70 N/mm².



Given data:

Load $P = 50KN$

Permissible shear stress $\tau_{max} = 70 N/mm^2$

To Find

size of weld 'h'

Soln

Step 1

Direct stress $\tau = \dfrac{\text{Load}}{\text{Area of weld}}$

$$\tau = \frac{P}{A}$$

Area of weld $= (2 \times 400) \times t$

$= 800\,t$ | $t = 0.707h$

$$A = 800 \times 0.707 \times h$$

$$\boxed{A = 565.6\,h}$$

$$\tau = \frac{50 \times 10^3}{565.6 \times h}$$

$$\boxed{\tau = \frac{88.4}{h}} \ N/mm^2$$

Step.2

Bending stress $\sigma_b = \dfrac{M_b}{Z}$ on <7.17>

M_b = bending Moment = Load x Distance

$$M_b = 50 \times 10^3 \times 300$$

$$\boxed{M_b = 15 \times 10^6 \ N.mm}$$

Z = Section Modulus

$Z = 2w \times t$ <~~PSG.DB 11.5~~>

$$Z_w = \frac{d^2}{3}$$ LPS4 D.B 11.57

$$Z_w = \frac{400^2}{3}$$

$$\boxed{Z_w = \cancel{21.33 \times 10^6}} \quad 53.33 \times 10^3$$

$$t = 0.707\, h$$

$$Z = Z_w \times t = \overset{53.33 \times 10^3}{\cancel{21.33 \times 10^6}} \times 0.707\, h$$

$$\boxed{Z = \cancel{15.08 \times 10^6\, h}} \quad 37.7 \times 10^3\, h.$$

$$\sigma_b = \frac{M_b}{Z} = \frac{15 \times 10^6}{\cancel{15.08 \times 10^6\, h}\ 37.7 \times 10^3\, h.}$$

$$\boxed{\sigma_b = \frac{\overset{397.89}{\cancel{0.99}}}{h}} \quad N/mm^2$$

<u>Step 3</u> Maximum shear stress

$$\tau_{max} = \frac{1}{2}\sqrt{[\sigma_b]^2 + 4\tau^2}$$

$$70 = \frac{1}{2}\sqrt{\left(\frac{\overset{397.8}{\cancel{0.99}}}{h}\right)^2 + 4\left(\frac{88.4}{h}\right)^2}$$

$$70 = \frac{1}{2}\sqrt{\frac{\cancel{0.99^2}\ 397.8^2}{h^2} + \frac{4 \times 88.4^2}{h^2}}$$

$$70 = \frac{1}{2h}\sqrt{\cancel{0.99^2}\ 397.8^2 + 4 \times 88.4^2}$$

$$\cancel{70 = \frac{1}{2h} \times 176.80}$$

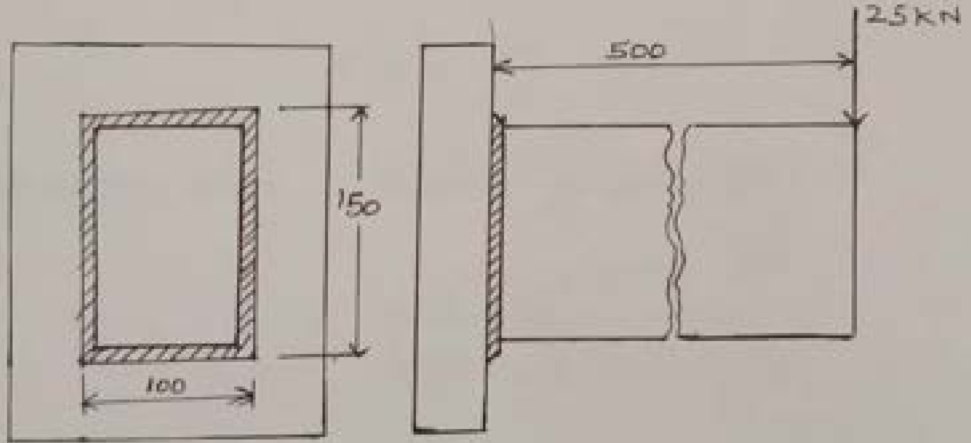
$$70 = \frac{88.401}{h}$$

$$h = \frac{88.401}{70}$$

$h =$

$\boxed{h = 1.262\ mm}$ $\boxed{h = 3.1\ mm}$

2. A beam of rectangular cross section is welded to a ~~suport~~ support by means of fillet welds as shown in Fig. Determine the size of welds, if the permissible shear stress in the weld is limited to 75 ~~N/mm²~~ MPa



<u>Given data</u>

Load $P = 25KN = 25\times10^3 N$

depth $d = 150mm$

width $b = 100mm$

Maximum shear stress $\tau_{max} = 75\ N/mm^2$ Length $L = 500mm$

<u>To find</u>

size of weld.

Soln

Step-I Direct Stress τ

$$\tau = \frac{\text{Load}}{\text{Area of weld}} = \frac{P}{A}$$

$A = [2b + 2d] \times t = [2 \times 100 + 2 \times 150] \times 0.707h$

$A = 353.5\,h$

$$\tau = \frac{25 \times 10^3}{353.5\,h}$$

$$\boxed{\tau = \frac{70.72}{h}} \quad N/mm^2$$

Step 2 Bending Stress σ_b <7.17>

$\sigma_b = \frac{M_b}{Z}$ <7.17>

M_b bending Moment = Load × Distance

$M_b = 25 \times 10^3 \times 500$

$\boxed{M_b = 12.5 \times 10^6 \ N.mm}$

Z = Section Modulus

$Z = Z_w \times t$

$Z_w = bd + \frac{d^2}{3}$ <PSG DB 11.6>

$Z_w = (100 * 150) + \frac{150^2}{3}$

$Z_w = 22.5 \times 10^3$

$t = 0.707\,h$

$Z = 22.5 \times 10^3 \times 0.707\,h$

$\boxed{Z = 15.9 \times 10^3\,h}$

$$\sigma_b = \frac{M}{Z} = \frac{12.5 \times 10^6}{15.9 \times 10^3 h}$$

$$\boxed{\sigma_b = \frac{785.79}{h}}$$

Step-3 Maximum shear stress

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

$$75 = \frac{1}{2} \times \sqrt{\left(\frac{785.79}{h}\right)^2 + 4\left(\frac{70.72}{h}\right)^2}$$

$$75 = \frac{1}{2} \times \sqrt{\frac{(785.79)^2}{h^2} + 4 \times \frac{(70.72)^2}{h^2}}$$

$$75 = \frac{1}{2h} \times \sqrt{(785.79)^2 + 4 \times (70.72)^2}$$

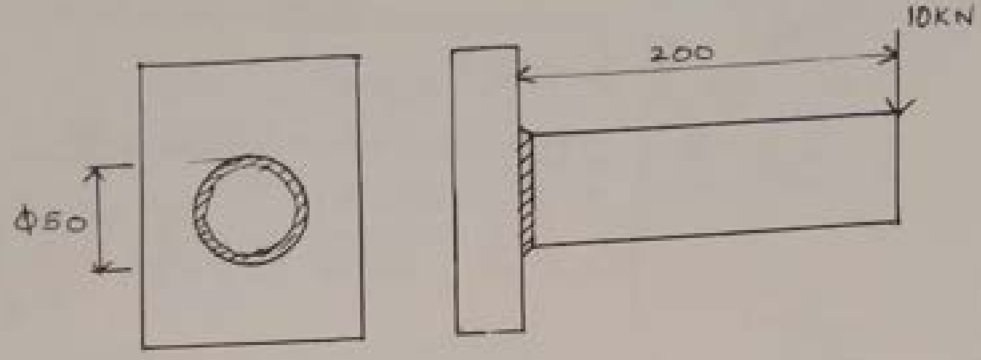
$$75 = \frac{1}{2h} \times 798.42$$

$$75 = \frac{399.21}{h}$$

$$h = \frac{399.21}{75}$$

$$\boxed{h = 5.32\ mm}$$

3. A circular beam, 50 mm in diameter is welded to a support by means of a fillet weld as shown in fig. Determine the size of weld, if the permissible shear stress in the weld is limited to 100 N/mm²



Given

Diameter of shaft $d = 50mm$

Length of shaft $L = 200mm$

Load $P = 10KN = 10 \times 10^3 N$

Maximum shear stress $\tau_{max} = 100 N/mm^2$

To find

Size of weld (h)

Soln

Step I — Direct shear stress

$$\tau' = \frac{Load}{Area}$$

$$Area = \pi d t$$

$A = \pi \times 50 \times 0.707\,h$

$A = 111.05\,h\ mm^2$

$$\tau = \frac{10 \times 10^3}{111.05\,h}$$

$$\boxed{\tau = \frac{90.04}{h}}$$

Step 2

Bending Stress σ_b

$$\sigma_b = \frac{M_b}{Z} \quad \langle 7.1 \rangle$$

M_b = Load × Distance

$M_b = 10 \times 10^3 \times 200$

$$\boxed{M_b = 2 \times 10^6\ N.mm}$$

Checkin

$Z = Z_w \times t$

It's wrong

$$Z_w = \frac{\pi d^2}{2} = \frac{\pi \times 50^2}{2}$$

$\boxed{Z_w = 3.926 \times 10^3}$ mm^3

$t = 0.707\,h$

$Z = 3.926 \times 10^3 \times 0.707\,h$

$$\boxed{Z = 2776.38\,h}$$

in Text Book

$\pi d^2/4$

$$Z_w = \frac{\pi d^2}{4} = 1.96 \times 10^3\,h$$

$$Z = \frac{\pi d^2 \times t}{4}$$

$$= \frac{\pi \times 50^2 \times 0.707\,h}{4}$$

$= 69409.56\,h\ mm^2$

$$\sigma_b = \left(\frac{28.81}{h}\right)$$

$Z = Z_w \times t$

Refer [P.S.G. D.B. 11.6]

$$Z_w = \frac{\pi d^2}{4}$$

$Z_w = 1962.5$

$$\sigma_b = \frac{P \times l}{Z_w \times t} = \frac{M_b}{Z}$$

$$\sigma_b = \frac{1440.72}{h}$$

Step (3)

$$\tau_{max} = \frac{1}{2}\sqrt{\sigma_b^2 + 4\tau^2}$$

$h = 7.25\ mm$

$$\sigma_b = \frac{M_b}{Z} = \frac{2\times10^6}{2776.38h}$$

$$\boxed{\sigma_b = \frac{720.25}{h}}$$

Step 3 Maximum shear stress

$$\left[\tau_{max} = \frac{1}{2}\sqrt{\sigma_b^2 + 4\tau^2}\right]$$

$$100 = \frac{1}{2}\times\sqrt{\left(\frac{720.25}{h}\right)^2 + 4\left(\frac{90.04}{h}\right)^2}$$

$$100 = \frac{1}{2}\times\sqrt{\frac{720.25^2}{h^2} + 4\times\frac{90.04^2}{h^2}}$$

$$100 = \frac{1}{2h}\times\sqrt{720.25^2 + 4\times(90.04)^2}$$

~~100~~ $100 = \frac{1}{2h}\times 742.42$

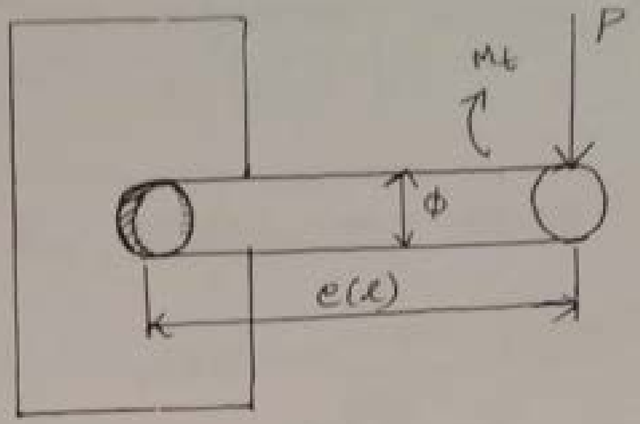
$$100 = \frac{371.21}{h}$$

$$h = \frac{371.21}{100}$$

Ans. $\boxed{h = 3.71\,mm}$ $h =$

Type III

Design of weld by means of torque by end Load.



e = l = eccentricity or Length

P = Load

M_t = Twisting Moment

ϕ = diameter.

Procedure.

Step - I

Direct Shear stress (or) ~~Direct~~ Shear Stress

$$\tau_1 = \frac{P}{A} = \frac{\text{Load}}{\text{Area of Load}}$$

Step - II

Secondary Shear Stress

$$\tau_2 = \frac{M_t \times r}{J} \quad \text{P54 } \langle 7.1 \rangle$$

~~τ_2~~

M_t = Twisting Moment = Twisting Load $\times \frac{d}{2}$

r = Radius ($d/2$)

$J = J_w \times t$ < P54 11.5 & 11.6 >

Step-3

Resultant shear stress

$$\tau = \sqrt{(\tau_1)^2 + (\tau_2)^2}$$

Step-4 bending stress σ_b

$$\sigma_b = \frac{M_b \times y}{I}$$ <PSG 7.1>

$\sigma_b = \frac{M_b}{Z}$ M_b = bending Moment

M_b = Load x Distance

y = $d/2$ (or) half of depth.

Z = section Modulus of weld

$Z = Z_w \times t$

Step-5

Maximum shear stress

(or) Permissible stress

$$\sigma_{b\,max} \text{ (or) } \tau_{max} = \frac{\sigma_b}{2} + \frac{1}{2}\sqrt{\sigma_b^2 + 4\tau^2}$$

σ_b = bending stress N/mm^2

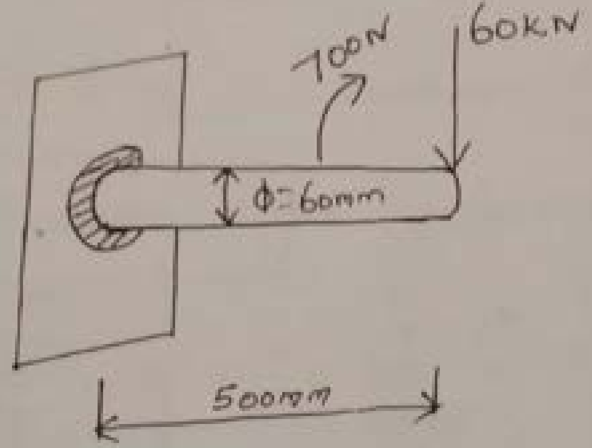
τ = Resultant shear stress N/mm^2

Problem

1. ~~The~~ / A 60 mm diameter solid shaft is to be welded to a flat plate by a fillet weld around the circumference of the shaft.

Problem

1. The 60mm diameter of solid shaft one end is welded to support ~~pla~~ the plate by means of filled weld. The other end end is loaded at torque as shown in fig. Find the size of weld. If ~~t~~he design shear stress of weld is 85MPa.



Given

diameter of shaft $d = 60mm$

Bending Load $P = 60KN$

Twisting Load $P_T = 700N$

Length of solid shaft $l = 500mm$

To Find:

size of weld 'h'

soln:

step-1 - Direct shear stress

$$\tau_1 = \frac{\text{Load}}{\text{Area of weld}} = \frac{P}{A}$$

$$\tau_1 = \frac{P}{\pi d t} = \frac{P}{\pi \times d \times 0.707 h}$$

$$\tau_1 = \frac{60 \times 10^3}{\pi \times 60 \times 0.707 h}$$

$$\boxed{\tau_1 = \frac{450.226}{h}}$$

step-2 Secondary shear stress

$$\tau_2 = \frac{M_t \times r}{J}$$

$$\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{L} \quad 27.17$$

$$\frac{T}{J} = \frac{\tau}{R}$$

$$\tau = \frac{T \times R}{J}$$

M_t = Twisting Load $\times \frac{d}{2}$

$$M_t = 700 \times \frac{60}{2}$$

$$\boxed{M_t = 21 \times 10^3} \text{ N.mm}$$

$$r = \frac{d}{2} = \frac{60}{2}$$

$$\boxed{r = 30mm}$$

J = Polar Moment of Inertia in weld

$$J = J_w \times t$$

$$J_w = \frac{\pi d^3}{4} \quad \langle \text{PSG } 11.6 \rangle$$

$$J_w = \frac{\pi \times 60^3}{4}$$

$$\boxed{J_w = 196.6 \times 10^3} \; mm^3$$

$$t = 0.707\,h$$

$$J = 196.6 \times 10^3 \times 0.707\,h$$

$$\boxed{J = 119.9 \times 10^3\,h}$$

$$\tau_2 = \frac{M_t \times r}{J} = \frac{21 \times 10^3 \times 30}{119.9 \times 10^3\,h}$$

$$\boxed{\tau_2 = \frac{5.25}{h}}$$

Step-3 Resultant shear stress

$$\tau = \sqrt{\tau_1^2 + \tau_2^2}$$

$$\tau = \sqrt{\left(\frac{450.226}{h}\right)^2 + \left(\frac{5.25}{h}\right)^2}$$

$$\tau = \sqrt{\frac{(450.226)^2}{h^2} + \frac{(5.25)^2}{h^2}}$$

$$\tau = \sqrt{\frac{202.703 \times 10^3}{h^2} + \frac{27.56}{h^2}}$$

$$\tau = \sqrt{\frac{(202.703 \times 10^3) + (27.56)}{h^2}}$$

$$\tau = \sqrt{\frac{202.731 \times 10^3}{h^2}}$$

$$\boxed{\tau = \frac{450.25}{h}}$$

Step ~~3~~ 4 Bending ~~M~~ stress σ_b

$$\sigma_b = \frac{M_b \times y}{I} = \frac{M_b}{Z}$$ PSG <7.1>

M_b = Load x Distance = $60\times10^3 \times 500$

$$\boxed{M_b = 30\times10^6}\ \text{N.mm}$$

$Z = Z_w \times t$

$$Z_w = \frac{\pi d^2}{4}$$ <PSG D.B 11.6>

$$Z_w = \frac{\pi \times 60^2}{4}$$

$$\boxed{Z_w = 2.8\times10^3}$$

$t = 0.707h$

$Z = 2.8\times10^3 \times 0.707h$

$$\boxed{Z = 1.99\times10^3\, h}$$

$$\sigma_b = \frac{30\times10^6}{1.99\times10^3\, h}$$

$$\boxed{\sigma_b = \frac{15\times10^3}{h}}$$

Step-5 Maximum Shear Stress

$$\tau_{max} = \frac{\sigma_b}{2} + \frac{1}{2}\sqrt{\sigma_b^2 + 4\tau^2}$$

$$85 = \frac{15\times10^3/h}{2} + \frac{1}{2}\sqrt{\left(\frac{15\times10^3}{h}\right)^2 + 4\left(\frac{450.25}{h}\right)^2}$$

$$85 = \frac{7.5\times10^3}{h} + \frac{1}{2}\sqrt{\frac{225.81\times10^6}{h^2}}$$

$$85 = \frac{7.5\times10^3}{h} + \frac{1}{2}\times\frac{15.02\times10^3}{h}$$

$$85 = \frac{7.5\times10^3}{h} + \frac{7.51\times10^3}{h}$$

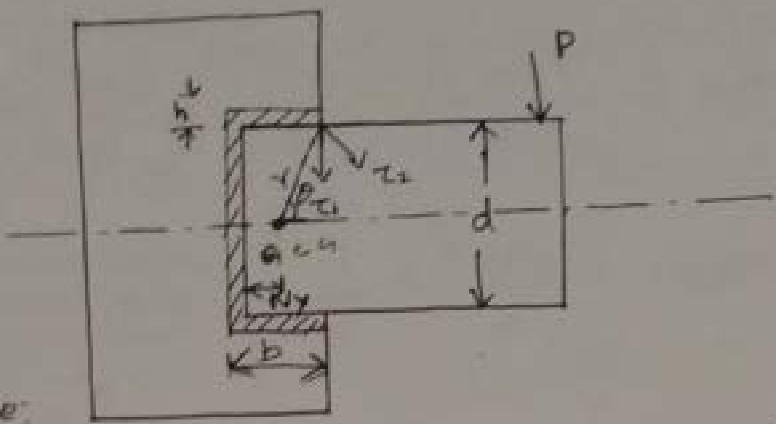
$$85 = \frac{(7.5\times10^3)+(7.51\times10^3)}{h}$$

$$85 = \frac{15.01\times10^3}{h}$$

$$h = \frac{15.01\times10^3}{85}$$

$$\boxed{h = 176.58\ mm}$$

Type. 4 Direct shear stress and Torsion.



Procedure:

Step-I Direct shear stress or Primary shear stress

$$\tau_1 = \frac{\text{Load}}{\text{Area of weld}}$$

Step-II secondary shear stress due to twisting Moment

$$\tau_2 = \frac{M_t \times r}{J}$$

M_t = Load × Eccentricity (or) Distance from C.G

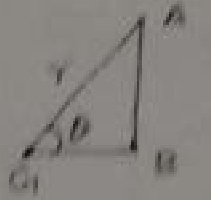
r = Radius of Twisting section. ~~r = Radius (or) depth/2~~

J = Polar Moment of Inertia = $J_w \times t$

J_w value Formula in <PSG 11.5 & 11.6>

Step-III Resultant Shear stress (or) Allowable (or) Permissible shear stress

$$\tau_{max} = \sqrt{\tau_1^2 + \tau_2^2 + 2\tau_1\tau_2\cos\theta}$$



AB = Radius (or) depth/2

$$GB = b - N_y$$

N_y value <PSG 11.5 & 11.6>

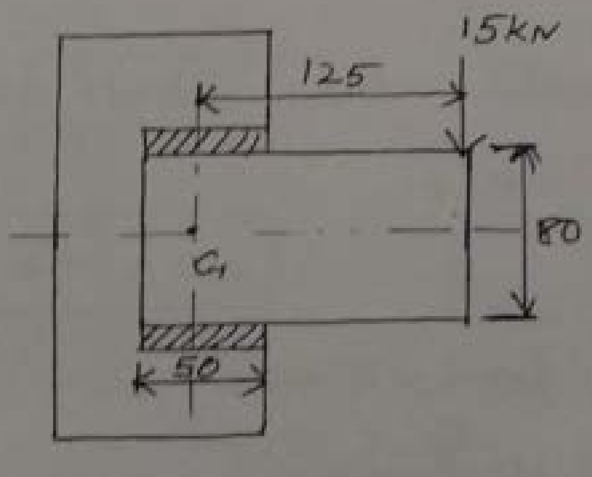
$$\cos\theta = \frac{GB}{r}$$

$$\theta = \cos^{-1}\left(\frac{GB}{r}\right)$$

$$r = \sqrt{AB^2 + GB^2}$$

Problem:

1. A bracket carrying a Load of 15kN is to be welded as shown in Fig. Find the size of weld weld required if the allowable shear stress is not to exceed 80 MPa.



Given:

Load $P = 15KN = 15 \times 10^3 N$

Allowable shear stress $\tau_{max} = 80MPa = 80 N/mm^2$

~~width of bar d = 80mm~~

depth of bar $d = 80mm$

To find:

size ~~length~~ of weld 'h'

Soln

Step I Direct shear stress (or) Primary shear stress

$$\tau_1 = \frac{\text{Load}}{\text{Area of weld}}$$

$$\tau_1 = \frac{P}{2b \times t} = \frac{15 \times 10^3}{2 \times 50 \times 0.707 h}$$

$$\boxed{\tau_1 = \frac{212.16}{h}}$$

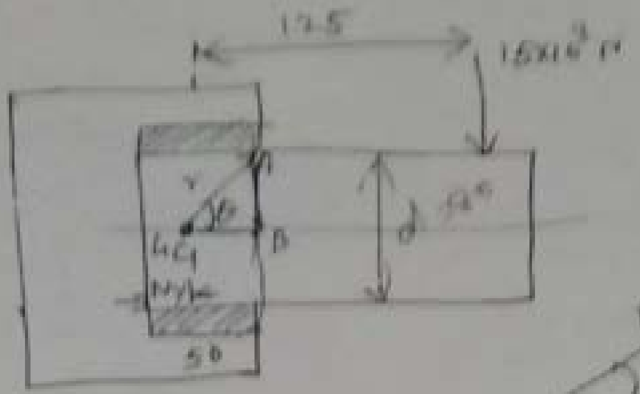
Step-II Secondary shear stress due to Twisting.

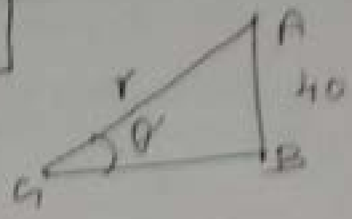
$$\tau_2 = \frac{M_t \times r}{J}$$

M_t = Load × Distance

$M_t = 15 \times 10^3 \times 125$

$\boxed{M_t = 1.875 \times 10^6 \text{ N-mm}}$





$$\gamma = \sqrt{AB^2 + GB^2}$$

$AB = 40mm$

$GB = \cancel{N_y}\ b - N_y$ | $b = 50mm$

$\cancel{b =}\ N_y = b/2$ [N_y formula is not mention in the Data book Pg 11.5]

$N_y = \frac{50}{2} = 25mm$

so assume $b/2$

$GB = 50 - 25$

$$\boxed{GB = 25\,mm}$$

$$\gamma = \sqrt{40^2 + 25^2}$$

$$\boxed{\gamma = 47.16\,mm}$$

$J = J_w \times t$

$$J_w = \frac{b^3 + 3bd^2}{6}$$ <PSG 11.5>

weld symbol

$$J_w = \frac{50^3 + 3 \times 50 \times 80^2}{6}$$

$$J_w = 180.83 \times 10^3$$

$$t = 0.707\,h$$

$$J = 180.83 \times 10^3 \times 0.707\,h$$

$$\boxed{J = 127.84 \times 10^3\,h}$$

$$\tau_2 = \frac{1.875 \times 10^6 \times 47.16}{127.84 \times 10^3\,h}$$

$$\boxed{\tau_2 = \frac{691.63}{h}}$$

$$\cos\theta = 0.53$$

$$\cos\theta = \frac{GB}{r}$$

$$\theta = \cos^{-1}\left(\frac{GB}{r}\right)$$

$$\theta = \cos^{-1}\left(\frac{25}{47.16}\right)$$

$$\theta = 57^\circ 59'$$

Step-III ~~Mo~~ Allowable Shear Stress

$$\tau_{max} = \sqrt{\tau_1^2 + \tau_2^2 + 2\tau_1\tau_2\cos\theta}$$

$$80 = \sqrt{\left(\frac{212.16}{h}\right)^2 + \left(\frac{691.63}{h}\right)^2 + 2 \times \left(\frac{212.16}{h}\right)\left(\frac{691.63}{h}\right) \times 0.53}$$

$$80 = \sqrt{\frac{45.01\times10^3}{h^2} + \frac{478.35\times10^3}{h^2} + \frac{155.54\times10^3}{h^2}}$$

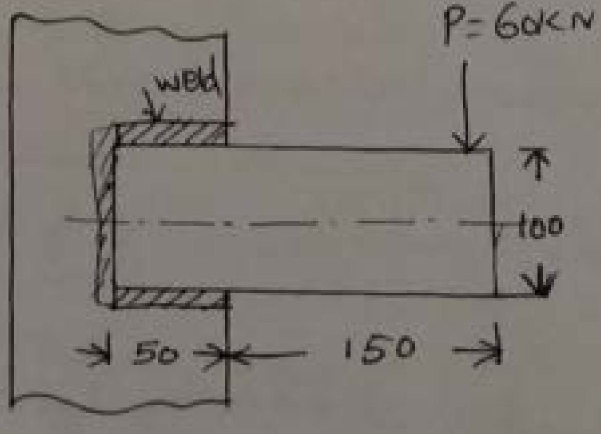
$$80 = \sqrt{\frac{678.90\times10^3}{h^2}}$$

$$80 = \frac{823.95}{h}$$

$$h = \frac{823.95}{80}$$

$$\boxed{h = 10.3\ mm}$$

2) A Rectangular steel plate is welded as a cantiLever to a vertical column and supports a single concentrated Load P. as shown in Fig. Determine the size of weld if shear stress in the same not to exceed 140MPa.



Given

Load $P = 60kN = 60 \times 10^3 N$

width of ~~wid~~ weld $b = \cancel{100}\ 50mm$

depth of weld $d = 100mm$

Maximum shear stress $\tau_{max} = 140MPa = 140 N/mm^2$

To Find:

size of weld 'h'

Soln

Step-I Primary shear stress

$$\tau_1 = \frac{\text{Load}}{\text{Area of weld}}$$

$$\tau_1 = \frac{P}{(2b+d)t} = \frac{60 \times 10^3}{(2 \times 50 + 100)0.707h}$$

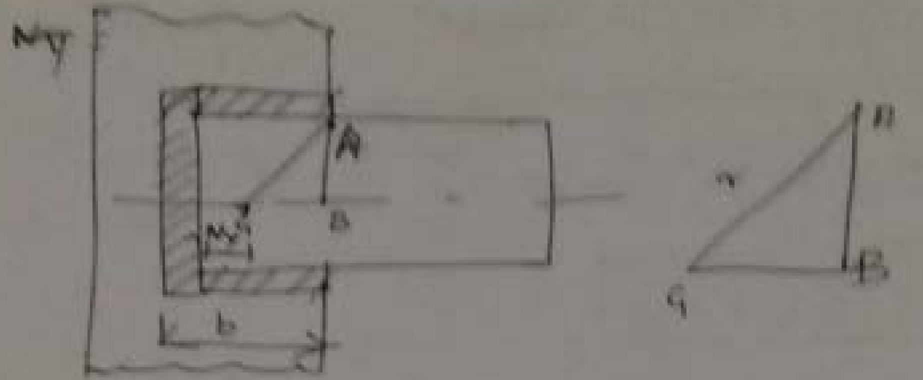
$$\boxed{\tau_1 = \frac{424.32}{h}}$$

Step-II Secondary shear stress

$$\tau_2 = \frac{M_t \times r}{J}$$

M_t = Load × eccentricity ~~of~~ from C.G

M_t = Load × $[150 + (b - N_y)]$ (or) Load × (150+4



$GB = b - N_y$

$$N_y = \frac{b^2}{2b+d} \quad \langle PSG\ 11.5 \rangle$$

$$N_y = \frac{50^2}{2\times 50+100}$$

$N_y = 12.5\ mm$

$GB = 50 - 12.5$

$\boxed{GB = 37.5\ mm}$

$M_t = 60\times10^3 \times [150 + 37.5]$

$\boxed{M_t = 9.75\times10^6\ N.mm}$ 11.25×10^6 N-mm

$$r = \sqrt{AB^2 + GB^2}$$

$AB = \frac{100}{2} = 50mm$

$$r = \sqrt{50^2 + 37.5^2}$$

$$\boxed{r = 62.5\ mm}$$

$J = J_w \times t$

$$J_w = \frac{(2b+d)^3}{12} - \frac{b^2(b+d)^2}{2b+d} \quad \langle PSG\ 11.5 \rangle$$

$$J_w = \frac{(2\times50+100)^3}{12} - \frac{50^2(50+100)^2}{2\times50+100}$$

$$J_w = \frac{666.66\times10^3}{\cancel{12}} - 281250$$

$$\boxed{J_w = \cancel{666.54\times10^3}}\ mm^4 \qquad \boxed{J_w = 385416.667\ mm^4}$$

$$J = J_w \times t = \cancel{666.54\times10^3}\ 385416.667 \times 0.707\,h$$

$$\boxed{J = \cancel{471.24\times10^3\,h}\ mm^4} \qquad \boxed{J = 272489.5836\,h}\ mm^4$$

$$\tau_2 = \frac{M_t \times r}{J} = \frac{9.75\times10^6 \times 62.5}{272.489\times10^3\,h}$$

$$\boxed{\tau_2 = \cancel{\frac{1286.48}{h}}} \qquad \boxed{\tau_2 = \frac{2236.32}{h}}$$

Step Maximum Shear stress

$$\tau_{max} = \sqrt{\tau_1^2 + \tau_2^2 + 2\tau_1\tau_2 \cos\theta}$$

$$\cos\theta = \frac{GB}{r} = \frac{37.5}{62.5}$$

$$\cos\theta = 0.6$$

$$140 = \sqrt{\left(\frac{424.32}{h}\right)^2 + \left(\frac{2236.32}{h}\right)^2 + 2\times\left(\frac{424.32}{h}\right)\left(\frac{\overset{2236.32}{\cancel{1286.44}}}{h}\right)\times 0.6}$$

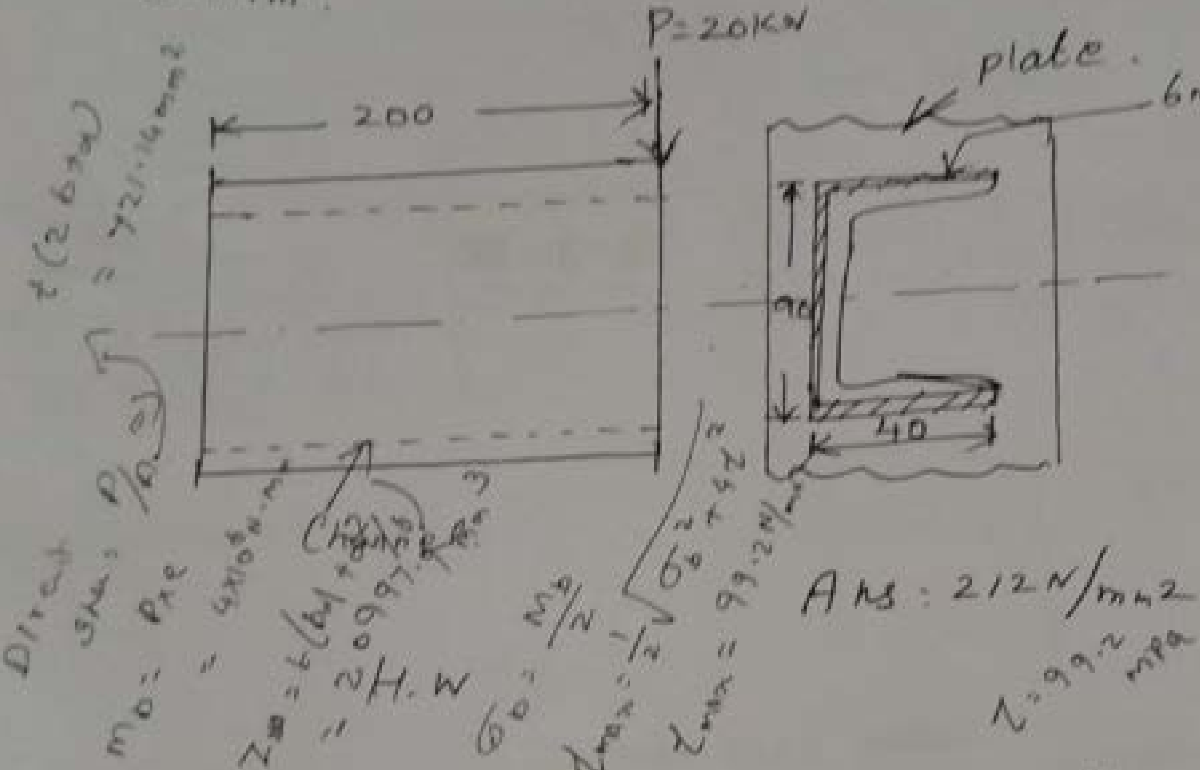
$$140 = \sqrt{\frac{2.49\times10^6}{h^2}}$$

$$140 = \frac{\cancel{1578}\,2513.935}{h}$$

$$h = \frac{\cancel{1578}\,2513.935}{140}$$

$$\boxed{h = \cancel{11.27}\,mm} \qquad \boxed{h = 17.95\ mm}$$

3) Find the Maximum shear stress Induced in the weld of 6mm size when a channel as shown in Fig. is welded to a plate and Loaded with 20KN Force at a distance of 200mm.

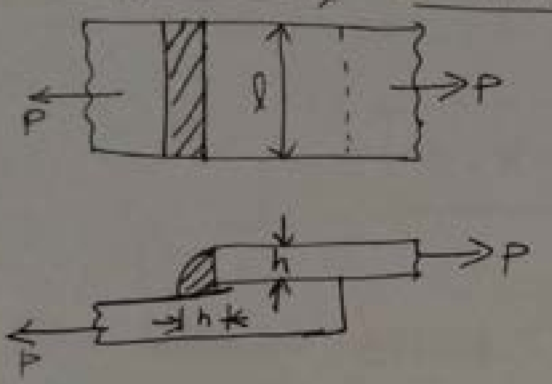


4) A plate of width 240mm is welded to a vertical plate by placing it on the vertical plate to from a Cantilever with a Projecting length of 480mm and overlap b/w the plates as 120mm. Fillet welding is done b/w the plates on all the three side. A vertical Load of 35KN is applied on the cantilever at its free end. parallel to the width. If the allowable stress on the weld is 94MN/m². determine the weld size

Strength of Transverse Fillet Welded Joints

The transverse fillet welds are designed for tensile strength. Let us consider a single and double transverse Fillet welds as shown in fig.

→ Single transverse Fillet weld:



Maxi tensile stress $\sigma_{tmax} = \frac{P}{A}$

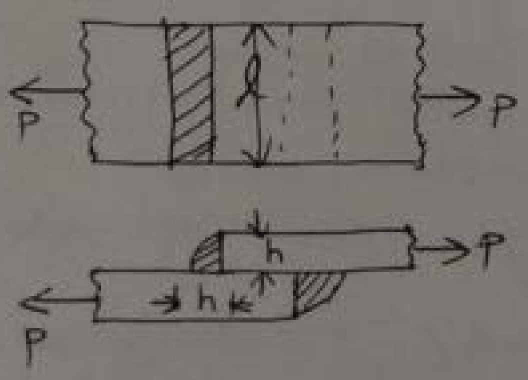
$$P = \sigma_{tmax} \times A$$

$A = l \times t$

$A = l \times 0.707h$

$$\boxed{P = \sigma_{tmax} \times 0.707\, h \times l}$$

→ Double transverse Fillet weld:



Maxi Tensile Stress $\sigma_{t_{max}} = \frac{\text{Load}}{\text{Area of weld}}$

$$\sigma_{t_{max}} = \frac{P}{A}$$

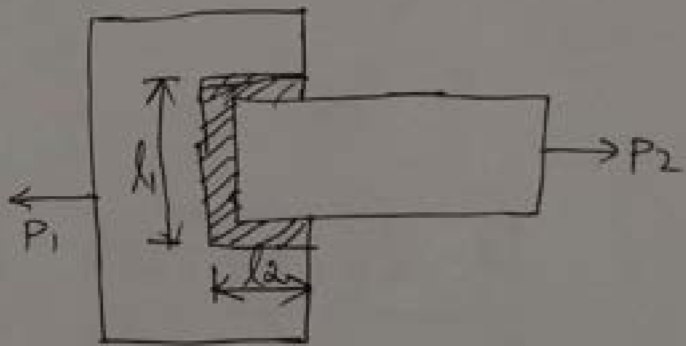
$A = 2 \times l \times t$

$A = 2 \times l \times 0.707\,h$

$$P = \sigma_{t_{max}} \times A$$

$$\boxed{P = \sigma_{t_{max}} \times 2 \times 0.707\,h \times l}$$

Combination of single and double parallel weld



l_1 = length of single fillet weld

l_2 = length of double parallel fillet weld

P_1 = Load on single transverse weld

P_2 = Load on Double transverse weld

$$\boxed{l_1 = d - t}$$

d = death in mm

t = thickness of weld

(or) thickness of plate

Maximum Load

$$\boxed{P = P_1 + P_2}$$

$h = t$

$P_1 = 0.707\, h\, l_1 \times \sigma_{tmax}$

$P_2 = 2 \times 0.707\, h \times l_2 \times \sigma_{tmax}$ (or)

If shear stress is given $P_2 = 2 \times 0.707 \times h \times l_2 \times \tau_{max}$

$$\boxed{P = 0.707\, h\, l_1 \times \sigma_{tmax} + 2 \times 0.707\, h\, l_2 \times \sigma_{tmax}}$$

~~Maxm Permi~~

Permissible stress: $\sigma_{t\,max} = \dfrac{\sigma_t}{K_t}$ = ~~Load~~

σ_t = tensile stress

K_t = stress concentration factor.

$P = \dfrac{\text{Load on tensile}}{\text{Area of plate}}$

$P = \dfrac{\sigma_t}{\text{Area of plate}}$

σ_t & τ_{max} is not given assume.

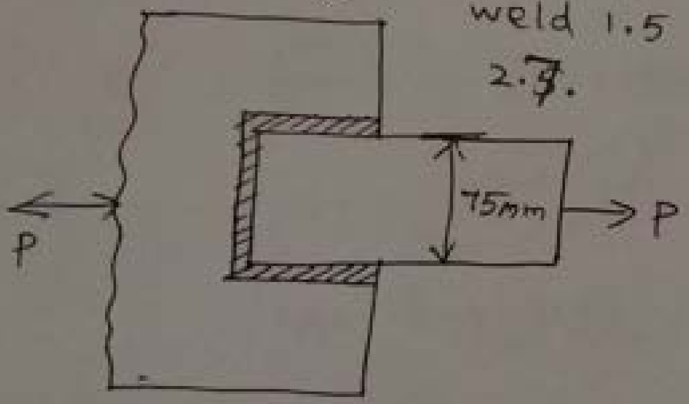
$\sigma_t = 70\ N/mm^2$

$\tau_{max} = 56\ N/mm^2$

Problem:

1. A plate 75 mm wide and 12.5 mm thick is Jointed with another plate by a single transverse weld and a double parallel fillet weld as shown in Fig. The maximum tensile stress and shear stress are 70 MPa and 56 MPa respectively.

Find the length of each parallel fillet weld, if the joint is subjected to both static and fatigue Loading. stress concentration factor for transverse weld 1.5 and parall fillet wed is 2.7.



Given:

depth of weld plate $d = 75mm$

thickness of weld plate $t = 12.5mm$

tensile stress $\sigma_t = 70MPa = 70 N/mm^2$

shear stress $\tau = 56MPa = 56 N/mm^2$

Stress Contentration factor k_t for single fillet $= 1.5$

To find:

Length of single and Double Parallel weld

l_1 & l_2

Soln

1. Length of single fillet weld l_1

$$l_1 = d - t$$

$$l_1 = 75 - 12.5$$

$$\boxed{l_1 = 62.5\ mm}$$

2. Length of double parallel fillet weld l_2

If total Load $P = P_1 + P_2$

$$P_1 = \sigma_{t_{max}} \times A \qquad \sigma_t = \frac{P}{A}$$

$P_1 = \sigma_{t\,max} \times$ Area of single filled weld

If K_t is given $\sigma_{t\,max} = \frac{\sigma_t}{K_t}$

$$\sigma_{t\,max} = \frac{70}{1.5}$$

$$\sigma_{t\,max} = 46.66\ N/mm^2$$

$$A = 1 \times l_1 \times t$$

$A = l_1 \times 0.707 h$ $\quad\quad h = t$

$A = 62.5 \times 0.707 \times 12.5$

$A = 552.34 \ mm^2$

$P_1 = 46.66 \times 552.34$

$\boxed{P_1 = 25.77 \times 10^3 \ N}$

P_2 = Load on double parallel fillet weld

P_2 = Stress × Area

$P_2 = 2 \times 0.707 \times h \times l_2 \times \tau_{max}$

$\tau_{max} = \frac{\tau}{K_t}$

$\tau_{max} = \frac{56}{2.7}$ $\quad\quad h = t$

$\tau_{max} = 20.7 \ N/mm^2$

$P_2 = 2 \times 0.707 \times 12.5 \times l_2 \times 20.7$

$\boxed{P_2 = 365.87 \, l_2}$

P = total Load

stress $\sigma_t = \frac{\text{Load}}{\text{Area of plate}} = \frac{P}{A}$

$P = \sigma_t \times A$

$P = 70 \times d \times t = 70 \times 75 \times 12.5$

$\boxed{P = 65.6 \times 10^3 N}$

$$65.6\times10^3 = 25.77\times10^3 + 365.87\, l_2$$

$$365.87\, l_2 = 65.6\times10^3 - 25.77\times10^3$$

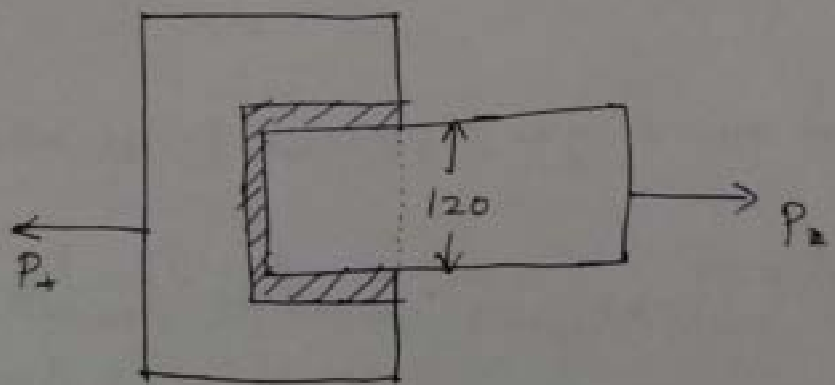
$$l_2 = \frac{65.6\times10^3 - 25.77\times10^3}{365.87}$$

$$\boxed{l_2 = 108.86\ mm}$$

2) Determine the length of the weld run for a plate of size 120mm wide and 15mm thick to be welded to another plate by means of

1. A single transverse weld ; and
2. Double parallel ~~weld~~ fillet welds when the joint is subjected to variable Load.

Given:



~~d~~

depth of plate [wide] $d = 120mm$

thickness of plate $t = \overset{15}{\cancel{150}}mm$

<u>To find</u>

Length of single and Double transverse fillet weld.

<u>Soln</u>

1. Length of single fillet weld l_1

$$l_1 = d - t$$

$$l_1 = 120 - 15$$

$$\boxed{l_1 = 105\ mm}$$

2. Length of double transverse fillet weld l_2

$$P = P_1 + P_2$$

$P_1 =$ Load on single transverse fillet weld

$$P_1 = \cancel{\sigma}\ A \times \sigma_{t\,max}$$

$$\sigma_{t\,max} = P/A$$

$$P = A \times \sigma_{t\,max}$$

$$P_1 = l_1 \times t \times \sigma_{t\,max}$$

Maximum tensile stress $\sigma_{t\,max} = \frac{\sigma_t}{K_t}$

σ_t is not given assume 70 N/mm² &

K_t for single ~~par~~ fillet weld = 1.5

$$\sigma_{t\,max} = \frac{70}{1.5}$$

$$\sigma_{t\,max} = 46.66\ N/mm^2$$

~~$P_1 = 10.5$~~ $P_1 = l_1 \times 0.707 \times h \times \sigma_{t\,max}$

$h = t$

$$P_1 = 105 \times 0.707 \times 15 \times 46.66$$

$$\boxed{P_1 = 51.96 \times 10^3\ N}$$

P_2 = Load on double fillet weld

P_2 = Area × τ_{max}

$P_2 = 2 \times l_2 \times 0.707\,h \times \tau_{max}$

τ_{max} = Maximum shear stress

$$\tau_{max} = \frac{\tau}{K_t}$$

τ = shear stress not given assume 56 N/mm²

and k_t for double parallel fillet weld = 2.7

$$\tau_{max} = \frac{56}{2.7}$$

$$\tau_{max} = 20.74 \text{ N/mm}^2$$

$$P_2 = 2 \times l_2 \times 0.707 \times 15 \times 20.74$$

$$\boxed{P_2 = \cancel{1.099 \times 10^3}\ 439.89\, l_2} \quad N$$

P = $\sigma_t \times$ Area of plate

$$P = 70 \times d \times t = 70 \times 120 \times 15$$

$$\boxed{P = 126 \times 10^3 \text{ N}}$$

$$P = P_1 + P_2$$

$$126 \times 10^3 = 51.96 \times 10^3 + \cancel{126}\ \cancel{1.099 \times 10^3}\ 439.89\, l_2$$

$$l_2 = \frac{126 \times 10^3 - 51.96 \times 10^3}{\cancel{1.099 \times 10^3}\ 439.89}$$

$$\boxed{l_2 = 168.31 \text{ mm}}$$

3) A plate 100 mm wide and 12.5mm thick is to be welded to another plate by means of fillet welds. The plates are subjected to a load of 50 KN. Find the length of the weld so that the maximum stress does not to exceed 56 MPa. Ea Consider the Joint first under static Loading and then under fatigue Loading

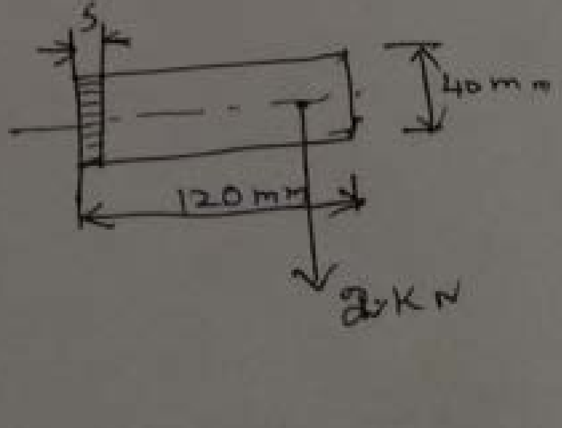
H.W

4) A welded Joint as shown in fig. is subjected to an eccentric Load of 2KN. Find the size of weld, if the Maximum Shear stress in the weld is 25Mpa

H.W.

note

$A = 2 \times l \times t$



UNIT-4

DESIGN OF ENERGY STORAGE IN DEVICES

Spring:-

Spring is the elastic body whose function is to loaded on spring compression and to recover the original shape when load is removed.

Application of spring:-

* Absorb or control energy due to either shock or vibration as in the car spring.
* Railway Buffers.
* Aircraft

Types of spring:-

* Helical spring.
* Conical Spring

* Helical spring.
  * compression coil helical spring
  * Tension coil helical spring.

  ~~conical springs~~
* Conical spring
* Torsional spring
* Leaf spring or laminated spring
* Disc spring
* Special purpose spring

DESIGN PROCEDURE OF HELICAL SPRING :-

(Refer DB Pg: 7.100)

Step 1 : Dimension of spring.

d - Diameter of wire

D - Mean diameter of spring

D_o - Outer diameter of spring

$$D_o = D + d$$

Shear stress $\tau = K_s \frac{8PD}{\pi d^3} = K_s \frac{8PC}{\pi d^2}$

P - axial load

If two load is given, $P = P_{max}$

K_s - Wahl stress factor

$$K_s = \frac{4C-1}{4C-4} + \frac{0.615}{C}$$

C - Spring index

$$C = \frac{D}{d}$$

Step 2 : Number of ~~spring~~ active coils (n)

By using the deflection relation:

$$y = \frac{8PD^3 n}{Gd^4} = \frac{8PC^3 n}{Gd}$$

where

n - number of coils

G - modulus of rigidity.

$$G = 8 \times 10^4 \ N/mm^2$$

P = load

If two load is given $P = P_{max} - P_{min}$.

Step 3 : Stiffness of spring 'q'.

$$q = \frac{Gd^4}{8D^3n} = \frac{Gd}{8C^3n} \quad (or) \quad \frac{W}{Y}$$

$q = ?$

Step 4 : Total number of coil. (n_t)

Refer DB pg.7.101

From Data book pg.no.7.101

For end condition, plane or plane ground

Step 5 : Solid Length of Spring

$$L_s = dn + d$$

Step 6 : Free length of spring

$$L_f = Pn + d. \quad (or) \text{ pitch value.}$$

$$L_f = L_s + Y$$

Step 7 : Pitch of coils.

$$P = \frac{L_f - L_s}{n_t} + d$$

Step 8 : Helix angle 'α'.

$$\alpha = \tan^{-1}\left(\frac{P}{\pi D}\right) \qquad P\text{-Pitch value}$$

Step 9: Workdone or energy. <7.100>

$$U = \frac{PY}{2}$$

P = load.

<u>Note</u>:-

Workdone (or) energy × No of Spring = KE

$$\frac{PY}{2} \times n = \frac{1}{2} mv^2$$

6. Derive the expression for Shear Stress for helical spring by using torsional expression $\frac{T}{J} = \frac{C\theta}{l} = \frac{\tau}{R}$

Where $J = \frac{\pi d^4}{32}$

T = Torque = Twisting load $(P) \times \frac{D}{2}$

$$R = \frac{d}{2}$$

$$\frac{P \times (D/2)}{\left(\frac{\pi d^4}{32}\right)} = \frac{\tau}{(d/2)}$$

$$\tau = \frac{8PD}{\pi d^3}$$

$$\tau = K_s \frac{8PD}{\pi d^3}$$

1. A helical coil spring is to be designed for a operating load of range is 90N to 135 N. The deflection of the spring for the load range is 7.5 mm. Assume the spring index of 10. The permissible shear stress of 480 N/mm² and modulus of rigidity is 0.8 × 10⁵ N/mm² for the material design the spring.

Given Data:-

$$P_{min} = 90\,N$$

$$P_{max} = 135\,N$$

$$y = 7.5\,mm$$

$$C = 10$$

$$\tau = 480\,N/mm^2$$

$$G = 0.8 \times 10^5\,N/mm^2$$

To Find:-

Design of Helical spring.

Solution:-

Refer DB Pg. 7.100

Step 1 : Dimension of spring.

d - diameter of wire

D - Mean diameter of spring

D_o - Outer diameter of spring

$$D_o = D + d$$

$$\tau = K_s \frac{8PC}{\pi d^2}$$

$$P = P_{max} = 135\text{ N}$$

$$K_s = \frac{4C-1}{4C-4} + \frac{0.615}{C}$$

$$= \frac{4(10)-1}{4(10)-4} + \frac{0.615}{10}$$

$$K_s = 1.14$$

$$\tau = 1.14 \; \frac{8 \times 135 \times 10}{\pi d^2}$$

$$480 = \frac{3919.03}{d^2}$$

$$d = 2.85\text{ mm}$$

say $\boxed{d = 3\text{ mm}}$

$$C = \frac{D}{d}$$

$$D = 10 \times 3$$

$$\boxed{D = 30\text{ mm}}$$

$$D_o = 30 + 3$$

$$\boxed{D_o = 33\text{ mm}}$$

Step 2 : Number of active coils (n)

By using the deflection relation

$$y = \frac{8PD^3n}{Gd^4}$$

$$P = P_{max} - P_{min} = 135 - 90$$

$$= 45\ N.$$

$$n = \frac{7.5 \times 0.8 \times 10^5 \times 3^4}{8 \times 45 \times 30^3}$$

$$\boxed{n = 5}$$

Step 3: Stiffness of spring 'q'

$$q = \frac{Gd^4}{8D^3 n}$$

$$= \frac{0.8 \times 10^5 \times 3^4}{8 \times 30^3 \times 5}$$

$$\boxed{q = 6\ N/mm}$$

Step 4: Total number of coil (n_t).
(Refer DB Pg. 7.101)

For end condition,

~~n_t = [illegible]~~

Assume square ~~and Ground~~ condition.

$$n_t = n + 2$$

$$= 5 + 2$$

$$\boxed{n_t = 7}$$

Step 5: Solid Length of spring.
consider square condition.

$$L_s = dn + 3d$$

$$= 3(5) + 3(3)$$

$$\boxed{L_s = 24\ mm}$$

Step 6 : Free length of Spring

$$L_f = Pn + 2d$$

$$L_f = L_s + y$$

$$= 24 + 7.5$$

$$\boxed{L_f = 31.5\,mm}$$

Step 7 : Pitch of coil

$$P = \frac{L_f - L_s}{n_t} + d$$

$$= \frac{31.5 - 24}{7} + 2$$

$$\boxed{P = 4.07\,mm}$$

Step 8 : Helix angle 'α'

$$\alpha = \tan^{-1}\left(\frac{P}{\pi D}\right)$$ P - pitch value

$$= \tan^{-1}\left(\frac{4.07}{\pi \times 30}\right)$$

$$\alpha = 2.47$$

Step 9 : Workdone or energy.

$$U = \frac{P_{max}\,y}{2}$$ P - load

$$= \frac{135 \times 7.5}{2}$$

$$\boxed{U = 506.25\ N \cdot mm}$$

2. A Rail wagon moving at a velocity of 1.5 m/s is brought to rest by a bumber consisting of two helical springs arranged in parallel. The mass of the wagon is 1500 kg. The spring are compressed by 150 mm in bringing the wagon to rest. The spring index can be taken as 6. The spring are made of oil hardened and tempered steel wire with ultimate tensile Strength of 1250 N/mm^2 and modulus of rigidity of 81,370 N/mm^2. The permissible Shear Stress for the spring wire can be taken as 50% of the ultimate tensile strength. Design the spring and calculate i) wire diameter

ii) Mean coil diameter

iii) Number of active coils.

iv) Total number of coil

v) Solid length

vi) Free length

vii) Pitch of coil.

viii) Required spring rate and

ix) Actual spring rate.

Given Data:-

$V = 1.5\ m/s$

$m = 1500\ kg$

no of spring $n = 2$

$C = 6$

$y = 150\ mm.$

$\sigma_u = 1250\ N/mm^2$

$G = 81370\ N/mm^2$

$\tau = 50\%\ \sigma_u$

$\tau = 0.5\ \sigma_u = 0.5 \times 1250$

$= 625\ N/mm^2$

To Find:-

Design of Helical Spring.

Solution:-

Step: Dimension of spring

d - diameter of wire

D - Mean ~~coil~~ diameter of spring

D_o - out

Solution:-

Step 1 : Dimension of Spring

d - diameter of wire

D - Mean diameter of spring

D_o - Outer diameter of Spring.

$$\tau = k_s \frac{8PC}{\pi d^2}$$

$$k_s = \frac{4C-1}{4C-4} + \frac{0.615}{C}$$

$$= \frac{4(6)-1}{4(6)-4} + \frac{0.615}{C}$$

$$k_s = 1.25$$

$$\frac{Py}{2} \times n = \frac{1}{2} m v^2$$

$$\cancel{\frac{P \times 150}{2} \times 2 = \frac{1}{2} \times 1500 \times (1.5)^2 \times (\ldots)^2}$$

$$\cancel{P \times 150 = 1687.5}$$

$$\frac{Py}{2} \times n = \frac{P \times 150}{2} \times 2$$

$$= P \times 150$$

$$\frac{1}{2} m v^2 = \frac{1}{2} \times 1500 (1.5)^2$$

$$= 1687.5 \text{ J}$$

$$= 1.687 \times 10^6 \text{ Nmm}$$

Now $P \times 150 = 1.687 \times 10^6$

$$P = 11.25 \times 10^3 \text{ N}$$

$$\tau = 1.25 \quad \frac{8 \times 11.25 \times 10^3 \times 6}{\pi \times d^2}$$

$$d^2 = \frac{1.25 \times 8 \times 11.25 \times 10^3 \times 6}{\pi \times 625}$$

$$d = 18.54 \text{ mm}$$

Say $\boxed{d = 20 \text{ mm}}$

$$C = \frac{D}{d}$$

$$D = 6 \times 20$$

$$\boxed{D = 120 \text{ mm}}$$

$$D_o = D + d = 120 + 20$$

$$\boxed{D_o = 140 \text{ mm}}$$

Step 2 : Number of active coils 'n'

$$y = \frac{8PD^3n}{Gd^4}$$

$$n = \frac{150 \times 81370 \times 20^4}{8 \times 11.25 \times 10^3 \times 120^3}$$

$$n = 12.56$$

say $\boxed{n = 13}$

Step 3 : Stiffness of spring :

$$q = \frac{Gd^4}{8D^3n}$$

$$= \frac{81370 \times 20^4}{8 \times 120^3 \times 13}$$

$$\boxed{q = 72.44 \ N/mm}$$

Step 4 : Total number of coils.

Refer DB pg. 7.101.

Assume square condition

$$n_t = n_t + 2$$

$$= 13 + 2$$

$$\boxed{n_t = 15}$$

Step 5 : Solid Length of Spring

$$L_s = dn + 3d$$

$$= (20 \times 13) + (3 \times 20)$$

$$\boxed{L_s = 320 \ mm}$$

Step 6 : Free length of spring.

$$L_f = Pn + 3d$$

$$L_f = L_s + y$$

$$= 320 + 150$$

$$\boxed{L_f = 470 \ mm}$$

Step 7 : pitch of coils.

$$p = \frac{L_f - L_s}{n_t} + d = \frac{470 - 320}{15} + 20$$

$$\boxed{P = 30 \ mm}$$

Required Spring rate :-

$$q_R = \frac{P}{y}$$

$$= \frac{11.25 \times 10^3}{150}$$

$$\boxed{q_R = 75 \text{ N/mm}}$$

3. A helical compression spring is used to absorb the ~~sa~~ shock. The initial compression of the Spring is 30mm and it is further compressed by 50mm while absorbing the shock. The spring is to be absorb 250 Joule of energy during the process. The spring index can be taken as 6. The spring is made of patterned and cold drawn steel wire with an ultimate tensile strength of 1500 MPa and Modulus of rigidity as 81370 MPa. The permissible shear stress for a spring wire should be taken as 30% Ultimate tensile strength. Design the spring and calculate

i) wire diameter

ii) Mean coil diameter

iii) Number of Active coils

iv) Free length

v) Pitch of the turn

Given Data:- $(1 J = 1 Nm)$

~~y~~

initial compression, $y_1 = 30$ mm.

Final compression, $y_2 = 80$ mm.

Energy absorbed, $U = 250$ J

$= 250 \times 10^3$ N mm

$C = 6$.

$\sigma_u = 1500 \text{ N/mm}^2$

$G = 81370$ MPa

$= 81370 \text{ N/mm}^2$

$\tau = 30\% \ \sigma_u$

$= 0.3 \times 1500$

$= 450 \text{ N/mm}^2$

To Find:-

i) wire diameter

ii) Mean coil diameter

iii) Number of Active coils

iv) Free length

v) Pitch of the turn

Design of Helical spring.

Solution:-

Step 1 : Dimensions of Spring:

d - ~~Dimension of spring~~ diameter of wire

D - Mean Diameter of spring

Do - outer diameter of spring

$$\tau = K_s \frac{8PC}{\pi d^2}$$

$$K_s = \frac{4C-1}{4C-4} + \frac{0.615}{C}$$

$$= \frac{4(6)-1}{4(6)-4} + \frac{0.615}{6}$$

$$K_s = 1.25$$

Find 'P'.

If two deflection and energy is given

$$y_1 = 30\,mm$$

$$\text{stiffness } q_1 = \frac{\text{load}}{\text{deflection}} = \frac{P_1}{y_1}$$

$$q = \frac{P_1}{30}$$

$$P_1 = 30 \times q$$

$$y_2 = 80\,mm$$

$$q = \frac{P_2}{80}$$

$$P_2 = 80 \times q$$

Average Force during compression $= \dfrac{P_1+P_2}{2}$

$$= \frac{30q + 80q}{2}$$

$$= \frac{110q}{2}$$

$$= 55q$$

Energy absorbed during shock

= Average Force × Second compression of spring.

$250 \times 10^3 = 55q \times 50$

$\boxed{q = 90.9\ N/mm}$

$P_1 = 30 \times 90.9$

$\boxed{P_1 = 2727\ N}$

$P_2 = 80 \times 90.9$

$\boxed{P_2 = 7272\ N}$

$P = P_{max} = 7272\ N.$

$$d^2 = \frac{1.25 \times 8 \times 7272 \times 6}{\pi \times 450}$$

$d = 18.75\ mm$

say $\boxed{d = 20\ mm}$

$C = \frac{D}{d}$

$D = 120\ mm$

$D_o = D + d = 120 + 20$

~~= 140 mm~~

$= 140\ mm.$

Step 2 : Number of active coils.

$$y = \frac{8PD^3n}{Gd^4}$$

$$P = P_{max} - P_{min}$$

$$= 7272 - 2727$$

$$= 4545 \text{ N}$$

$$n = \frac{20 \times 81370 \times 20}{8 \times 4545 \times 6^3}$$

$$n = 16.57$$

$$\boxed{n = 17}$$

Step 3 : Actual Stiffness of spring 'q'

$$q = \frac{Gd}{8C^3n}$$

$$q = \frac{81370 \times 20}{8 \times 6^3 \times 17}$$

$$\boxed{q = 55.39 \text{ N/mm}}$$

Step 4 : Total no of coil 'n_t'

Assume square condition

$$n_t = n + 2 = 17 + 2$$

$$\boxed{n_t = 19}$$

Step 5 : Solid length of spring

$$L_s = dn + 3d = (20 \times 19) + (3 \times 20)$$

$$\boxed{L_s = 440 \text{ mm}}$$

Step 6: Free length of spring

$$L_f = pn + d$$

$$L_f = L_s + y$$

$$= 440 + 80$$

$$\boxed{L_f = 520 \text{ mm}}$$

Step 7: Pitch of coils

$$p = \frac{L_f - L_s}{n_t} + d$$

$$= \frac{520 - 440}{19} + 20$$

$$\boxed{P = 24.4 \text{ mm}}$$

Step 8: Helix angle.

$$\alpha = \tan^{-1}\left(\frac{p}{\pi D}\right)$$ p - pitch value

$$= \tan^{-1}\left(\frac{24.4}{3.14 \times 120}\right)$$

$$\alpha = 3.70$$

4. Design a helical spring for a spring load safety valve for the following condition

Diameter of valve seat = ~~60mm~~ 65 mm.

operating pressure = 0.7 N/mm^2

maximum pressure when the valve blows of freely = 0.75 N/mm^2

Maximum lift of the valve when the pressure raises from 0.7 to 0.75 N/mm^2 = 3.5 mm

Maximum allowable stress $= 550\ N/mm^2$

Modulus of rigidity $= 84\ kN/mm^2$

Spring index $= 6$.

Draw a neat sketch of a free spring showing the main dimensions.

Gn Data:-

Diameter of valve seat $D_1 = 65\ mm$

$$P_{max} = 0.75 N/mm^2$$

$$P_{min} = 0.7\ N/mm^2$$

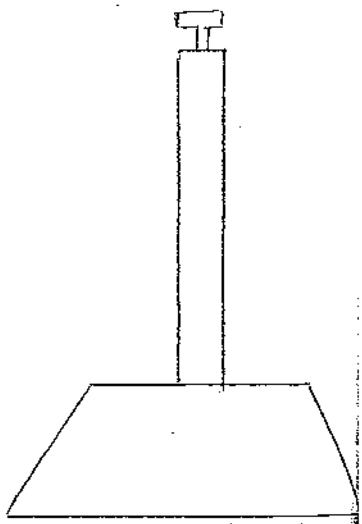
$$\tau = 550\ N/mm^2$$

$$G = 84\ kN/mm^2$$

$$= 84 \times 10^3\ N/mm^2$$

$$C = 6.$$

$$y = 3.5\ mm$$



To Find:-

Design of helical spring

Solution:-

Step 1 : Dimension of spring (Refer DB pg 7.100)

d - diameter of wire

D - Mean diameter of spring

D_o - Outer diameter of spring

$$\tau = K_s \frac{8PC}{\pi d^2}$$

$$K_s = \frac{4C-1}{4C-4} + \frac{0.615}{C}$$

$$= \frac{4(6)-1}{4(6)-4} = \frac{0.615}{6}$$

$$\boxed{K_s = 1.25}$$

Load $P_{max} = P_{r max} \times \text{Area}.$

$$= 0.75 \times \frac{\pi}{4} 65^2$$

$$= 2488.73 \text{ N}.$$

$$P_{min} = P_{r min} \times \text{Area}$$

$$= 0.7 \times \frac{\pi}{4} 65^2$$

$$P_{min} = 2322.8 \text{ N}.$$

$$\cancel{d^2 = \frac{550 \times \pi}{1.25 \times 8}}$$

$$d^2 = \frac{1.25 \times 8 \times 2488 \times 6}{\pi \times 550}$$

$$d = 9.29$$

$$\boxed{d = 10 \text{mm}}$$

$$C = \frac{D}{d}$$

$$\boxed{D = 60 \text{mm}}$$

$$D_o = D + d$$

$$= 60 + 10$$

$$\boxed{D_o = 70 \text{mm}}$$

Step 2 : Number of active coils

$$y = \frac{8PD^3 n}{Gd^4}$$

$$P = P_{max} - P_{min}$$

$$= 2488.73 - 2322.8$$

$$= 165.93 \text{ N}$$

$$n = \frac{3.5 \times 84 \times 10^3 \times 10^4}{8 \times 165.93 \times 60^3}$$

$$n = 10.25$$

$$\boxed{n = 11 \text{ nos}}$$

Step 3 : Stiffness of spring 'q'

$$q = \frac{Gd}{8c^3 n}$$

$$= \frac{84 \times 10^3 \times 10}{8 \times 6^3 \times 11}$$

$$\boxed{q = 44.19 \text{ N/mm}}$$

Step 4 : Total no of coils 'n_t'

Assume square condition

$$n_t = n + 2$$

$$= 11 + 2$$

$$\boxed{n_t = 13}$$

Step 5 : Solid length of spring

$$L_s = dn + 3d$$

$$= (10 \times 11) + (3 \times 10)$$

$$\boxed{L_s = 140 \text{ mm}}$$

Step 6 : Free length of spring

$$L_f = L_s + y$$

$$= 140 + 3.5$$

$$\boxed{L_f = 143.5 \text{ mm}}$$

Step 7 : Pitch of coils

$$p = \frac{L_f - L_s}{n_t} + d$$

$$= \frac{143.5 - 140}{13} + 10$$

$$\boxed{P = 10.26 \text{ mm}}$$

Step 8 : Helix angle

$$\alpha = \tan^{-1}\left(\frac{P}{\pi D}\right)$$

$$= \tan^{-1}\left(\frac{10.26}{\pi \times 60}\right)$$

$$\alpha = 3.11$$

Step 9 : workdone or energy

$$U = \frac{P_{max}\, y}{2}$$

$$= \frac{2488.73 \times 3.5}{2}$$

$$U = 4355.2 \text{ Nmm}$$

DESIGN OF SPRING SUBJECTED TO UNDER VARYING LOADS :-

If factor of safety is given, Find the coil diameter d

(Refer DB pg 7.102)

$$\frac{1}{n} = \frac{\tau_m - \tau_a}{\tau_y} + \frac{2\tau_a}{\tau_e}$$

where τ_m - mean shear stress

$$\tau_m = \frac{8 K_{sh} P_m D}{\pi d^3} = \frac{8 K_{sh} P_m C}{\pi d^2}$$

τ_a - amplitude shear stress

$$\tau_a = \frac{8 K_s P_a D}{\pi d^3} = \frac{8 K_s P_a C}{\pi d^2}$$

P_m = mean load

$$= \frac{P_{max} + P_{min}}{2}$$

P_a = amplitude load

$$= \frac{P_{max} - P_{min}}{2}$$

K_s = Wahl stress factor.

$$= K_{sh} K_c \text{ (or) } K_s = \frac{4C-1}{4C-4} + \frac{0.615}{C}$$

K_{sh} = direct shear factor

$K_{sh} = 1 + \frac{0.5}{C}$

K_c = Curvature Factor

τ_o = endurance shear stress

$= 0.5 \tau_y$

where τ_y = Yield shear stress

$\tau_y = 0.405\, \sigma_u$ (For light service).

σ_u = ultimate tensile strength.

1. A helical compression spring made of oil tempered carbon steel is subjected to a load which varies from 400 N to 1000 N. The spring index is 6 and design factor of safety is 1.25. If the yield stress in shear is 770 MPa and endurance stress in shear is 350 MN/m². Find the size of spring wire, diameter of the spring, Number of turns of the spring and free length of spring.

The compression of the spring at the maximum load 30 mm. The modulus of rigidity for spring material may be taken as 80 KN/mm² and design the helical spring.

Gn Data:-

$P_{max} = 1000\ N$

$P_{min} = 400N$

FOS (or) $n = 1.25$

$C = 6$, $K_c = 1.15$

$\tau_y = 770\ MPa = 770\ N/mm^2$

τ_e (or) $\tau_o = 350\ MN/m^2 = 350 \times 10^6\ N/m^2$

$= 350\ N/mm^2$

$y = 30mm$

$G = 80kN/mm^2$

$= 80 \times 10^3\ N/mm^2$

To Find:-

i) Size of spring wire.

ii) diameter of spring

iii) Number of turns of spring

iv) Free length of spring.

Solution:-

Step 1 : Dimension of spring.

If factor of safety is given

$$\frac{1}{n} = \frac{\tau_m - \tau_a}{\tau_y} + \frac{2\tau_a}{\tau_e}$$

(Refer DB pg. No. 7.102)

$$\tau_m = \frac{8 K_{sh} P_m C}{\pi d^2}$$

$$P_m = \frac{P_{max} + P_{min}}{2}$$

$$= \frac{1000 + 400}{2}$$

$$P_m = 700 \text{ N}$$

$$K_s = K_{sh} K_c$$

$$K_{sh} = 1 + \frac{0.5}{C}$$

$$= 1 + \frac{0.5}{6}$$

$$\boxed{K_{sh} = 1.08}$$

$$K_s = 1.08 \times 1.15$$

$$\boxed{K_s = 1.24}$$

$$\tau_m = \frac{8 \times 1.08 \times 700 \times 6}{\pi \times d^2}$$

$$\boxed{\tau_m = \frac{11550.82}{d^2}}$$

$$\tau_a = \frac{8 K_s P_a C}{\pi d^2}$$

$$P_a = \frac{P_{max} - P_{min}}{2} = \frac{1000 - 400}{2}$$

$$= 300 \text{ N}$$

$$\tau_a = \frac{8 \times 1.24 \times 300 \times 6}{\pi \times d^2}$$

$$\boxed{\tau_a = \frac{5683.74}{d^2}}$$

$$\frac{1}{1.25} = \frac{11550.82 - 5683.74}{d^2 \times 770} + \frac{2(5683.74)}{d^2 \times 350}$$

$$\frac{1}{1.25} = \frac{40.09}{d^2}$$

$$d^2 = 40.09 \times 1.25$$

$$d = 7.07 \text{ mm}$$

$$\boxed{d = 7 \text{mm}}$$

$$C = \frac{D}{d}$$

$$D = 6 \times 7$$

$$\boxed{D = 42 \text{ mm}}$$

$$D_o = D + d = 42 + 7$$

$$\boxed{D_o = 49 \text{ mm}}$$

Step 2 : Number of active coils, n

$$y = \frac{8PD^3n}{Gd^4}$$

$$P = P_{max} - P_{min} = 1000 - 400$$

$$= 600 \text{N}$$

$$n = \frac{30 \times 80 \times 10^3 \times 7^4}{8 \times 600 \times 42^3}$$

$$n = 16.20$$

$$\boxed{n = 16 \text{ nos}}$$

Step 3: Stiffness of spring

$$q = \frac{Gd}{8C^3 n}$$

$\frac{N}{mm^2}$ $\times mm$

$$= \frac{80 \times 10^3 \times 7}{8 \times 6^3 \times 16}$$

$$q = 20.25 \text{ N/mm}$$

Step 4: Total number of coils, n_t

Assume square condition

$$n_t = n + 2$$

$$= 16 + 2$$

$$\boxed{n_t = 18}$$

$$L_s = dn + 3d$$

$$= (7 \times 16) + (3 \times 7)$$

$$L_s = 133 \text{ mm}$$

$$L_f = L_s + y$$

$$= 133 + 30$$

$$L_f = 163\ mm$$

$$P = \frac{L_f - L_s}{n_t} + d$$

$$= \frac{163 - 133}{18} + 7$$

$$P = 8.66$$

$$\alpha = \tan^{-1}\left(\frac{P}{\pi D}\right)$$

$$= \tan^{-1}\left(\frac{8.66}{3.14 \times 42}\right)$$

$$\alpha = 3.75^\circ$$

$$U = \frac{PY}{2}$$

$$U = \frac{1000 \times 30}{2}$$

$$U = 15 \times 10^3\ Nmm$$

DESIGN OF LEAF SPRING:-

Multi leaf spring are used to suspension of ~~bogque~~ car, trucks and railway Multi leaf spring consist of a series of flat plates. The Flat plates are called leaves of the Spring. The leaves are graduated length.

The leaf at top has maximum length. The length gradually decrease from the top leaf to bottom leaf.

Length of the spring = 2L

Load acting on the Spring = 2P

Ratio of Total depth and breadth = $\frac{nt}{b}$

where, n - number of leaf

t - thickness of spring

b - breadth of leaf

Formula used (Refer DB Pg. 7.104)

Bending stress $\sigma_b = \frac{6PL}{nbt^2}$

Deflection of the Spring $y = \frac{6PL^3}{Enbt^3}$

Where, E is the Young's modulus of the material.

initial space, h (or) $x = \frac{2PL^3}{Enbt^3}$

or

Nip (h)

Load exerted on the clipping Bolt Assembled, $P_b = \frac{2n_e n_g P}{n(2n_g + 3n_e)}$

$n = n_e + n_g$

where, n_e - number of extra full length leaves

n_g - number of graduated leaves.

1. A locomotive semi ~~[illegible]~~ eliptical laminated spring has an overall length of 1 m and substains a load of 70kN at its centre. The spring has three full length leaves and 15 graduated leaves with a central band of 100mm width. All the leaves are to be stressed to 400MPa. when full loaded the ratio of total spring depth to that of width is 2. The Young's modulus of 210 kN/mm^2. Determine i) thickness and width of leaves ii) The initial gap that should provided between the full length and graduated leaf band load is applied.

iii) The load exerted on the band after the spring is assembled.

Gin Data:-

$$2L = 1\,m = 1000\,mm ,\Rightarrow$$

$$2P = 70\,KN$$

$$P = 35\,KN = 35\times10^{3}\,N$$

$$n_e = 3$$

$$n_g = 15$$

$$l = 100\,mm$$

$$\sigma_b = 400\,MPa = 400\,N/mm^2$$

$$\frac{nt}{b} = 2$$

$$E = 210\,KN/mm^2 = 210\times10^{3}\,N/mm^2$$

To Find:-

i) Thickness and width of leaves.

ii) Initial gap provided between full length and graduated leaf. (x)

iii) Load exerted (P_b)

Solution:-

i) Thickness and width of leaves

$$\frac{nt}{b} = 2$$

$$n = n_e + n_g = 3+15$$

$$\boxed{n = 18}$$

$$\frac{18t}{b} = 2$$

$$\boxed{b = 9t}$$

If Bending Load is given

$$\sigma_b = \frac{6PL}{nbt^2}$$

$$400 = \frac{6 \times 35 \times 10^3 \times L}{18 \times 9t \times t^2}$$

$2L = 1000$ – Length of central band

$= 1000 - l$

$= 1000 - 100$

$2L = 900$

$$\boxed{L = 450 \text{ mm}}$$

$$t^3 = \frac{6 \times 35 \times 10^3 \times 450}{18 \times 9 \times 400}$$

$t = 11.34$ mm

Say $\boxed{t = 12 \text{mm}}$

$b = 9t = 9 \times 12$

$$\boxed{b = 108 \text{ mm}}$$

ii) initial gap or Nip

$$x = \frac{2PL^3}{Enbt^3}$$

$$= \frac{2 \times 35 \times 10^3 \times 450^3}{210 \times 10^3 \times 18 \times 108 \times 12^3}$$

$$\boxed{x = 9 \text{ mm}}$$

iii) Load exerted

$$P_b = \frac{2n_e n_g P}{n(2n_g + 3n_e)}$$

$$= \frac{2 \times 3 \times 15 \times 35 \times 10^3}{18((2 \times 15) + (3 \times 3))}$$

$$\boxed{P_b = 4487.17 \text{ N}}$$

2. Design a leaf spring for a truck to the following specification. Maximum load of spring is 140 KN, Number of Spring = 4, Material = Chromium - Vanadium, spring Permissible tensile stress 600 MPa, Maximum number of leaves = 10, span of spring = 1000 mm, Permissible deflection = 80 mm, Young's modulus = 2×10^5 MPa.

Gvn Data! -

$$2P = 140 \text{ kN}$$

$$P = 70 \text{ kN} = 70 \times 10^3 \text{ N}$$

No of leaves $n = 10$

$$\sigma = 600 \text{ MPa} = 600 \text{ N/mm}^2$$

$$2L = 1000 \text{ mm}$$

$$L = 500 \text{ mm}$$

$$y = 80 \text{ mm}$$

$$E = 2 \times 10^5 \text{ MPa}$$

$$= 2 \times 10^5 \text{ N/mm}^2$$

To Find

i) b, t

ii) x

iii) P_b

Solution! -

i) Thickness and width of leaves.

$$\frac{nt}{b} = 2$$

$$\frac{10\,t}{b} = 2$$

$$\frac{t}{b} = 0.2$$

$$\boxed{b = 5t}$$

If Bending load gr

$$\sigma_b = \frac{6PL}{nbt^2}$$

$\frac{bt}{b}l = 3$

$$600 = \frac{6 \times 70 \times 10^3 \times 500}{10 \times 5t \times t^2}$$

~~Determining~~

$2L = 1000 - 80$

$2L = 920$

$L = 460.$

$$t^3 = \frac{6 \times 70 \times 10^3 \times 460}{10 \times 5 \times 600}$$

$$\boxed{t = 18.60 \text{ mm}}$$

$b = 5 \times 19.12$

$$\boxed{b = 93.02 \text{ mm}}$$

ii) $$x = \frac{2PL^3}{Enbt^3} = \frac{2 \times 70 \times 10^3 \times 460^3}{2 \times 10^5 \times 10 \times 93.02 \times 18.6^3}$$

$= 11.38 \text{ mm}.$

iii) $$P_b = \frac{2 n_e n_g P}{n(2n_g + 3n_e)}$$

UNIT - 5

DESIGN OF BEARINGS.

Bearing :-

Bearing is a mechanical elements that permit relative motion between two parts such as the shaft and housing wit minimum friction.

Functions :-

* Free rotation of shaft
* Support the shaft (or) the axle an holds it in the correct position.
* The bearing takes of the forces that act on the shaft.

Classification :-

i) Depending upon the direction of the load

* Radial bearing
  - Load act on the bearing perpendicular to the direction of motion of the moving element.
* Thrust Bearing.
  - The load act along the axi

of rotation

ii) Depending upon the nature of contact

* Sliding contact bearing

The slides take place the surfaces of contact between the moving element and the fixed element.

* Rolling contact bearing

The steel bars or rollers, the interpost between the moving and fixed element.

Types of sliding contact bearing

* Full Journal bearing (360°)

* Partial Journal bearing (120° or 180°)

Hydro dynamic Lubricated Bearing:-

The thick flim bearings are those, in which the working surface are completely separated from each other by the lubricant.

* Boundry lubricated Bearing

- Thin film bearings partial contact.

* Zero film Bearing - without any lubricant.

DESIGN PROCEDURE OF JOURNAL BEARING:-

Step 1 : Dimensions of bearing (D, L).

(Refer DB pg. 7.31)

$$\frac{L}{D} = ?$$

* If machine is not given assume centrifugal pump

* Diameter of Journal is not given Assume D = 100 mm.

* Power is given $P = \frac{2\pi N T}{60 \times 1000}$

* Torque $T = \frac{\pi}{16} \tau D^3$. ; D = ?

Step 2 : Pressure developed (or) Check for Bearing pressure.

* Allowable Bearing pressure $[P_a]_{all}$ in kgf/cm^2

Induced Bearing pressure, $P_{ind} = \frac{load}{A} = \frac{W}{D \times L}$

where, W - load in kgf

L, D in cm.

$P_{indu} < [P]_{all}$

Design is safe

Step 3 : Viscosity of oil. (Z)

(Refer DB pg 7.31)

$$\frac{Zn}{P} = ?$$

$\Rightarrow Z = ?$

Where, Z in cps (centipoise)

$P = P_{induced}$

n - Speed in rpm.

Step 4 : Select grade of oil (SAE)

(Refer DB Pg No 7.41)

* t_o = operating temperature in bearing ~~oil~~ in °C.

* If t_o is not given Assume 60°C to 90°C.

Step 5 : Bearing module :-

From basic theory $\left[\frac{Zn}{P}\right]_{induced} \geq \frac{1}{3}\left[\frac{Zn}{P}\right]_{all}$

(DB Pg. 7.31)

∴ Bearing operated on hydrodynamic lubrication.

Step 6 : coefficient of friction μ.

(Refer DB Pg. No. 7.34)

From McKEES EQUation.

$$\mu = \frac{33.25}{10^{10}}\left(\frac{Zn}{P}\right)\left(\frac{D}{C}\right) + k$$

$\left[\frac{Zn}{P}\right]_{all} \Rightarrow$ (Refer DB Pg. 7.31)

D - Diameter of Journal in cm

C - Diametral clearance (Refer DB Pg 7.32)

C (diameteral clearance) in micron from table Pg. No. 7.32

1 micron $= 10^{-6}$ meter.

K is constant from the graph. (Pg. No. 7.34).

Step 7: Heat generated. (H_g) (or) Power lost (Refer DB Pg. 7.34).

Heat generated $H_g = \mu W V$ in kgf m/min.

W - load in kgf

V - velocity in m/min.

$V = \pi D N$ m/min.

Step 8: Heat dissipated, H_d (Refer DB Pg. 7.34

$$H_d = \frac{(\Delta t + 18)^2 L D}{K} \quad \text{kgf m/min}$$

Where, L and D in meter

K - Constant heat dissipation (Pg. 7.35).

Δt - average temperature rise.

$$\Delta t = \frac{t_o - t_a}{2}$$

$$H_d = CA(t_o - t_a)$$

Step 9: Artificial Cooling is required or not?

If $H_g > H_d$, Aritifical cooling is required

Step 10: mass of oil (m)

$$m = \frac{H_g - H_d}{C' \Delta t_o} \text{ in kgf/min}$$

Where, C' - specific heat of oil (DB Pg. 7.34)

$= 17100$ kgf cm / kgf°c

$= 171$ kgf m / kgf°c

Δt_o - increase in temperature of oil

If it is not given $\Delta t_o = 10°c$

Step 11: Selection of Bearing material

Refer DB Pg. 7.30

Step 12: Diameter of bearing D_b

$$D_b = D + C$$

where, D - Diameter of Journal

C - Diametoral clearance

Note: -

* $1 \, NS/m^2 = 1 \, kg/ms$ (or) PaS

* $1 \, CP = 1 \times 10^{-3}$ PaS

$= 10^{-9}$ MPaS.

NOTES

1) Gerber curve:- Gerber curve is a parabola drawn between endurance limit and ultimate tensile strength.

Soderberg line: Line Joining σ_y on mean stress axis and σ_{-1} on stress amplitude axis is called Soderberg line Diagram used for ductile material

Goodman line : Line Joining σ_u on mean stress axis and σ_{-1} on stress amplitude axis is called as Goodman line Diagram used for brittle material

What are the different

2) Method to reduce stress concentration

Santh partitions

* Avoiding sharp corners
* Providing fillets.

Santhu

Santh

* Use of multiple holes instead of single hole
* Undercutting the shoulder parts.

Define

3) Critical speed of shaft:- The speed at which the shaft runs so that the additional deflection of the shaft from the axis of rotation becomes infinite is known as critical or whirling speed

4) Limitation of welding:-

* It has poor vibration damping characteristics
* Welding results distortion of parts which induces residual stresses

5) Theory of bonded Joints:- A rivet is a short cylindrical bar with a head integral to it. The cylinder portion of the rivet is called shank or body and lower portion of the shank is known as tail. The riveted Joints are widely used for Jointing light metals

NOTES

6) Self locking of power screw ~~torque~~

If the friction angle (ϕ) is greater than helix angle (α) of the power screw, the torque required to lower the load will be positive which indicates an effort applied to lower the load. This type of screw is known as Self-locking screw. The efficiency is less than 50%.

7) Pitch: It is the axial distance from a point on one thread to corresponding point on the next thread gadgets

Lead :- It is the distance moved by the screw in one turn

8) By what material threaded fasteners are made?

Steel is the material of which most of the fasteners are made. For improving their properties alloy steel like nickel steel, Ni-Cr Steel, Cr-V Steel are prefer

9) Caulking :- The edge of one plate is pressed tightly against the plate on which it rests by means of a caulking tool.

Fullering :- The operation is similar to caulking operation except that fullering makes used of a tool having thickness at the end equal to the plate thickness and it carries an angle of 80° at the end.

10) D/B coupling & clutch :- A coupling is a device used to make permanent or semi permanent connection where as a clutch permits rapid connection or disconnection at will of the operator.

1. Design a Journal bearing for a centrifugal pump for a following Data

Diameter of Journal = 150 mm

Load of Bearing = 40 kN

Speed of Journal = 900 rpm.

Gn Data:-

$D = 150\ mm = 0.15\ m = 15\ cm$

$W = 40\ kN = 40 \times 10^3\ N$

$= 40 \times 10^2\ kgf$

$N = 900\ rpm$

Solution:-

Step 1 : Dimension of bearing

For centrifugal pump (Refer DB pg. 7.31)

$\frac{L}{D} = 1.0 - 2.0$

Take $\frac{L}{D} = 1.5$

$L = 1.5 \times 15$

$\boxed{L = 22.5\ cm}$

Step 2 : pressure developed or check for bearing pressure.

$[P]_{all} = 7 - 14\ kgf/cm^2$ (Refer pg. 7.31)

Take $[P]_{all} = 14\ kgf/cm^2$

$P_{ind} = \frac{W}{D \times L} = \frac{40 \times 10^2}{22.5 \times 15}$

$= 11.85\ kgf/cm^2$

$P_{induced} < [P]_{all}$, Design is safe

Step 3 : Viscosity of oil

$$\frac{Zn}{P} = 2844.5$$

$P = P_{Induced}$

$$Z = \frac{2844.5 \times 11.85}{900}$$

$$= 37.45 \text{ cps}$$

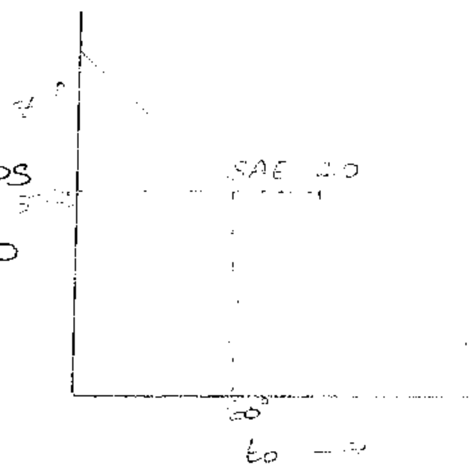
Step 4 : Selection of grade (Refer DB pg. 7.41)

From the graph, x-axis temperature assume 60°c to 90°c

Take $t_0 = 60^\circ c$

Y axis $Z = 37.45$ cps

Grade of oil SAE 40 is selected.



Step 5 : Bearing module

From Basic theory, $\left.\frac{Zn}{P}\right]_{Ind} > \frac{1}{3}\left[\frac{Zn}{P}\right]_{all}$

$$\frac{37.45 \times 900}{11.85} > \frac{1}{3}(2844.5)$$

$$2844.3 > 948.16$$

Condition is satisfied, so bearing is operated on hydro dynamic lubrication.

Step 6 : Coefficient of friction

From McKEES Equation

$$\mu = \frac{33.25}{10^{10}}\left(\frac{Zn}{P}\right)\left(\frac{D}{C}\right) + k$$

C - diameteral clearance (Refer DB pg. 7.32)

C = 125 to 200 micron for D = 150 mm

Take $c = 150$ micron

$= 150 \times 10^{-6}$ m

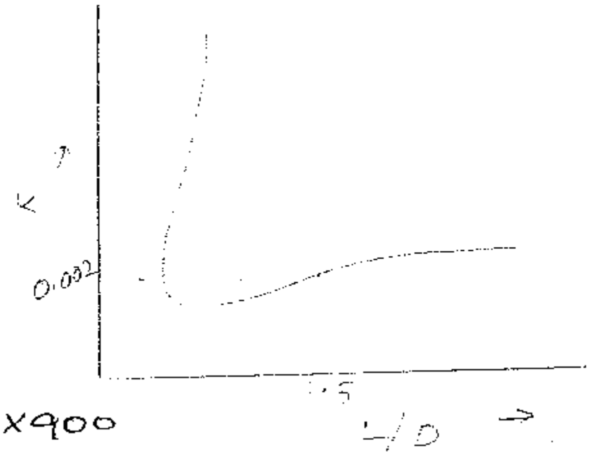
$= 15 \times 10^{-3}$ cm.

K – constant (Refer DB Pg. 7·34)

$K = 0.002$

$$\mu = \frac{33.25}{10^{10}}(2844.5)\left(\frac{15}{15\times10^{-3}}\right) + 0.002$$

$$\boxed{\mu = 0.0114}$$



Step 7: Heat generated.

$H_g = \mu W V$

$$= \frac{0.0114 \times 40 \times 10^{2} \times \pi \times}{0.15 \times 900}$$

$= 20.18 \times 10^{3}$ kgf m/min.

Step 8: Heat dissipated, H_d

$$H_d = \frac{(\Delta t + 18)^2 LD}{k}$$

Assume $t_a = 30^\circ$

$$\Delta t = \frac{t_o - t_a}{2} = \frac{60 - 30}{2}$$

$= 15$

$k = 437$ for heavy construction (Rf DB Pg. 7·35)

$$H_d = \frac{(15+18)^2 \times 0.15 \times 0.225}{437}$$

$H_d = 0.084$ kgf /min

Step 9: Artifical Cooling is required or not?

$$H_g = 20.18 \times 10^3 \text{ kgf m/min}$$

$$H_d = 0.084 \text{ kgf m/min}$$

$H_g > H_d$, Artifical cooling is required.

Step 10: Mass of oil (m)

$$m = \frac{H_g - H_d}{C' \Delta t_o}$$

Assume $\Delta t_o = 10°C$

$$m = \frac{20.18 \times 10^3 - 0.084}{171 \times 10}$$

$$m = 11.80 \text{ kgf/min.}$$

Step 11: Selection of Bearing material

(Refer DB Pg. 7.30)

Rubber material is selected for the the application of marine propellers, pumps, Turbine

Step 12: Diameter of Bearing.

$$D_b = D + C$$

$$= 15 + 15 \times 10^{-3}$$

$$= 15.015 \text{ cm.}$$

2. Design a Journal bearing for centrifugal pump from the following data

Load on the Journal = 20,000 N

Speed of the Journal = 900 rpm

Type of oil SAE = 10 for which absolute viscosity at 55°c = 0.017 kg/m sec, ambiant temperature of oil = 15°c, Maximum bearing pressure for the pump = 1.5 N/mm^2.
calculate also mass of the lubricating oil required for artifical cooling is rise of temperature of oil limited to 10°c. Heat dissipation coefficient 1232 W/m^2/°c

Gen Data:-

$$W = 20{,}000\,N = 2000\ kgf$$

$$N = 900\ rpm$$

Type of oil SAE = 10

$$t_o = 55°c$$

$$Z = 0.017\ kg/m\,sec$$

$$= 17\,cP$$

$$t_a = 15°c$$

$$[P]_{all} = 1.5\,N/mm^2 = 15\ kgf/cm^2$$

$$\Delta t = 10°c$$

$$C = 1232\ W/m^2/°c$$

To Find:-

mass of oil (m)

Solution:-

Step 1 : Dimension of Journal bearing <7.31>

For centrifugal pump

$$\frac{L}{D} = 1 - 2$$

Take $\frac{L}{D} = 1.5$

Diameter is not given assume $D = 100\,mm$

$D = 10\,cm$

$L = 1.5 \times 10$

$\boxed{L = 15\,cm}$

Step 2 : pressure developed (or) Check for bearing pressure.

$[P]_{all} = 15\,kgf/cm^2$ (Given).

$P]_{ind} = \frac{W}{D \times L}$

$= \frac{2000}{10 \times 15}$

$= 13.33\,kgf/cm^2$

$P]_{ind} < [P]_{all}$, Design is safe.

Step 3 : Viscosity of oil

$Z = 0.017\,kg/ms$ (Given).

$= 17\,cps$

Step 4 : Selection of grade

Oil SAE = 10 (Given)

Step 5 : Bearing module.

From Basic theory $\left.\frac{Zn}{P}\right]_{ind} \geq \frac{1}{3}\left[\frac{Zn}{P}\right]_{all}$

$\frac{17 \times 900}{13.33} > \frac{1}{3}(2844.5)$

$1147.7 > 948.16$

Condition is is satisfied so the bearing is operated on hydro dynamic lubrication.

Step 6 : Coefficient of friction <7.34>

From McKEES Equation.

$$\mu = \frac{33.25}{10^{10}}\left(\frac{Zn}{P}\right)\left(\frac{D}{c}\right)+k$$

c - diametral clearance <7.32>

$C = 100$ to 175 microns for $D = 100$ mm

$C = 150$ micron

$= 150 \times 10^{-6}$ m

$C = 15 \times 10^{-3}$ cm.

k = constant

$= 0.0025$

$$\mu = \frac{33.25}{10^{10}}(2844.5)\left(\frac{10}{15\times10^{-3}}\right) + 0.0025$$

$$\boxed{\mu = 8.79 \times 10^{-3}}$$

Step 7 : Heat generated (or) Power loss

$H_g = \mu W V$ kgf m/min

$= 8.79 \times 10^{-3} \times 2000 \times \pi \times 0.10 \times 900$

$= 4.97 \times 10^{3}$ kgf m/min.

$= 828.33$ W

Step 8 : Heat dissipated

$H_d = CA\left[\frac{t_o - t_a}{2}\right]$

$= C \times L \times D\left[\frac{t_o - t_a}{2}\right]$

$= 1232 \times 0.15 \times 0.10\,(20^{\circ})$

$H_d = 369.6$ W

$$H_d = \frac{(\Delta t + 18)^2 LD}{K}$$

$$= \frac{(20+18)^2 \times 0.15 \times 0.1}{\cancel{0.0025}437}$$

$\cancel{2866}$

$= 0.0049$ kgf m/min

Step 9 : Artifical coolings is required or not.

$$H_g = 4.97 \times 10^3 \text{ kgf m/min}$$

$$\cancel{= 4.97 \times 10^2 \text{ W}} \quad 228.32 \text{ W}$$

$$H_d = 369.6 \text{ W}$$

$H_g > H_f$, Artifical cooling is required.

Step 10 : mass of oil (m)

$$m = \frac{H_g - H_d}{c' \Delta t_o}$$

$$c' = 171 \text{ kgf m/kgf}^\circ\text{C}$$

$$m = \frac{4.97 \times 10^3 - 369.6}{171 \times 10} \quad 0.049$$

S

$$m = 0.26 \text{ kgf/min}$$

Design of Journal bearing Subjected to Radial load Hydro dynamic bearing (360°, 180°, 120°, 60°).

DESIGN PROCEDURE :-

Step 1 : Dimension of Journal bearing

$\frac{L}{D} = ?$ <7.31>

* If machine is not given assume centrifugal pump.

* Diameter of Journal is not given Assume $D = 100mm$.

* Power is given $P = \frac{2\pi NT}{60 \times 1000}$

* Torque $T = \frac{\pi}{16} \tau D^3$; $D = ?$

Step 2 : Pressure developed (or) check for bearing pressure.

Allowable Bearing pressure $[P]_{all}$ in kgf/cm^2

induced bearing pressure, $P_{ind} = \frac{Load}{A} = \frac{W}{L \times D}$

W - load in kgf

L, D in cm

$P_{ind} < [P]_{all}$

Design is safe.

Step 3 : Viscosity of oil.

$Z = ?$ $\langle 7.31 \rangle$

Step 4 : Sommer field number

$\langle 7.34 \rangle$

$$S = \left(\frac{Z'n'}{P}\right)\left(\frac{D}{C}\right)^2$$

where, $P = P_{induced}$

Z' - absolute viscosity

$$Z' = \frac{Z}{9.81 \times 10^7} \text{ kgf sec/cm}^2.$$

n' - speed in rps.

D - Diameter of Journal

C - Diameteral clearance

$C = 2 \times$ radial clearance

$= 2 \times C_r$

Step 5 : Dimensionless performance parameters

(Refer DB Pg. 7.36, 7.37, 7.38, 7.39)

* Coefficient of friction variable.

$$= \mu \frac{D}{C}$$

* Flow variable $= \dfrac{4q}{DCn'L}$

* Flow ratio $= \dfrac{q_s}{q}$

* Pressure ratio $= \frac{P}{P_{max}}$

where, $P = P_{induced}$

* Temperature rise variable $= \frac{\rho c' \Delta t_o}{P}$

* Minimum oil film thickness $= \frac{2h_o}{c}$

Step 6 : Heat generated (or) power loss in friction.

$$H_g = \mu W V \quad \text{in kgf m/min}$$

Step 7 : Heat dissipated H_d

$$H_d = \frac{(\Delta t + 18)^2 LD}{k} \quad \text{kgf m/min}$$

$$H_d = CA\,\Delta t_o$$

$$\Delta t_o = \frac{t_o - t_a}{2}$$

Step 8 : Amount of artifical cooling required

$$= H_g - H_d$$

Step 9 : mass of lubricating oil.

$$m = \frac{H_g - H_d}{c'\Delta t_o} \quad \text{kgf/min.}$$