STELLA MARY'S COLLEGE OF ENGINEERING

(Accredited by NAAC, Approved by AICTE - New Delhi, Affiliated to Anna University Chennai)

Aruthenganvilai, Azhikal Post, Kanyalumari District, Tamilnadu - 629202.

CE8395 - STRENGTH OF MATERIALS FOR MECHANICAL ENGINEERS

(Anna University: R2017)

Prepared By

Dr.F.Michael Raj, ME., Ph.D,

Professor

DEPARTMENT OF MECHANICAL ENGINEERING

2020-2021 (EVEN)

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Aruthenganvilai, Kallukatti Junction Azhikal Post, Kanyakumari District-629202, Tamil Nadu.

DEPARTMENT OF MECHANICAL ENGINEERING

COURSE MATERIAL

REGULATION	**2017**
YEAR	**II**
SEMESTER	**04**
COURSE NAME	**STRENGTH OF MATERIALS FOR MECHANICAL ENGINEERS**
COURSE CODE	**CE8395**
NAME OF THE COURSE INSTRUCTOR	**Dr.F.Michael Raj**

CE8395 - STRENGTH OF MATERIALS FOR MECHANICAL ENGINEERS

SYLLABUS:

UNIT I - STRESS, STRAIN AND DEFORMATION OF SOLIDS 9

Rigid bodies and deformable solids – Tension, Compression and Shear Stresses – Deformation of simple and compound bars – Thermal stresses – Elastic constants – Volumetric strains –Stresses on inclined planes – principal stresses and principal planes – Mohr’s circle of stress.

UNIT II - TRANSVERSE LOADING ON BEAMS AND STRESSES IN BEAM 9

Beams – types transverse loading on beams – Shear force and bending moment in beams – Cantilevers – Simply supported beams and over – hanging beams. Theory of simple bending– bending stress distribution – Load carrying capacity – Proportioning of sections – Flitched beams – Shear stress distribution.

UNIT III - TORSION **9**

Torsion formulation stresses and deformation in circular and hollows shafts – Stepped shafts– Deflection in shafts fixed at the both ends – Stresses in helical springs – Deflection of helical springs, carriage springs.

UNIT IV - DEFLECTION OF BEAMS **9**

Double Integration method – Macaulay's method – Area moment method for computation of slopes and deflections in beams - Conjugate beam and strain energy – Maxwell's reciprocal theorems.

UNIT V - THIN CYLINDERS, SPHERES AND THICK CYLINDERS **9**

Stresses in thin cylindrical shell due to internal pressure circumferential and longitudinal stresses and deformation in thin and thick cylinders – spherical shells subjected to internal pressure – Deformation in spherical shells – Lame's theorem.

TEXT BOOKS:

1. Bansal, R.K., "Strength of Materials", Laxmi Publications (P) Ltd., 2016.
2. Jindal U.C., "Strength of Materials", Asian Books Pvt. Ltd., New Delhi, 2009

REFERENCES:

1. Egor. P.Popov "Engineering Mechanics of Solids" Prentice Hall of India, New Delhi, 2002.
2. Ferdinand P. Been, Russell Johnson, J.r. and John J. Dewole "Mechanics of Materials", Tata McGraw Hill Publishing 'co. Ltd., New Delhi, 2005.
3. Hibbeler, R.C., "Mechanics of Materials", Pearson Education, Low Price Edition, 2013.
4. Subramanian R., "Strength of Materials", Oxford University Press, Oxford Higher Education Series, 2010.

Course Outcome Articulation Matrix

Course Code / CO No	Program Outcome												PSO		
	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3
CE8395 / C214.1	3	1	3	3	0	0	0	0	0	0	1	3	3	3	0
CE8395 / C214.2	3	3	3	3	0	0	0	0	0	0	1	3	3	3	0
CE8395 / C214.3	3	3	3	3	0	0	0	0	0	0	2	3	3	3	0
CE8395 / C214.4	3	3	3	3	0	0	0	0	0	0	1	3	3	3	0
CE8395 / C214.5	3	3	3	3	0	0	0	0	0	0	2	3	3	3	0
Average	3	3	3	3	0	0	0	0	0	0	1	3	3	3	0

Unit Introduction

When an external force acts on a body, the body tends to undergo some deformations.

* Due to the cohesion b/w the molecules, the body resists deformation.

* This resistance by which material of the body opposes the deformation is known as "Strength of material".

Unit - I

Stress, strain & Deformation of solids:-

Stress:-

* The force of resistance per unit area, offered by a body against deformation is known as stress.

* The external force acting on the body is called the load or force.

* In simple words,it's defined as the "Internal resistance which the body offers to meet with the load is called stress.

* Mathematically stress is written as

$$\sigma = \frac{P}{A}.$$

where σ = stress

P = External force or load and

A = Cross-Sectional area.

Unit of stress = N/m^2 (or) N/mm^2.

Strain(ε) :-

When a body is subjected to some external force. there is some change of dimension of the body. The ratio of the change of dimension of the body to the original dimension is known as strain.

strain is dimensionless.

Types of strains:-

1. Tensile 2. Compressive & 3. shear.

Types of Stresses :-

(i) Normal stress :- It is the stress which acts in a direction perpendicular to the area.

* It is represented by (σ).

* The normal stress is further divided into
  (a) Tensile stress.
  (b) Compressive stress and
  (c) Shear stress

Tensile stress :-

* The stress induced in a body, when subjected to two equal & opposite pulls.

* As a result of which there is an increase in length, is known as Tensile stress.

$$\therefore \text{Tensile stress} = \sigma = \frac{\text{Tensile load } (P)}{A}$$

$$\therefore \sigma = \frac{P}{A}$$

Tensile strain :-

The ratio of increase in length to the original length is known as tensile strain.

It is denoted by "e".

$$e = \frac{\text{Increase in length}}{\text{Original length}} = \frac{\delta L}{L}$$

Compressive stress :-

The stress induced in a body, when subjected to two equal & opposite pushes.

∴ The compressive stress is given by

$$\sigma = \frac{\text{Resisting force (R)}}{\text{Area (A)}} = \frac{P}{A}$$

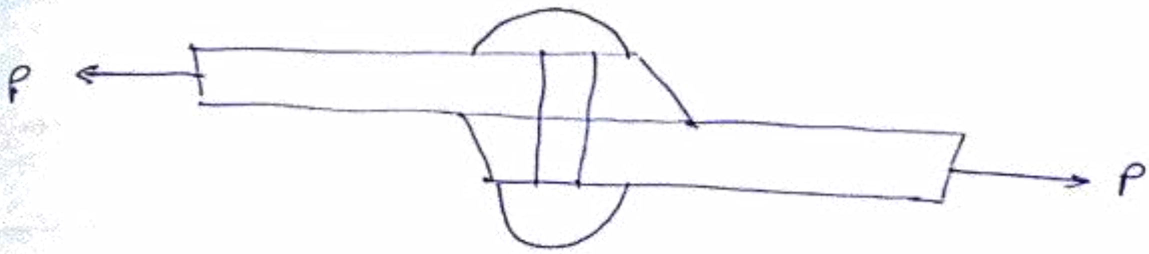
Compressive strain :-

$$e = \frac{\text{Decrease in length}}{\text{Original length}} = \frac{\delta L}{L}$$

Shear Stress :-

* The stress induced in a body, when subjected to two equal & opposite forces which are acting tangentially across the resisting section.

* As a result of which the body tends to shear off across the section is known as shear stress.



It is denoted by the symbol 'τ'.

$$\text{Shear stress, } \tau = \frac{\text{Shear resistance}}{\text{Shear area}} = \frac{R}{A} \Rightarrow \frac{P}{A}.$$

Shear strain(ϕ) :-

$$\phi = \frac{\text{Transversal displacement}}{\text{Distance AD}}$$

$$\phi = \frac{\delta L}{h}.$$

Elasticity :-

* When an external force acts on a body, the body tends to undergo some deformation.

* If the external force is removed & the body comes back to its original shape & size (which means the deformation disappears completely), the body is known as elastic body

* This property, by virtue of which certain materials return back to their original position after the removal of the external force is called elasticity.

Elastic limit :-

* The body will regain its previous shape & size only when the deformation caused by the external force is within a certain limit.

* Thus, there is a limiting value of force up to & within which, the deformation completely disappears on the removal of the force

* The value of stress corresponding to the limiting force is known as the elastic limit of the material

Hooke's law & Elastic Modulii.

* Hooke's law states that when a material is loaded within elastic limit, the stress is propotional to the strain is a constant within elastic limit.

* This constant is known as modulus of elasticity or Modulus of rigidity or Elastic modulii.

Young's Modulus (or) Modulus of Elasticity :-

The ratio of tensile stress or Compressive Stress to the Corresponding strain is a constant.

This ratio is known as Young's modulus or Modulus of elasticity & its denoted by E.

$$E = \frac{\text{Tensile stress}}{\text{Tensile strain}} \text{ (or) } \frac{\text{Compressive stress}}{\text{Compressive strain}}.$$

$$E = \frac{\sigma}{\varepsilon}.$$

Shear Modulus (or) Modulus of Rigidity

The ratio of shear stress to the corresponding shear strain within the elastic limit, is known as modulus of rigidity or modulus of shear. It's denoted by C (or) G (or) N.

$$C \text{ or } G \text{ or } N = \frac{\text{Shear Stress}}{\text{Shear strain}} = \frac{\tau}{\phi}.$$

Factor of Safety:-

It's defined as the ratio of ultimate tensile stress to the working (or permissible) stress. Mathematically it's written as

$$\text{Factor of Safety} = \frac{\text{Ultimate Stress}}{\text{Permissible stress}}$$

1. Longitudinal Strain:-

When a body is subjected to an axial tensile load, there is an increase in the length of the bar. But at the same time, there is a decrease in other dimensions of the body at right angles to the line of action of the applied load.

$$\text{Longitudinal strain} = \frac{\delta L}{L}$$

δL = Increase in length.

L = Length of the body.

Lateral Strain:-

The strain at right angles to the direction of applied load is known as lateral strain.

The length of the bar will increase while the breadth & depth will decrease.

$$\text{lateral strain} = \frac{\delta b}{b} \text{ or } \frac{\delta d}{d}.$$

$\delta b \rightarrow$ decrease in breadth. $\delta d \rightarrow$ decrease in depth.

Poisson's ratio:-

The ratio of the lateral strain to the longitudinal strain is a constant for a given material. when the material is stressed within the elastic limit. It's generally denoted as 'μ'.

$$\mu = \frac{\text{Lateral strain}}{\text{longitudinal strain}}.$$

Relationship b/w stress & strain (2D).

$$e_1 = \frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E}.$$

$$e_2 = \frac{\sigma_2}{E} - \mu \frac{\sigma_1}{E}.$$

where, $e_1 \rightarrow$ Total strain in x direction.

$e_2 \Rightarrow$ Total strain in y direction.

$\sigma_1 \rightarrow$ Normal stress in x-direction.

$\sigma_2 \rightarrow$ Normal stress in y direction.

For (iv) Relationship b/w σ & e :-

$$e_1 = \frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E} - \mu\frac{\sigma_3}{E}$$

$$e_2 = \frac{\sigma_2}{E} - \mu\frac{\sigma_3}{E} - \mu\frac{\sigma_1}{E}$$

$$e_3 = \frac{\sigma_3}{E} - \mu\frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E}$$

Pb :- ①

A tensile test was conduted on a mild ste[el]

The following data was obtained from the te[st]

Dia. of steel bar = 3 cm.

Gauge length of bar = 20 cm.

Load at elastic limit = 250 kN

Extension at a load of 150 kN = 0.21 mm

Max. load = 380 kN.

Total expansion = 60 mm.

Diameter of the rod at the failure = 2.25 cm.

Determine (a) Young's mod[ulus]

b) stress at elastic limi[t]

c) Percentage elongatio[n]

d) Percentage decre[ase] in area

Soln:-

Area of the rod: $A = \frac{\pi}{4}D^2$.

$$= \frac{\pi}{4}(3\times10^{-2})^2$$

$$= 7.0685\times10^{-4}\ m^2.$$

$$\text{Stress} = \frac{\text{Load}}{\text{Area}} = \frac{150 \times 1000}{7.0685 \times 10^{-4}} \Rightarrow 21220.9 \times 10^{+4}\ N/m^2$$

$$\text{strain} = \frac{\text{Increase in length.}}{\text{Original length}} = \frac{0.21 \times 10^{-3}}{20 \times 10^{-2}} = 0.00105.$$

$$\text{Young's modulus. } E = \frac{\text{Stress}}{\text{strain}} \Rightarrow \frac{21220.9 \times 10^{4}}{0.00105}$$

$$= 20209523 \times 10^{4}\ N/m^2 \Rightarrow 202.095\ GN/m^2$$

b) <u>Stress at elastic limit</u>

$$\text{Stress} = \frac{250 \times 10^3}{7.0685 \times 10^{-4}} = 35368 \times 10^4\ N/m^2.$$

$$= 353.68\ MN/m^2.$$

c) <u>% elongation :-</u>

$$\%\ \text{elongation} = \frac{\text{Total increase}}{\text{Original length}}$$

$$= \frac{60 \times 10^{-3}}{20 \times 10^{-3}} \times 100 = 30\%.$$

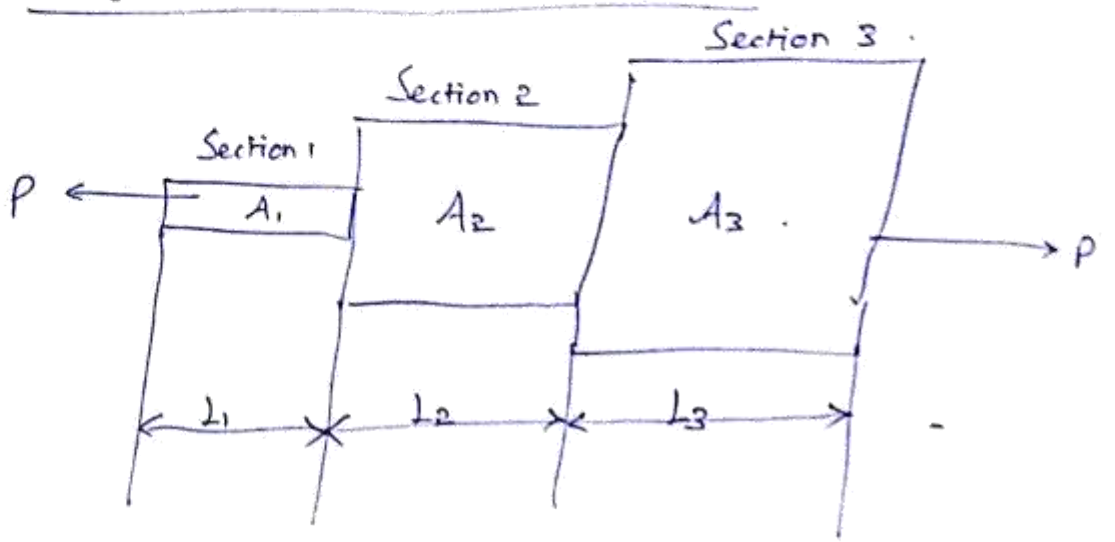
d) <u>Percentage decrease Area</u>

$$\%\ \text{decrease} = \frac{(\text{original Area} - \text{Area at failure})}{\text{Original area}} \times 100.$$

$$= \left[\frac{\frac{\pi}{4}(3 \times 10^{-2})^2 - \frac{\pi}{4}[2.25 \times 10^{-2}]^2}{\frac{\pi}{4}(3 \times 10^{-2})^2} \right] \times 100.$$

$$= 43\%.$$

Analysis of Bars of Varying Sections :-



P → Axial load acting on the bar.

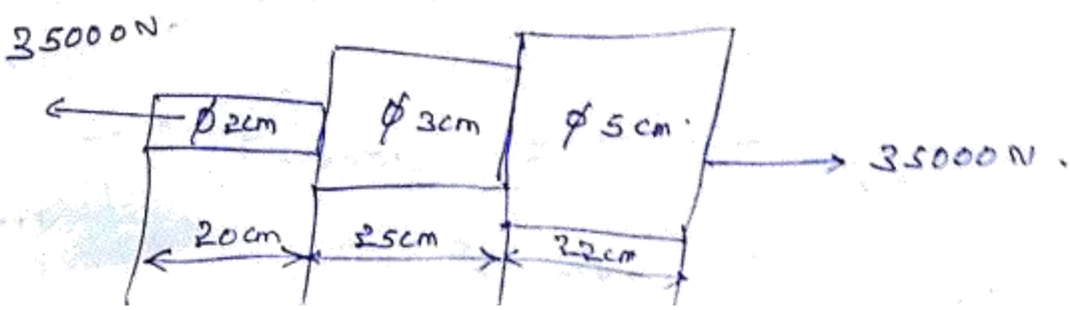
$L_1, L_2 \& L_3$ → Length of section ①, ② & ③.

A_1, A_2, A_3 → Cross-sectional area of section ①, ② & ③

Then the total change in length of the bar is given by,

$$dL = P\left[\frac{L_1}{E_1 A_1} + \frac{L_2}{E_2 A_2} + \frac{L_3}{E_3 A_3}\right]$$

Pb ② An axial pull of 35000 N is acting on a bar consisting of three lengths as shown in fig. If the Young's modulus $= 2.1 \times 10^5$ N/mm². Determ (i) Stress in each section & (ii) Total expansion of the b



Soln

Stress in Section ①, $\sigma_1 = \dfrac{35000}{\frac{\pi}{4}(20)^2}$

$= 111.408\ N/mm^2$.

Stress in Section ②, $\sigma_2 = \dfrac{35000}{\frac{\pi}{4}(30)^2}$

$= 49.5146\ N/mm^2$.

Stress in Section ③, $\sigma_3 = \dfrac{35000}{\frac{\pi}{4}(50)^2}$

$= 17.825\ N/mm^2$

Total expansion :-

$$dL = \frac{P}{E}\left[\frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3}\right].$$

$$= \frac{35000}{2.1\times 10^5}\left[\frac{200}{(\frac{\pi}{4})(20)^2} + \frac{250}{\frac{\pi}{4}(30)^2} + \frac{220}{\frac{\pi}{4}(50)^2}\right]$$

$$= \frac{35000}{2.1\times 10^5}(6.366 + 3.536 + 1.120)$$

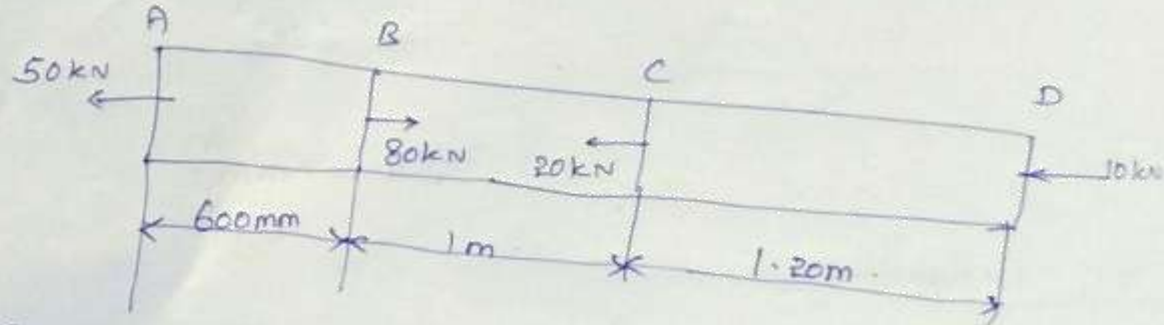
$dL = 0.183\ mm$.

——————— x ———————

Principles of Superposition:-

When a no. of loads are acting on a body. the resulting strain, according to the principle of superposition. will be the algebric sum of strains caused by individual loads.

Pb ⑤ A brass bar, having cross-sectional area of $1000\,mm^2$, is subjected to axial forces as shown in fig



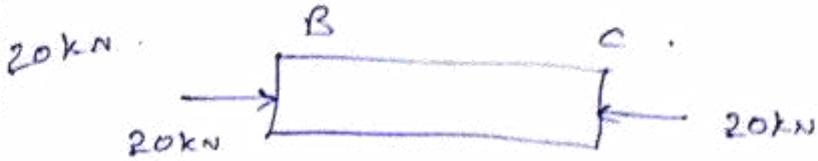
Find the total elongation of the bar, Take $E = 1.05 \times 10^5$

Soln:

The force of 80KN acting at B is split up into three forces of 50KN, 20KN & 10KN.

Then AB of the bar is subjected to a tensile load of 50KN

Part BC is subjected to a compressive load of 20kN.



Part BD is subjected to a compressive load of 10kN.

Part AB

$\therefore$ Increase in length of AB $= \dfrac{P_1}{AE} \times L_1$

$$= \frac{50 \times 1000 \times 600}{1000 \times 1.05 \times 10^5} = 0.2857.$$

Part BC

$$= \frac{P_2}{AE} \times L_2 \Rightarrow \frac{20 \times 1000 \times 1000}{1000 \times 1.05 \times 10^5}$$

$$= 0.1904.$$

Part BD

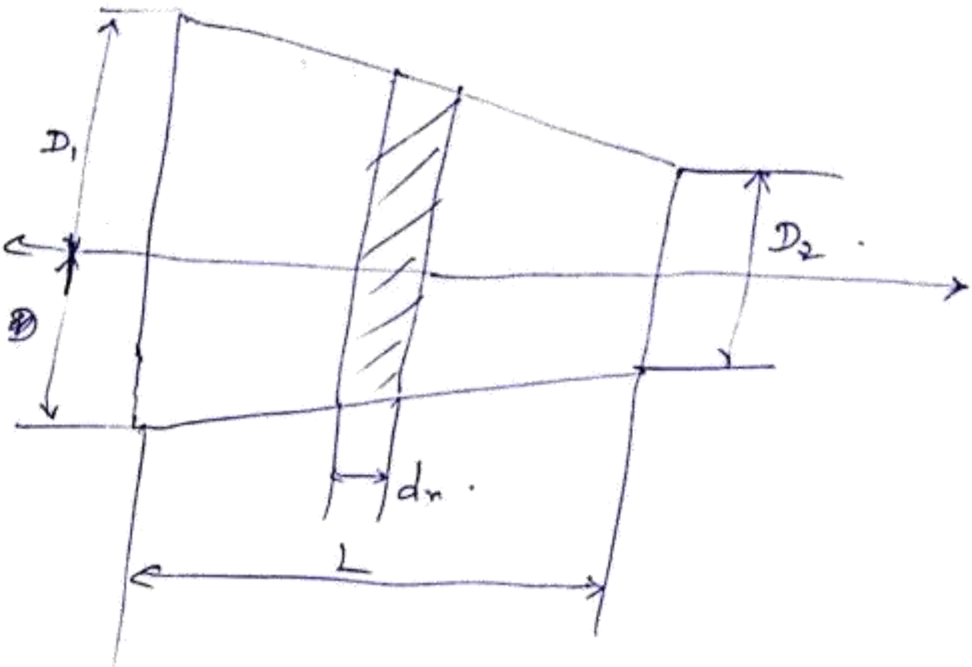
$$= \frac{P_3}{AE} \times L_3 \Rightarrow \frac{10 \times 1000 \times 2200}{1000 \times 1.05 \times 10^5} = 0.2095.$$

Total elongation $= 0.2857 - 0.1904 - 0.2095.$

$$= -0.1142 \text{ mm}.$$

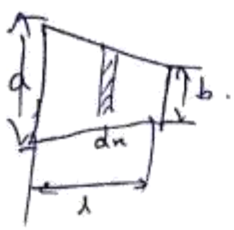
$\therefore$ (-ve) sign shows.

Analysis of Uniformly Tapering Circular rod:-



Total extension, $dL = \frac{4PL}{\pi . E \cdot D^2}$ (or) $\frac{4PL}{\pi E D_1 D_2}$

Analysis of Uniformly Tapering rectangular bar :-



Total extension : $\frac{PL}{E t (a-b)} \log_e \frac{a}{b}$.

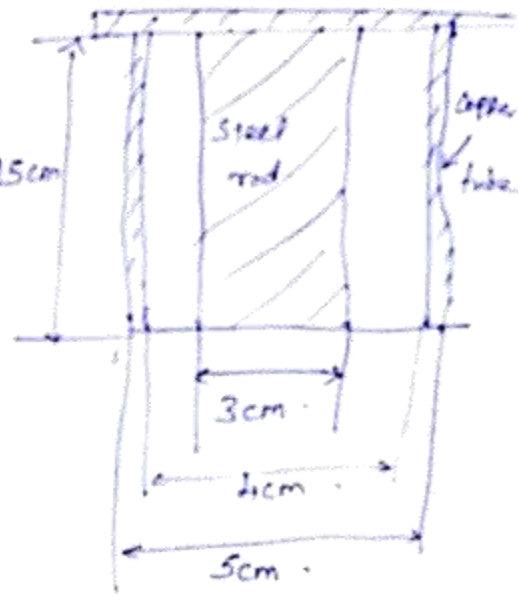
Analysis of bars of Composite Section:-

$$P = \sigma_1 A_1 + \sigma_2 A_2 .$$

$$\frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2} .$$

$\frac{E_1}{E_2}$ ~~ratt~~ is called modular ratio.

A steel rod of 3cm dia is enclosed centrally in a hollow copper tube of external dia 5cm & internal dia 4cm. The composite bar is then subjected to an axial pull of 45000 N. If the length of each bar is equal to 15cm Determine. (i) The stresses in the rod & load carried by each bar. Take E for steel = 2.1×10^5 N/mm² & for copper = 1.1×10^5 N/mm².



Soln:
Dia of steel

Area of steel rod $A_s = \frac{\pi}{4}(30)^2$

$= 706.86 \text{ mm}^2$

Exterior Area of copper tube

$$A_c = \frac{\pi}{4}[50^2 - 40^2] = 706.86 \text{ mm}^2$$

$$\frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c} \Rightarrow \sigma_s = \frac{E_s}{E_c} \times \sigma_c$$

$$= \frac{2.1 \times 10^5}{1.1 \times 10^5} \times \sigma_c = 1.909\sigma_c$$

Stress = $\frac{\text{Load}}{\text{Area}}$ ∴ Load = $\frac{\text{Stress}}{\text{Area}}$

Total load = Load on steel + Load on copper.

$\sigma_s \times A_s + \sigma_c \times A_c = P$.

$1.909\,\sigma_c \times 706.86 + \sigma_c \times 706.86 = 45000$.

$\sigma_c\,(1.909 \times 706.86 + 706.86) = 45000$.

$2056.25\,\sigma_c = 45000$.

$$\sigma_c = \frac{45000}{205625} = 21.88\ N/mm^2$$

Sub. the value of σ_c in eqn.

$\sigma_s = 1.909 \times 21.88$

$= 41.77\ N/mm^2$.

Load carried by each bar.

Load = Stress x Area.

∴ Load carried by steel rod.

$P_s = \sigma_s \times A_s$.

$= 41.77 \times 706.86 = 29525.5N$.

Load carried by copper tube.

$P_c = 45000 - 29525.5$.

$= 15474.5\ N$.

Thermal Stresses:-

Thermal stresses are the stresses induced in a body due to change in temperature.

Thermal strain, $e = \frac{\text{Extension Prevented}}{\text{Original length}}$

$$= \frac{dL}{L} \Rightarrow \frac{\alpha . T . L}{L} \Rightarrow \alpha . T .$$

$\alpha \rightarrow$ Co-efficient of linear expansion

$dL \rightarrow$ Extension of rod due to rise of temp.

Thermal stress, σ = Thermal strain × E

$$= \alpha . T . E .$$

Thermal stress is also known as temperature stress.

Thermal stresses in Composite Bars:-

$$\alpha_s T + \frac{\sigma_s}{E_s} = \alpha_b \times T - \frac{\sigma_b}{E_b} .$$

Volumetric strain -

$$e_v = \frac{\delta v}{v} \Rightarrow$$ $\delta v \rightarrow$ change in volume, $v \rightarrow$ original volume

1) Volumetric strain of a rectangular bar which is subjected to an axial load P in the direction of its length

$$e_v = \frac{\delta L}{L}(1 - 2\mu).$$

2) Volumetric strain of a rectangular bar subjected to three forces which are mutually perpendicular

$$\frac{dv}{v} = \frac{1}{E}(\sigma_x + \sigma_y + \sigma_z)(1 - 2\mu).$$

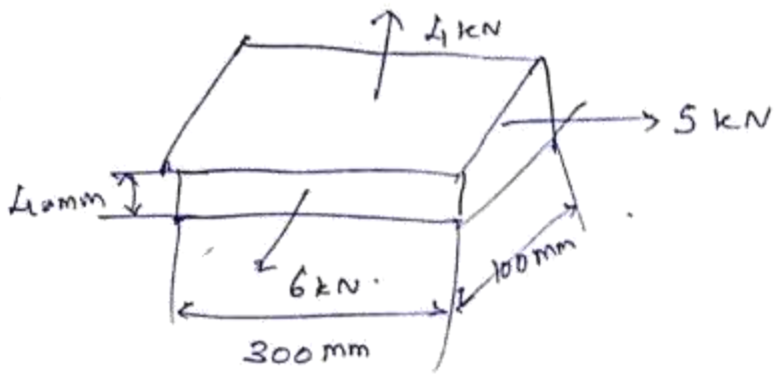
3) Volumetric strain on a cylindrical rod.-

$$e_v = \frac{\delta L}{L} - \frac{2\delta d}{d}$$

where $\frac{\delta L}{L}$ is the strain of length &

$\frac{\delta d}{d}$ is the strain of diameter.

A metallic bar 300 mm × 100 mm × 40 mm is subjected to a force of 5 kN (tensile), 6 kN (tensile) & 4 kN (tensile) along x, y & z directions. Determine the change in volume of the blocks. Take $E = 2 \times 10^5\ N/mm^2$, $\mu = 0.25$.



Soln:-

Dimensions of the bar $x = 300mm$, $y = 100mm$ & $z = 40mm$

$V = x \times y \times z = 12,00,000 mm^3$.

Load in x-direction = 5 kN = 5000 N.

y-direction = 6 kN = 6000 N.

z-direction = 4 kN = 4000 N.

Value of $E = 2 \times 10^5\ N/mm^2$.

$\mu = 0.25$.

$\therefore \sigma_x = \frac{\text{load in X-direction}}{y \times z}$ $\quad \sigma_y = \frac{\text{load in Y-dirn}}{x \times z}$

$= 1.25\ N/mm^2$. $\quad = 0.5\ N/mm^2$

$\sigma_z = \frac{\text{load in z direction}}{x \times y} = 0.133\ N/mm^2$.

$$\therefore \frac{dv}{v} = \frac{1}{E}(\sigma_x + \sigma_y + \sigma_z)(1-2\mu)$$

$$= \frac{1}{2\times10^5}(1.25 + 0.5 + 0.113)(1-2\times0.2$$

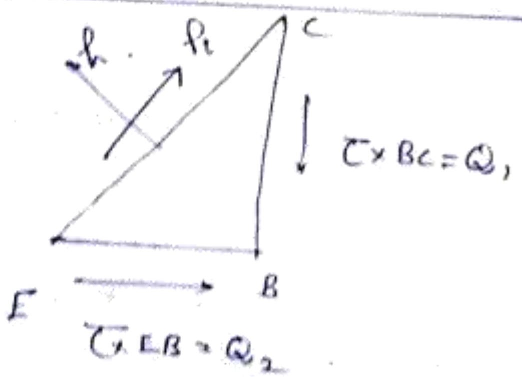
$$dv = \frac{1.883}{4\times10^5} \times 12,00,000.$$

$$= 5.649\,mm^3.$$

Bulk Modulus :-

$$k = \frac{\text{Direct Stress}}{\text{Volumetric Strain}} = \frac{\sigma}{\left(\frac{dv}{v}\right)}.$$

Stresses on Inclined Sections when the element is subjected to simple shear stresses :-



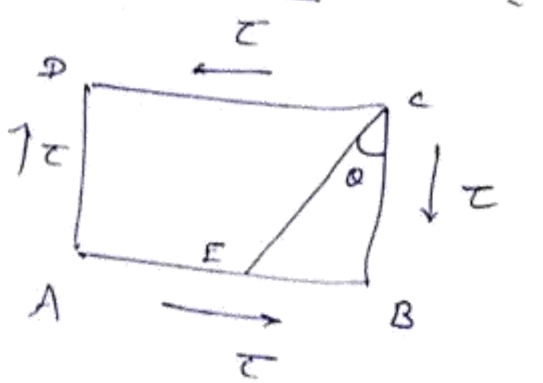


Fig. shows a rectangular block ABCD which is in a state of simple shear & hence subjected to a set of shear stressess of intensity τ on AB, CD & AD & CB.

Let the thickness of the block normal to the plane of the paper is unity.

It's required to find normal & tangential stresses across an inclined plane CE, which is having inclination θ with the face CB.

Consider the equillibrium of the triangular piece CEB of thickness unity. The forces acting on triangular piece CEB are shown in fig.

(i) Shear force on face CB

Q_1 = Shear stress × area of face CB.

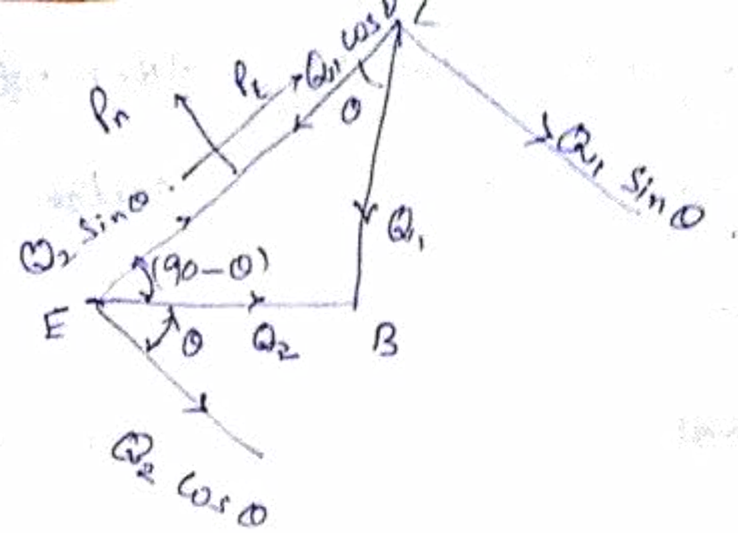
$= \tau \times BC \times 1$

$= \tau \times BC$ acting along CB.

(ii) Shear force of face EB.

Q_2 = Shear stress × area of face EB.

$= \tau \times EB \times 1 = \tau \times EB$ acting along EB.



$$P_n - Q_1 \sin\theta - Q_2 \cos\theta = 0.$$

$$P_n = Q_1 \sin\theta + Q_2 \cos\theta$$

$$= \tau \times BC \times \sin\theta + \tau \times EB \times \cos\theta.$$

$$P_t - Q_1 \cos\theta + Q_2 \sin\theta = 0.$$

$$P_t = Q_1 \cos\theta - Q_2 \sin\theta.$$

$$= \tau \times BC \cos\theta - \tau \times EB \times \sin\theta$$

$$\therefore \quad \sigma_n = \tau \sin 2\theta.$$

$$\sigma_t = \tau \cos 2\theta.$$

Principal Planes & Principal stresses:-

The planes, which have no shear stress, are known as principal planes.

Here the principal planes are the planes of zero shear stress.

These planes carry only normal stresses.

The normal stresses acting on a principal plane are known as principal stresses.

Obliquity :-

The angle made by the resultant stress with normal to the oblique plane is known as obliquity. Denoted by ϕ

$$\tan\phi = \tau / \sigma_n$$

Maximum Shear stress :-

The shear stress will be maximum when $\sin 2\theta = 1$, or $2\theta = 90^\circ$ or 270°

$$\therefore \theta = 45^\circ \text{ or } 135^\circ$$

And maximum shear stress $(\tau)_{max} = \dfrac{\sigma_1 - \sigma_2}{2}$

Principal Planes :- Member subjected to Direct stresses in two mutually

$$\sigma_n = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2}\cos 2\theta$$

→ **Direction**

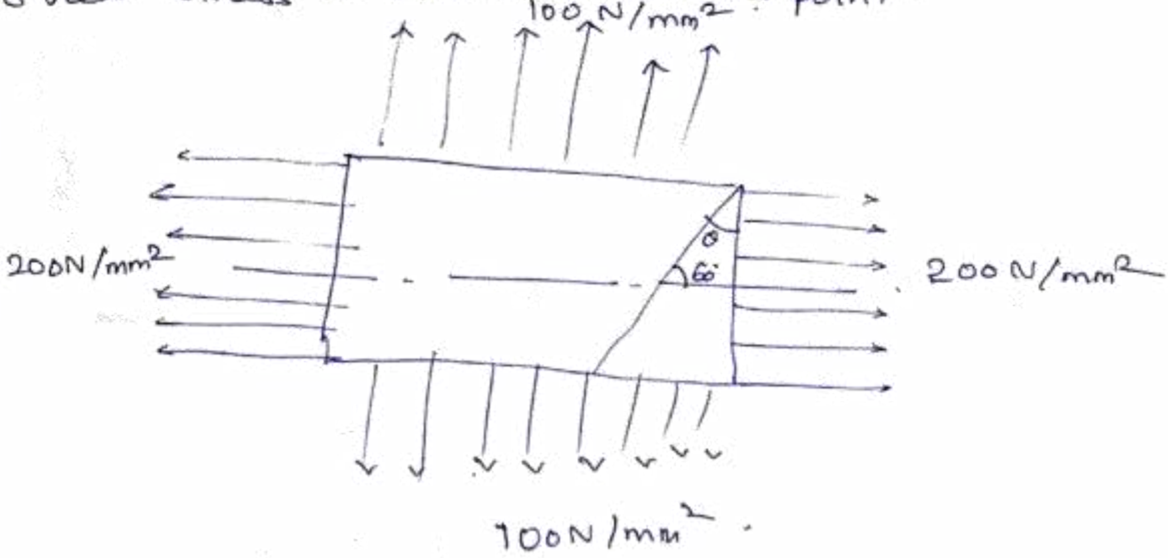
$$\sigma_t = \frac{(\sigma_1 - \sigma_2)}{2} . \sin 2\theta .$$

The resultant stresses.

$$\sigma_R = \sqrt{\sigma_n^2 + \sigma_t^2}$$

———— x ————

The stresses at a point in a bar are 200 N/mm² tensile & 100 N/mm² compressive. Determine the resultant stress in magnitude & direction on a plane inclined at 60° to the axis of major stress. Also determine the maximum intensity of shear stress in the material at point.



Major principal stress, $\sigma_1 = 200 N/mm^2$.

Minor Principal stress, $\sigma_2 = -100 N/mm^2$.

Angle of the plane, which it makes with the major principal Stress = 60°.

Angle $\theta = 90° - 60° = 30°$.

Resultant stress in magnitude & direction:-

$$\sigma_n = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2} \cos 2\theta .$$

$$= 125 \text{ N/mm}^2 .$$

$$\sigma_t = \frac{\sigma_1 - \sigma_2}{2} \sin 2\theta = 129.9 \text{ N/mm}^2 .$$

resultant $\sigma_R = \sqrt{\sigma_n^2 + \sigma_t^2} = \sqrt{125^2 + 129.9^2}$

$$= 180.27 \text{ N/mm}^2 .$$

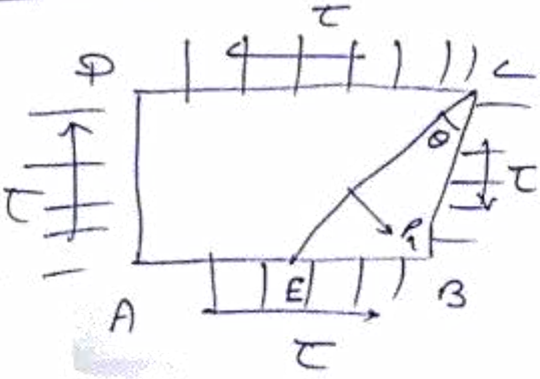
$$\tan\phi = \frac{\sigma_t}{\sigma_n} = 1.04 .$$

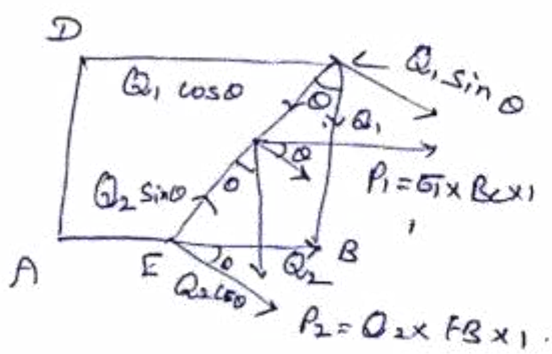
$$\phi = \tan^{-1}(1.04) = 46.6' .$$

Max. shear stress is given by

$$(\sigma_t)_{max} = \frac{\sigma_1 - \sigma_2}{2} = 150 \text{ N/mm}^2 .$$

A member subjected to Direct Stresses in two mutually ⊥r directions accompanied by simple Shear stress:-





$$\sigma_n = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2}\cos 2\theta + \tau \sin 2\theta$$

$$\sigma_t = \frac{\sigma_1 - \sigma_2}{2}\sin 2\theta - \tau \cos 2\theta$$

for Principal planes.

$$\tan 2\theta = \frac{2\tau}{\sigma_1 - \sigma_2}$$

Major Principal stress

$$= \frac{\sigma_1 + \sigma_2}{2} + \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$$

Minor Principal stress

$$= \frac{\sigma_1 + \sigma_2}{2} - \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$$

Maximum shear stress

$$\tan 2\theta = \frac{\sigma_2 - \sigma_1}{2\tau}$$

Max shear stress $= \frac{1}{2}\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}$

Mohr's Circle :-

It's a graphical method of finding normal, tangential & resultant stresses on an oblique plane.

Mohr's Circle can be drawn for the following cases

(i) A body subjected to two mutually ⊥r principal tensile stresses of unequal intensities

(ii) A body subjected to two mutually ⊥r principal stresses which are unequal & unlike (i.e one is tensile & other is compressive)

(iii) A body subjected to two mutually ⊥r principal tensile stresses accompanied by a simple shear stress.

———— × ————

Unit - 2

Transverse loading on Beams & Stresses in B

Shear force & Bending Moment Diagrams:-

* A shear force diagram is one which sh the variation of the shear force along the le of the beam.

* Bending moment diagram is one which s the variation of the bending moment along the length of the beam.

Types of Beams:-

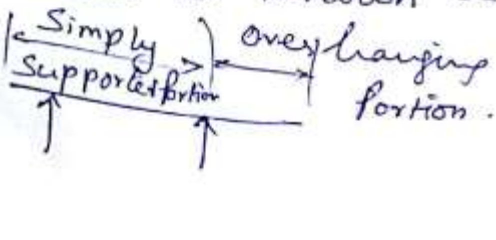
The following are the important types of b

1. Cantilever beam. 

2. Simply supported beam 

3. Overhanging beam

If the end portion of a beam is extended beyond support, such beam is known as over hanging beam.



4. Fixed beams:-

A beam whose both ends are fixed or built in walls, is known as fixed beam.

5. Continuous beams:-

A beam which is provided more than two supports.

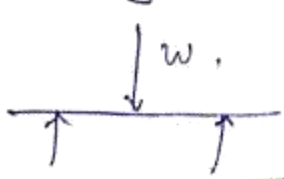
Types of loads:-

A beam is normally horizontal & the loads acting on beams are generally vertical.
The following are the important types of load acting on a beam.

1. Concentrated or Point load.
2. Uniformly distributed load and
3. Uniformly varying load.

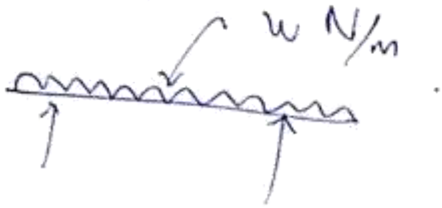
Concentrated or Point load:-

A conc. load is one which is considered to act at a point,



Uniformly distributed load:-

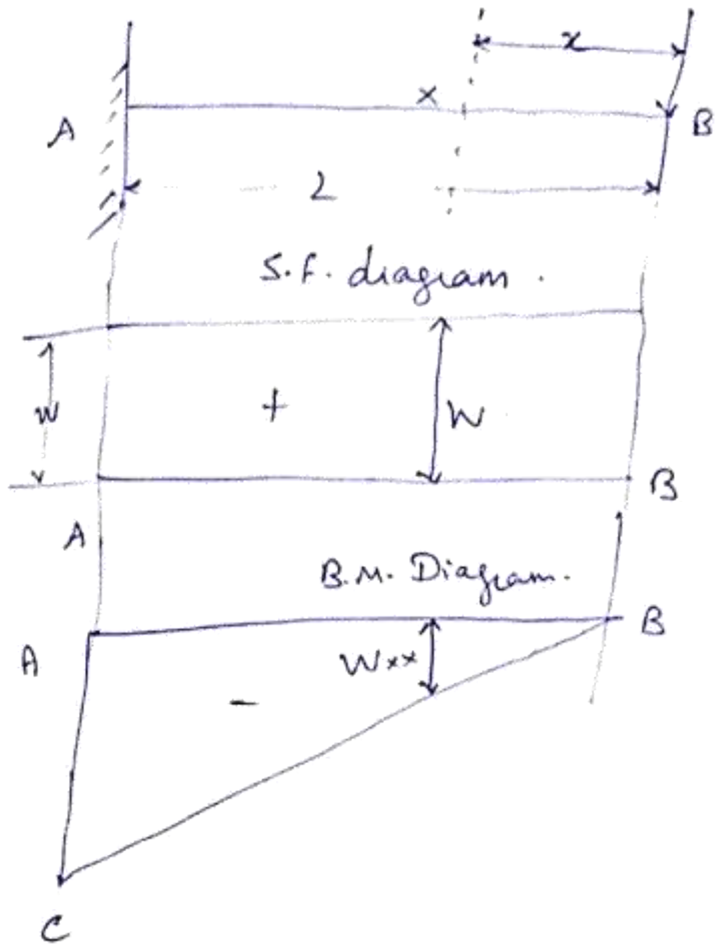
* A uniformly distributed load is one which is spread over a beam in such a manner that rate of loading w is uniform along the length.
* The rate of loading is expressed as w N/m run.
* It's represented as UDL.
* For solving the numerical problems, the total UDL is converted into a pt. load acting at the centre of uniformly distributed load.



Uniformly Varying load:-

* A uniformly varying load is one which is spread over a beam in such a manner that rate of loading varies from pt. to pt. along the beam, in which load is zero at one end & increases uniformly to the other end. Such load is known as triangular load.

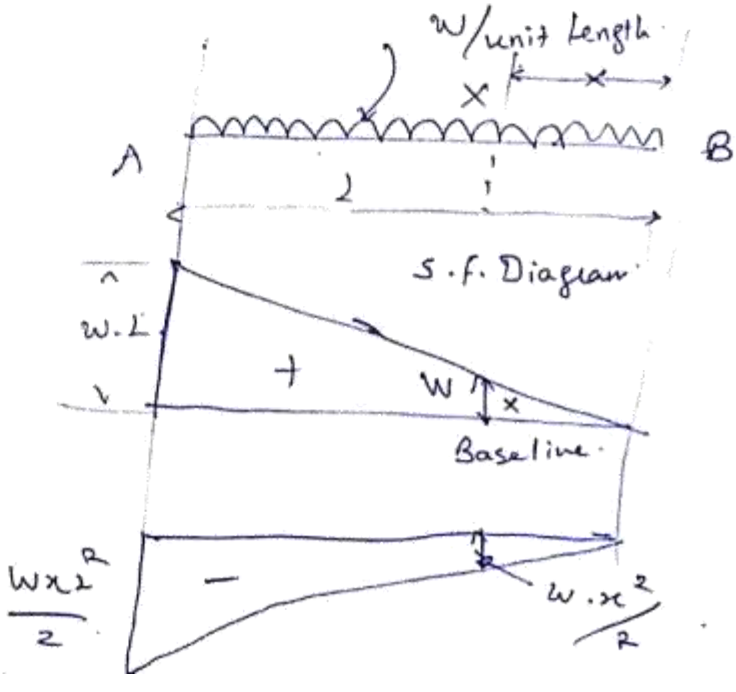
Shear force & Bending Moment diagrams for a cantilever with a point load at the free end :-



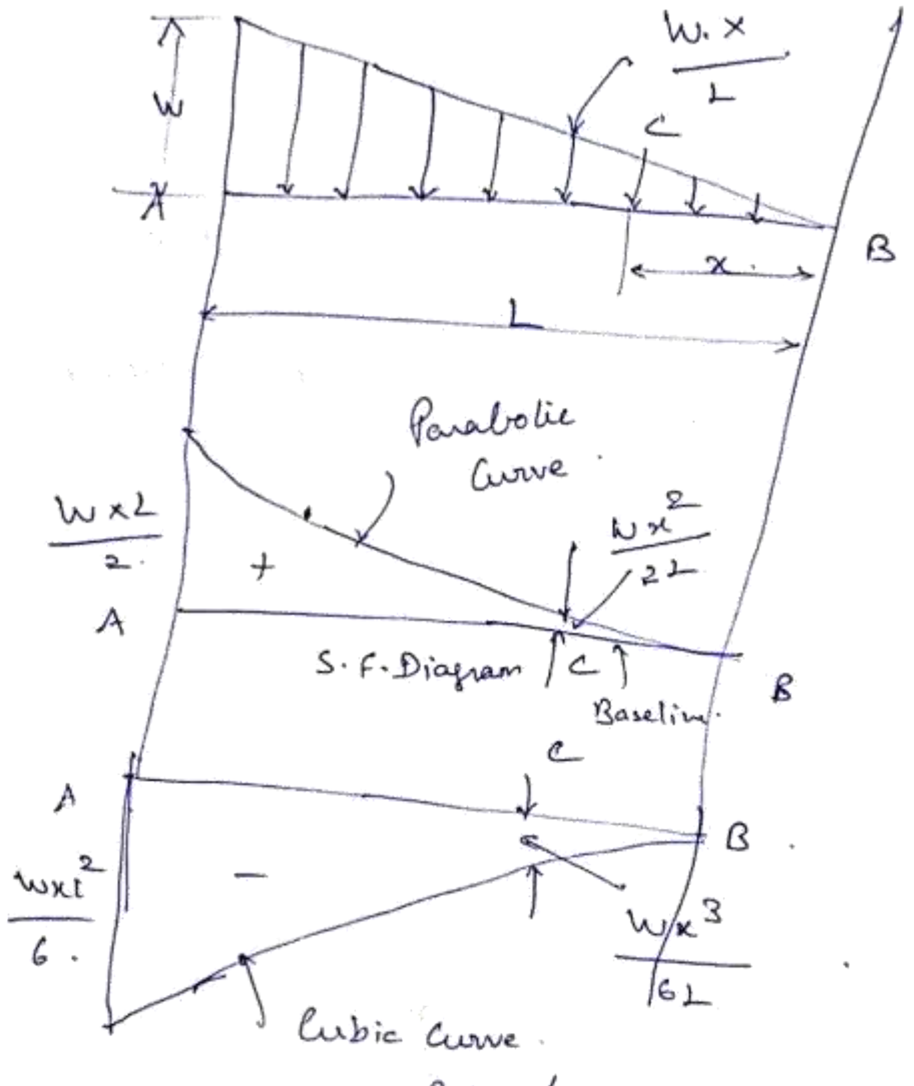
$F_x = +W.$

$M_x = -W x.$

Shear force & Bending Moment Diagrams for a Cantilever with a Uniformly distributed load:-



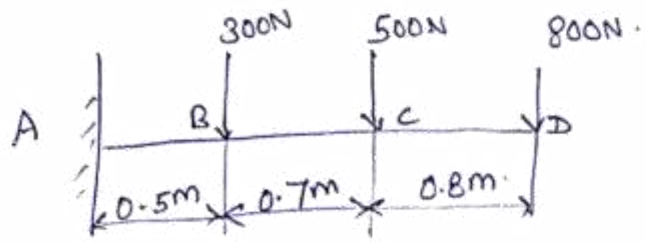
Shear force & Bending Moment diagrams for a Cantilever carrying a gradually varying load.



B.M. diagram.

x.

Pb.① A cantilever beam of length 2m carries the point loads as shown in figure. Draw the shear force & B.M. diagrams for the cantilever beam.



Shear force calculation:-

S.F at D. $F_D = 800$ N.

S.F at C, $F_C = 800 + 500 = 1300$N.

S.F. at B, $F_B = 800 + 500 + 300 = 1600$N.

S.F. at A, $F_A = 1600$N.

Bending Moment Diagram:-

The bending moment at D is zero.

(i) The bending moment at any section b/w C & D at a distance x & D is given by

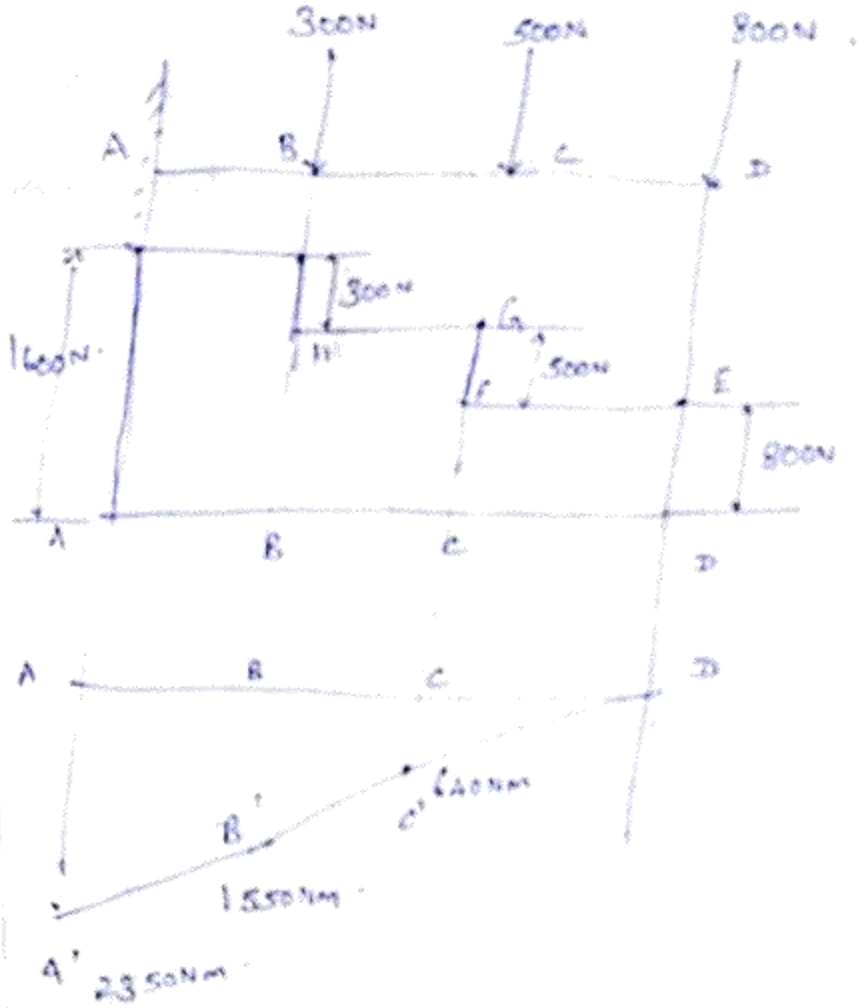
$$M_x = -800 \times x$$

B.M at C, $M_C = -800 \times 0.8 = -640$Nm

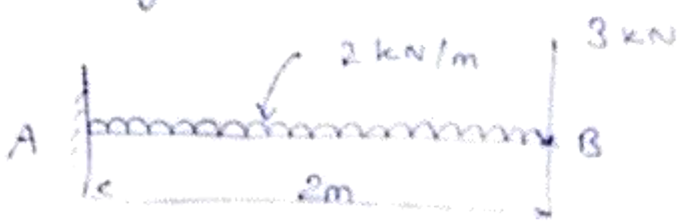
B.M. at B. $M_B = -800 \times 1.5 - 500(0.7) = -1550$Nm.

B.M at A. $M_A = -800 \times 2 - 500(1.2) - 300(0.5)$

$= -2350$Nm



Pb 2 A cantilever of length 2m carries a UDL of 2kN/m length over the whole length & a pt. load of 3kN & the free end. Draw the S.F & BM diagrams for the cantilever.



Shear force at B = 3kN.

$F_x = 3.0 + w \cdot x$

$= 3.0 + 2 \times x$

The above eqn. shows that shear force follows a straight line law.

At B, $x = 0$, hence $F_B = 3.0 kN$

At A, $x = 2$ hence $F_A = 3 + 2 \times 2 = 7 kN$

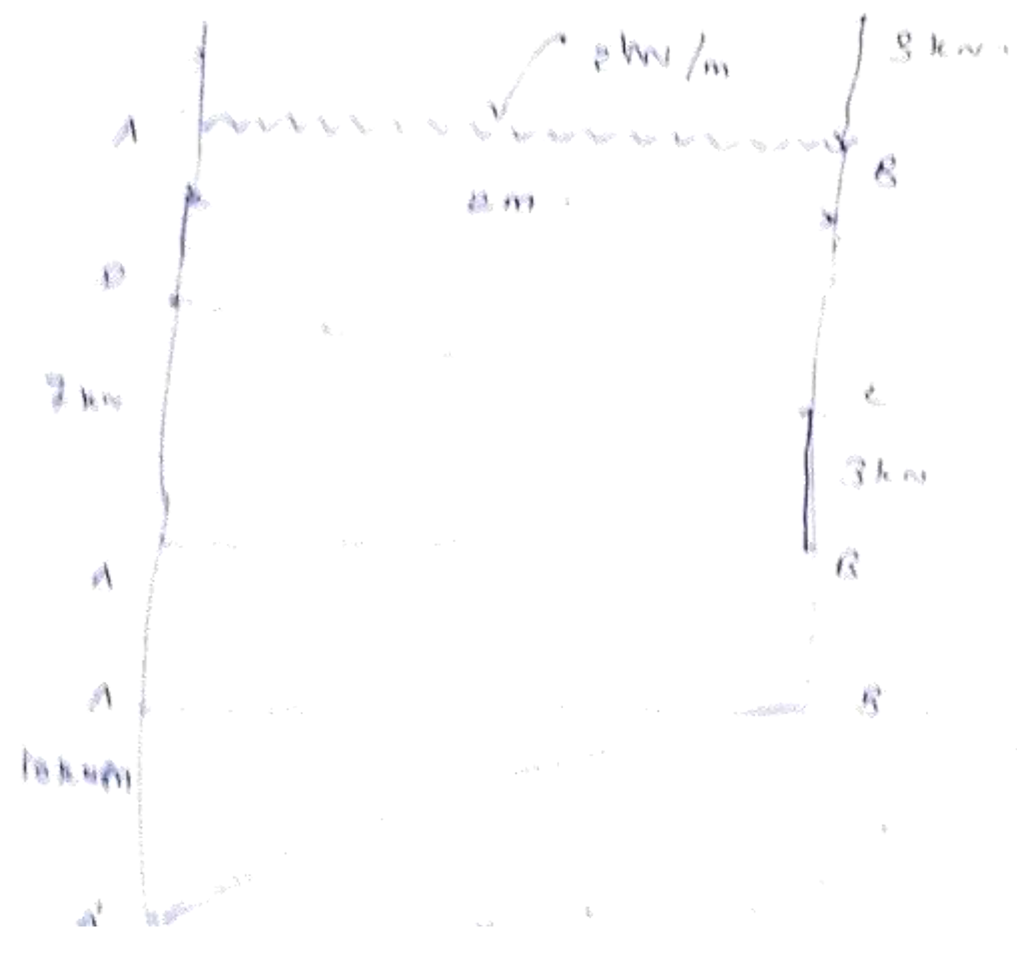
The Bending moment at any section at a distance x from the free end B is given by,

$$M_x = -\left(3x + 2x \cdot \frac{x}{2}\right)$$

$$= -\left(3x + \frac{2x^2}{2}\right) = -(3x + x^2)$$

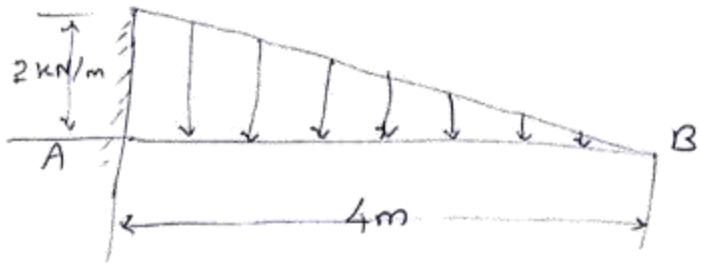
At B, $x = 0$, hence, $M_B = -(3 \times 0 + 0^2) = 0$

At A, $x = 2m$, hence, $M_A = -(3 \times 2 + 2^2) = -10 kNm$



Pb. 3

A cantilever beam of length 4m carries UVL, zero at the free end to 2kN/m at the fixed end. Draw the S.F & B.M. diagrams.

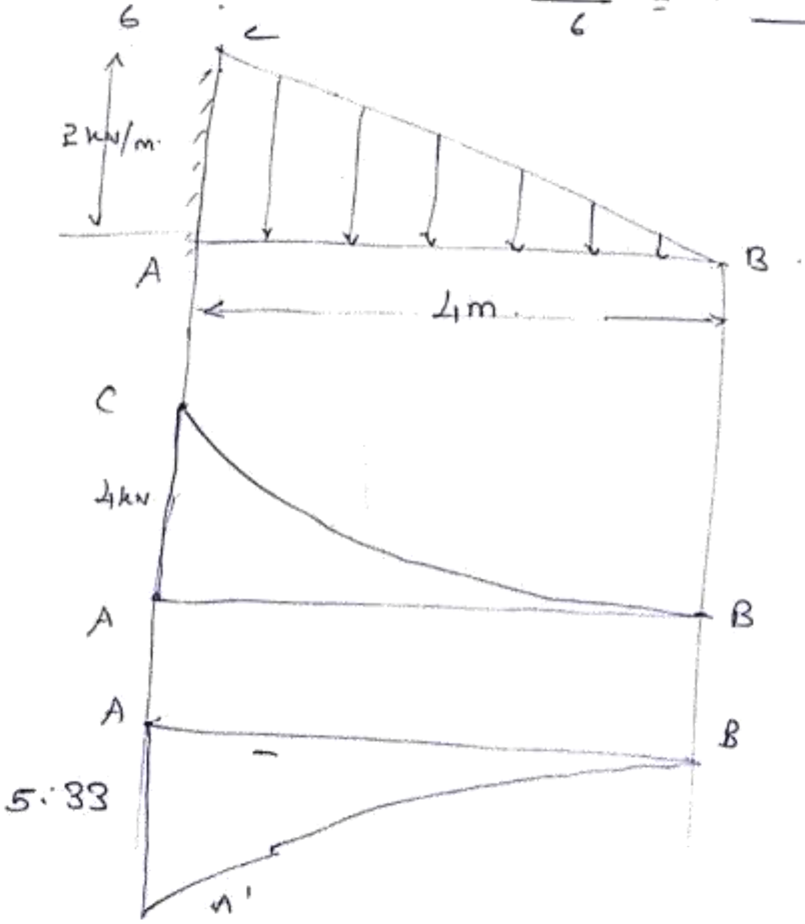


Shear force is zero at B. The shear force at C will be equal to the area of load diagram.

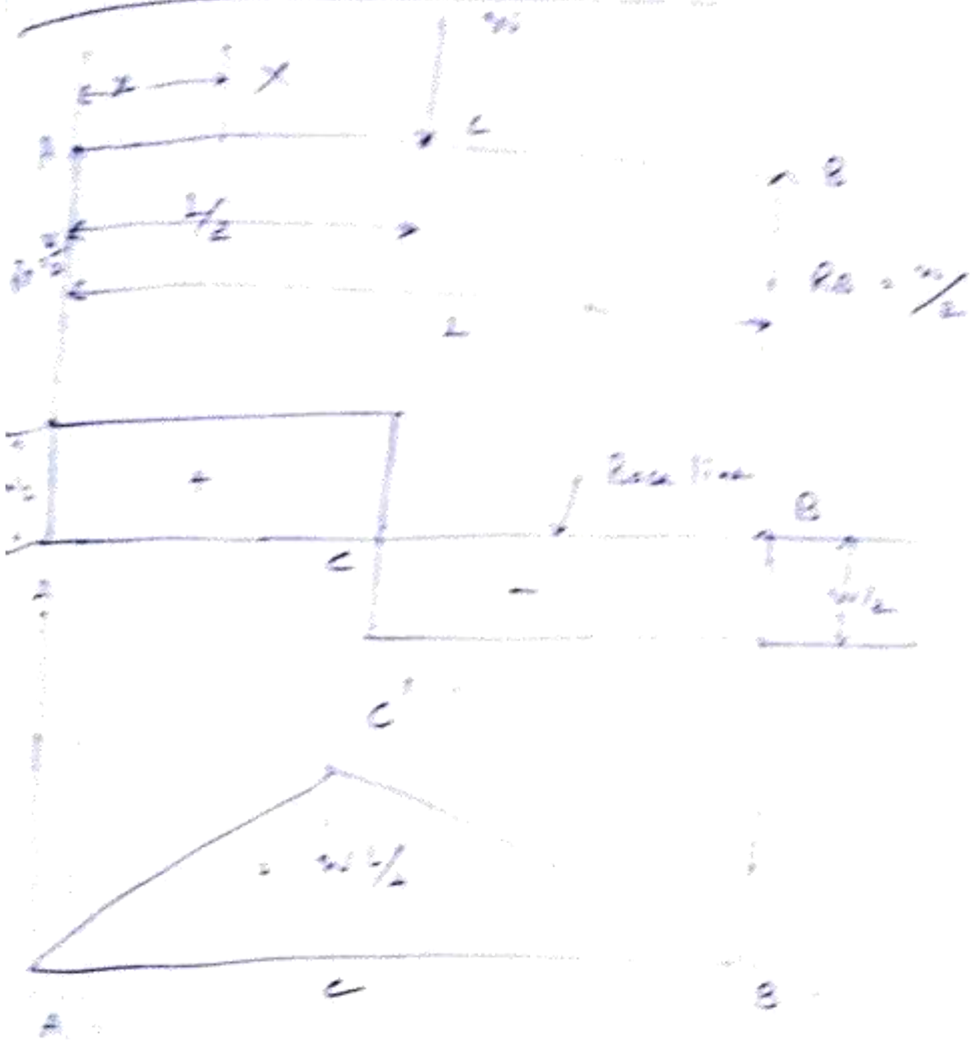
$$\text{Shear force at } C = \frac{4\times2}{2} = 4\text{ kN}.$$

The B.M. at B is zero, The B.M. at A is equal to

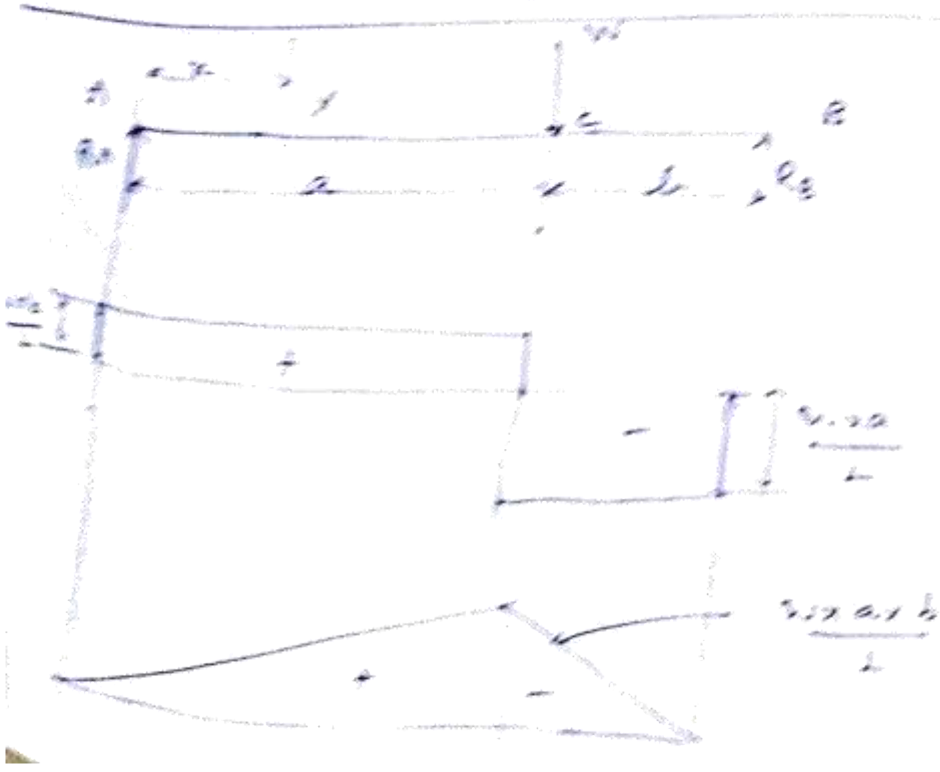
$-\frac{wl^2}{6}$: $M_A = -\frac{wL^2}{6} = -\frac{2\times4^2}{6} = -5.33$ kNm



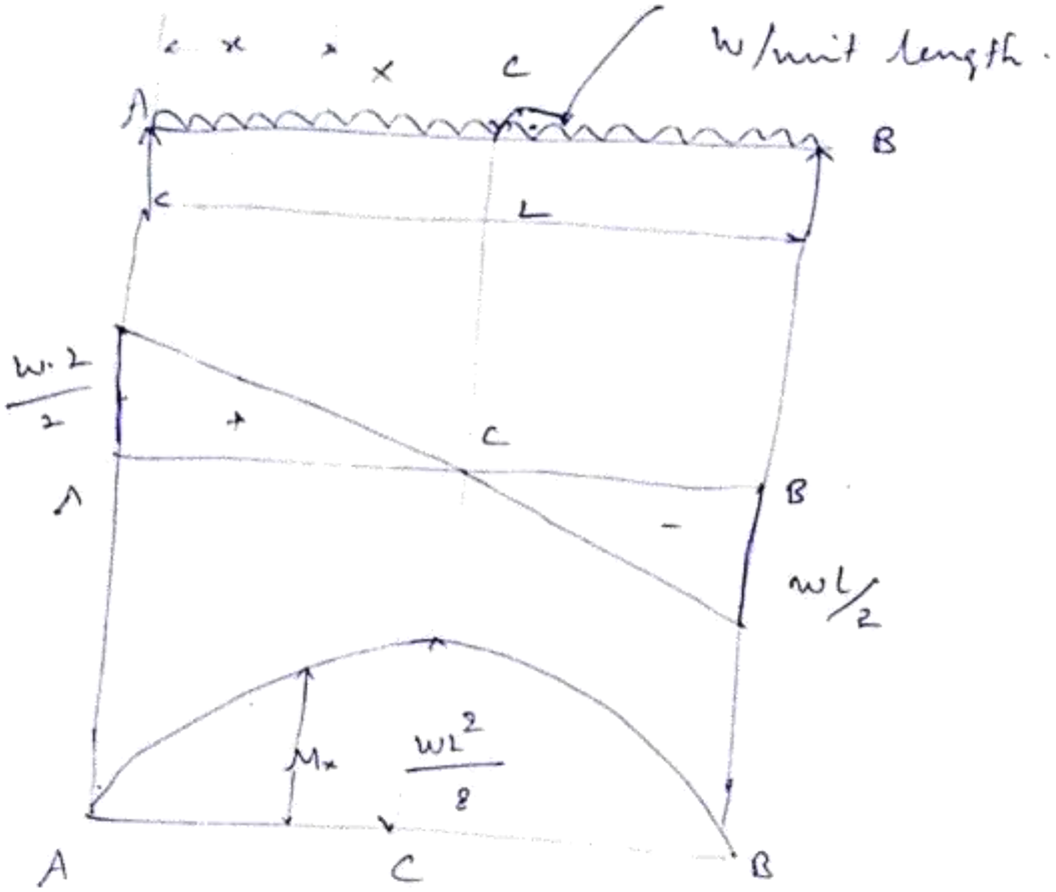
Shear force & Bending Moment diagrams for a SSB with a Point load @ midpoint



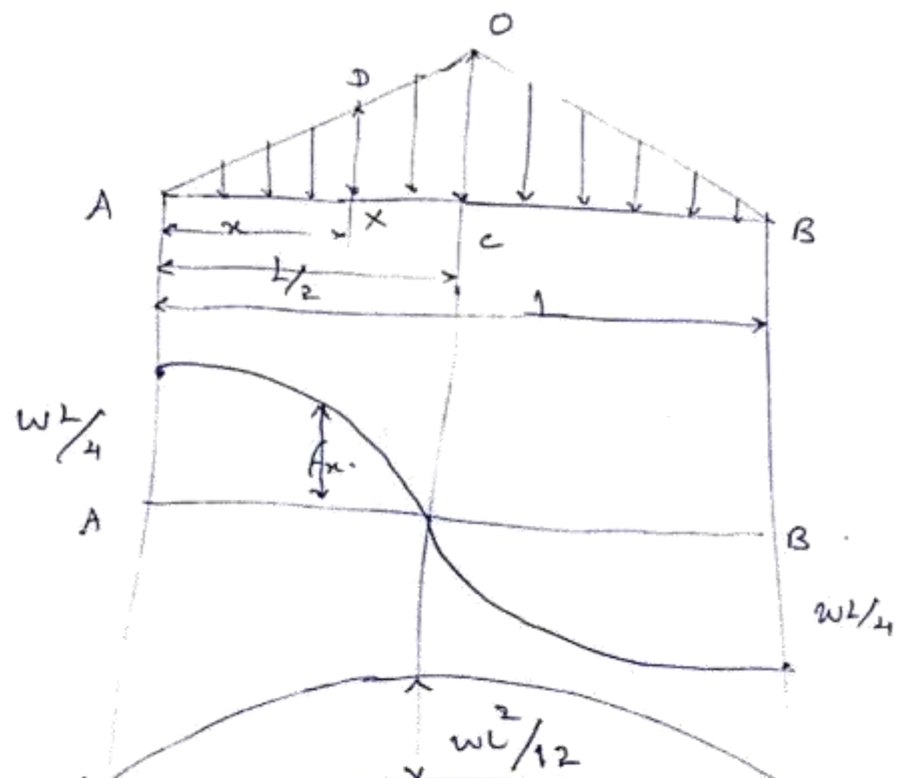
Shear force & B.M. Diagrams for a SSB with Eccentric Pt. load



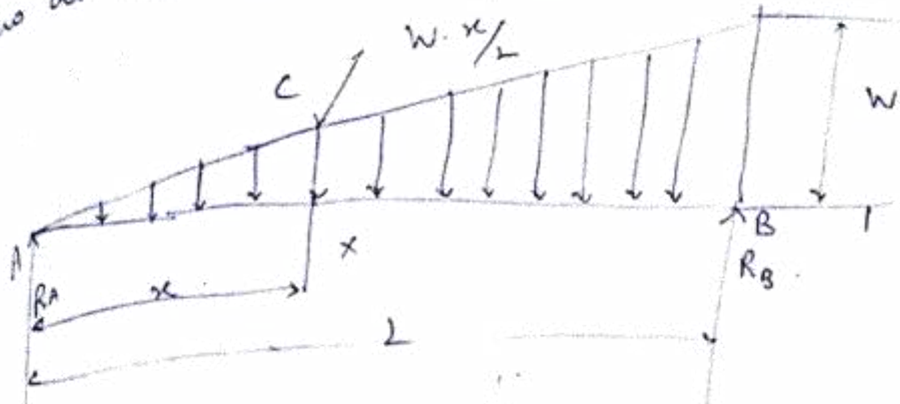
S.F & B.M. Diagrams for SSB Carrying UDL :-

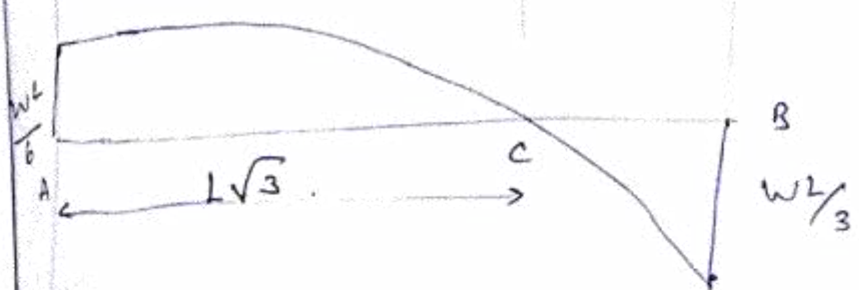


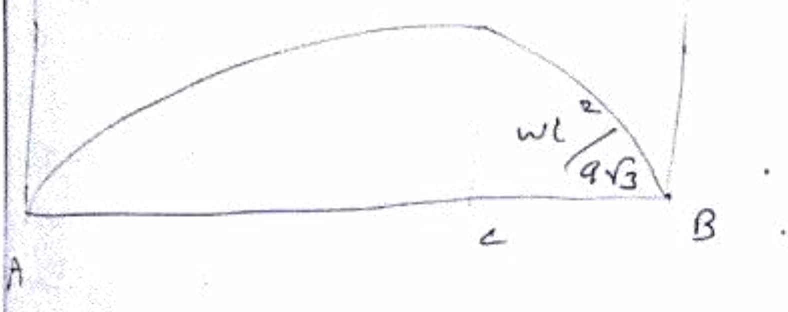
S.F. & B.M. Diagrams for SSB Carrying UVL from zero at each end to w/unit length at the centre



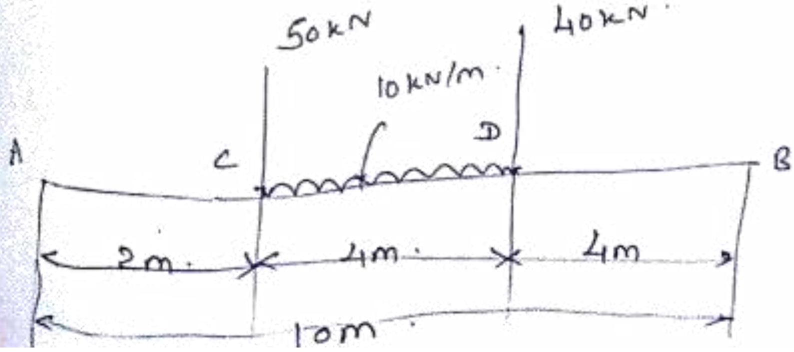
S.F & B.M Diagrams for a SSB carrying UVL from zero at one end to w/unit length at the other end.







Pb.4 A simply supported beam of length 10m, carries the UDL & two pt. loads as shown in figure. Draw the S.F & B.M diagram for the beam. Also calculate the bending moment.



First calculate the reactions R_A & R_B.

Taking moments of all forces about A, we get

$R_B \times 10 = 50 \times 2 + 10 \times 4 \times \left(2 + \frac{4}{2}\right) + 40(2+4)$.

$= 500$. $\boxed{R_B = 50\ kN}$

R_A = Total load on beam $- R_B$

$= (50 + 10 \times 4 + 40) - 50 \Rightarrow 80\ kN$

S.F. Diagram

S.F. at A, $F_A = R_A = 80\ kN$

S.F. just on R.H.S of $C = R_A - 50 = 30\ kN$

S.F. Just on L.H.S of $D = R_A - 50 - 10 \times 4 = -10\ kN$

S.F. just on R.H.S. of $D = R_A - 50 - 10 \times 4 - 40 = -50$

S.F. at $B = -50\ kN$

Now shear force at $E = R_A - 50 - 10(x-2)$

$= 50 - 10x$

But shear force at $E = 0$

$50 - 10x = 0$; $x = 5m$

B.M Diagram

B.M at C, $M_C = R_A \times 2 = 160$ kNm.

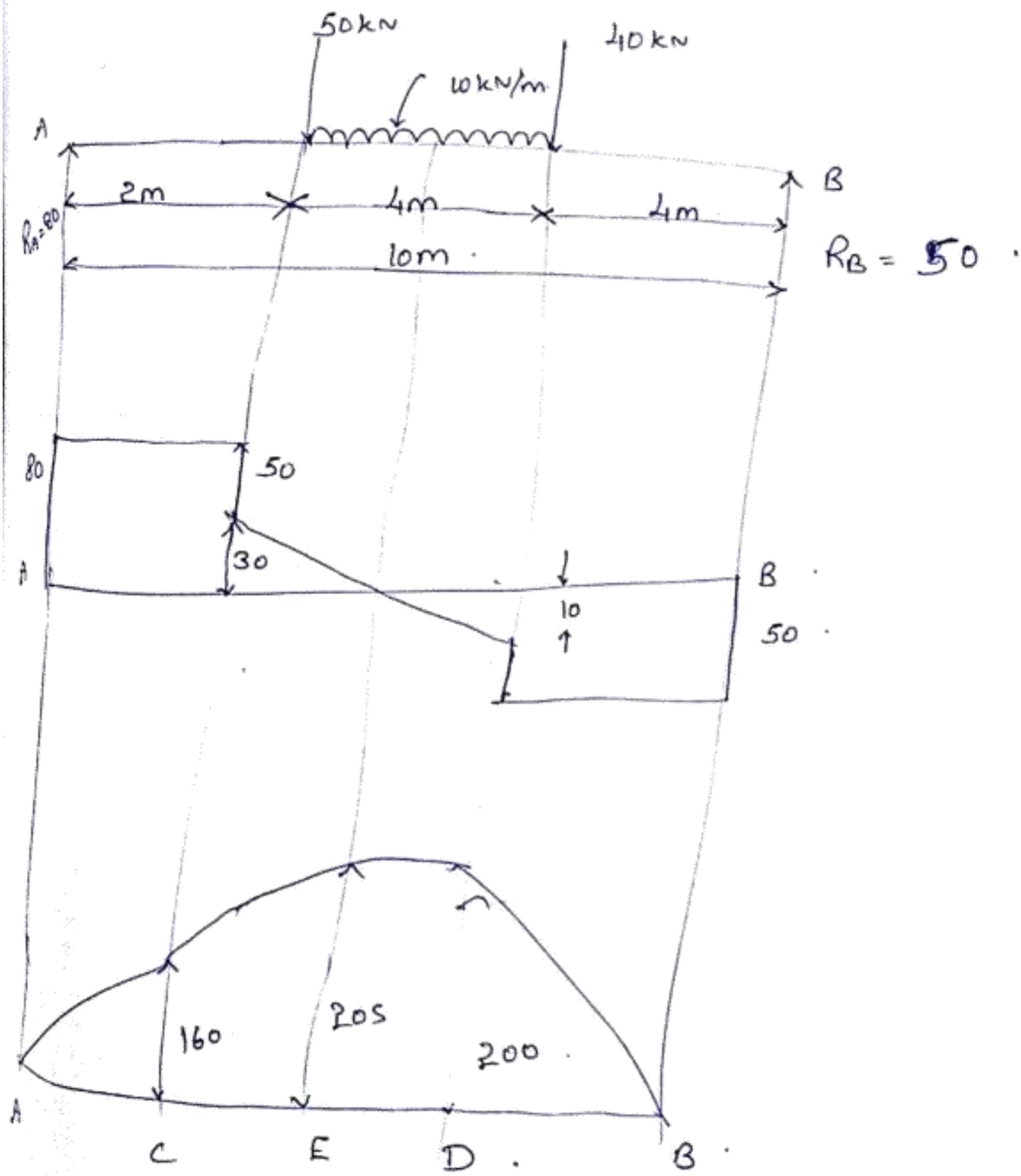
B.M. at D, $M_D = R_A \times 6 - 50 \times 4 - 10 \times 4 \times 4/2$

$= 200$ kNm.

At E, $x = 5$m & hence BM at E.

$M_E = R_A \times 5 - 50(5-2) - 10(5-2) \times \left(\frac{5-2}{2}\right)$

$= 205$ kN/m.



S.F & B.M Diagrams for Over hanged beams :-

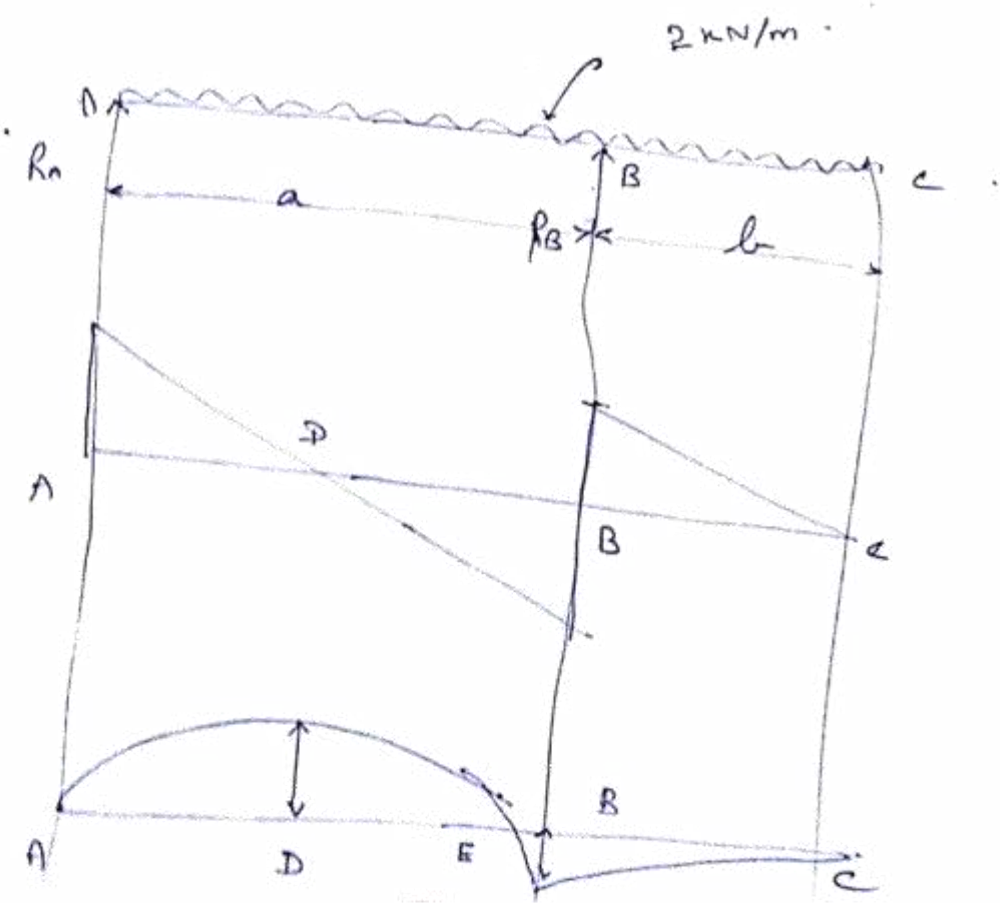
* If the end portion of a beam is extended beyond the support, such beam is known as Overhanging beam.

* B.M is +ve b/w the two supports whereas B.M is −ve for over hanging portion.

* Hence at some pt., the B.M is zero after changing its sign from +ve to −ve.

Point of Contraflexure :-

It's the pt. where B.M is zero after changing its sign from +ve to −ve or vice versa.



Bending ... Beams :-

when some external load acts on a beam, the shear force & bending moments are setup at all sections of the beam.

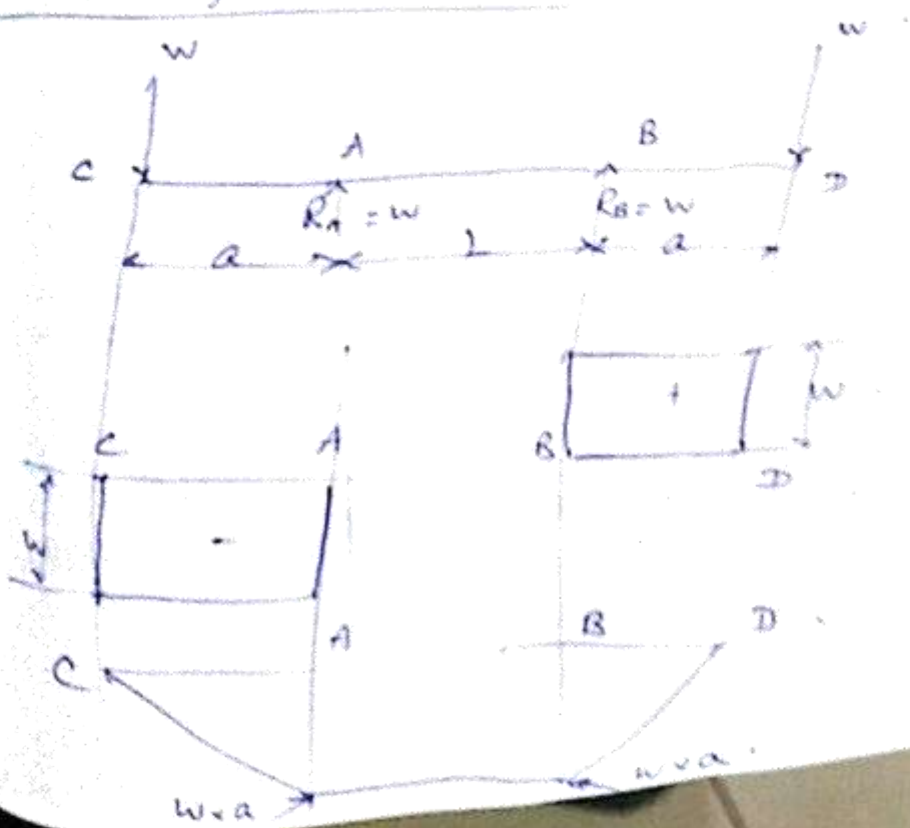
Due to the shear force & bending moment, the beam undergoes certain deformation.

The material of the beam will offer resistance or stresses against these deformations.

These stresses with certain assumptions can be calculated.

The stresses introduced by bending moments are known as bending stresses.

Pure bending (or) Simple bending :-



Theory of simple bending with assumptions ma

1. The material of the beam is homogeneo isotropic.

2. The value of Young's modulus of elasticity the same in tension & compression.

3. The transverse sections which were plane before bending remain plane after bending also

4. The beam is initially straight & all longi filaments bend into circular area with a common centre of curvature.

5. The radius of curvature is large compar with the dimensions of the cross-section

6. Each layer of the beam is free to expa or contract, independently of the layer, above or below

Section Modulus :-

Section modulus is defined as the ratio of moment of inertia of a section about the neutral axis to the distance of the outermost layer from the neutral axis. It's denoted by symbol Z.

$$Z = \frac{I}{y_{max}}$$

I = M.O.I about neutral axis

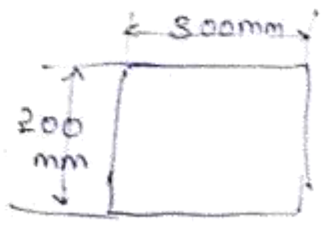
y_{max} = Distance of the outermost layer from the neutral axis

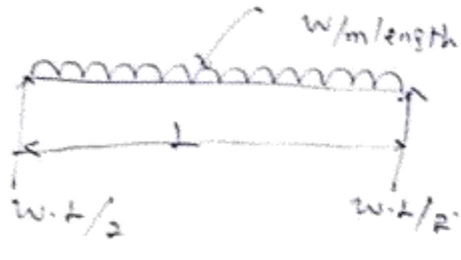
$$\frac{M}{I} = \frac{\sigma_{max}}{y_{max}}$$

$$M = \sigma_{max} \cdot \frac{I}{y_{max}}$$

$$\boxed{M = \sigma_{max} \cdot Z}$$

A rectangular beam 200mm deep & 300mm wide is SSB over a span of 8m. What UDL/m the beam may carry, if the bending stress is not to exceed 120 N/mm²





Given:-

Depth of beam d = 200mm

Width of beam b = 300mm

Length of beam L = 8m

Max. bending stress, $\sigma_{max} = 120\ N/mm^2$

$$Z = \frac{bd^2}{6} = \frac{300 \times 200^2}{6} = 2{,}000{,}000\, mm^3$$

Max. B.M. for a SSB carrying UDL as shown in fig

$$M = \frac{wl^2}{8} = w \times 8^2/8 \Rightarrow 8w.\, Nm$$

$$= 8000 \cdot w \cdot Nmm.$$

$$M = \sigma_{max} \cdot Z$$

$$8000w = 120 \times 20{,}00{,}000$$

$$\boxed{w = 30 kN/m.}$$

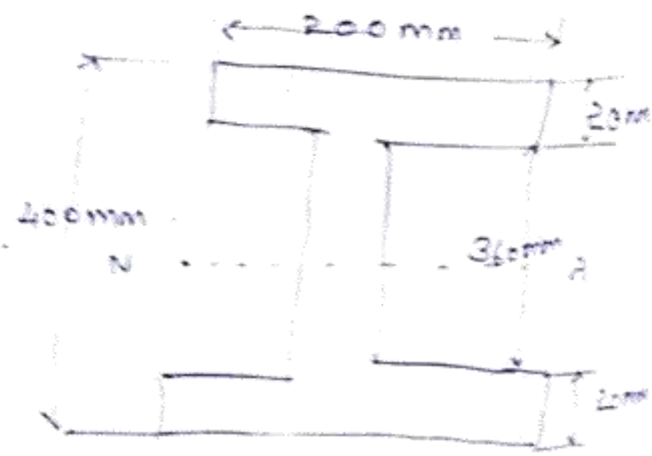
A rolled steel joist of I section has the dimensions as shown in fig. This beam of I section carries a UDL of 40kN/m run on a span of 10m. Calculate the max. stress produced due to bending.

Given:

UDL. $w = 40kN/m$

$= 40{,}000 N/m.$

Span $l = 10m.$



Moment of Inertia about the neutral axis

$$= \frac{200 \times 400^3}{12} - \frac{(200-10) \times 360^3}{12}$$

$$= 32794 6666\, mm^4.$$

B.M is given by.

$$M = \frac{wl^2}{8} = \frac{40.000 \times 10^2}{8} \Rightarrow 5 \times 10^8 \ N/mm$$

Now using the relation,

$$\frac{M}{I} = \frac{\sigma}{y}.$$

$$\sigma = \frac{M}{I} \times y.$$

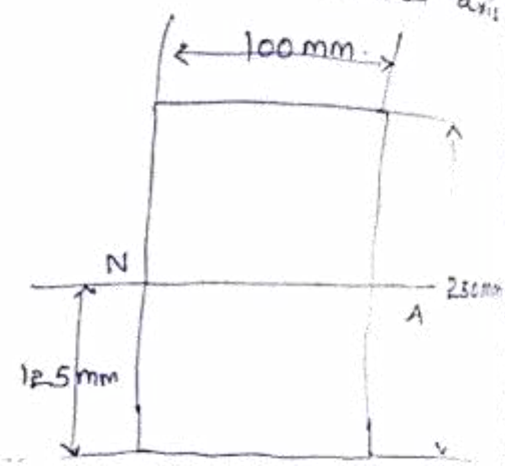
$$\sigma_{max} = 304.92 \ N/mm^2.$$

Shear stresses in beams :-

The following are the important sections over which the shear stress distribution is to be obtained.

1. Rectangular section.
2. Circular section.
3. I. section
4. T. sections and
5. Miscellaneous sections

A rectangular beam 100mm wide & 250mm deep is subjected to a maximum shear force of 50kN. Determine (i) Average shear stress. (ii) Maximum shear stress & (iii) Shear stress at a distance of 25mm above the neutral axis.



Given

Width, $b = 100mm$

Depth, $d = 250mm$

Maximum shear force, $F = 50kN = 50,000N$

(i) Average shear stress is given by

$$\tau_{avg} = \frac{F}{Area} = \frac{50,000}{b \times d} = \frac{50,000}{100 \times 250} = 2N/mm^2$$

(ii) Max. shear stress is given by eqn

$$\tau_{max} = 1.5 \times \tau_{avg}$$

$$= 1.5 \times 2 \Rightarrow 3 N/mm^2.$$

(iii) The shear stress at a distance y from N.A.

$$\tau = \frac{F}{2I}\left(\frac{d^2}{4} - y^2\right).$$

$$= \frac{50,000}{2I}\left(\frac{250^2}{4} - 25^2\right)$$

$$= \frac{50,000 \times 12}{2 \times 100 \times 250^3} \times 15,000 \text{ N/mm}^2$$

$$= 2.88 \text{ N/mm}^2$$

Unit - III

Torsion

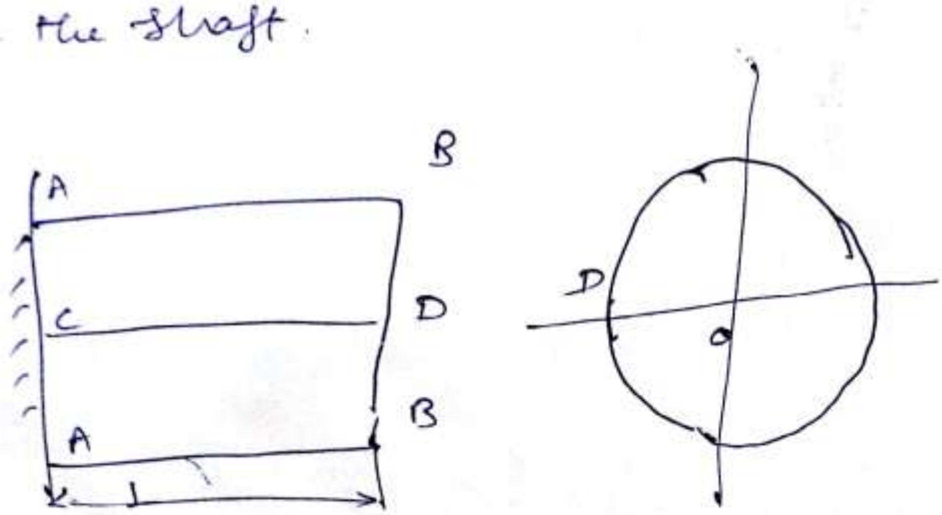
A shaft is said to be in torsion, when equal & opposite torques are applied at the two ends of the shaft

Due to the application of the torques at the two ends, the shaft is subjected to twisting moment.

It causes the shear stresses & shear strains in the material of the shaft.

Torque :-

It's the product of the force applied (tangentially to the ends of a shaft) & radius of the shaft.



R → Radius of shaft

l → Length of shaft

T → Torque applied at the end BB.

τ → Shear stress induced at the surface of the shaft due to torque T.

C → Modulus of rigidity of the material of the shaft.

ϕ = ∠DCD' also equal to shear strain.

$$\frac{\tau}{R} = \frac{C\theta}{L} = \frac{q}{r}.$$

Assumptions made in derivation of shear stress produced in a circular shaft subjected to torsion.

The derivation of shear stress produced in a circular shaft subjected to torsion is based on the following assumptions.

1. The material of the shaft is uniform throughout.
2. Twist along the shaft is uniform.
3. The shaft is of uniform circular section.
4. Cross-section of the shaft, which are plane before twist remain plain after twist.
5. All radii which are straight before twist remain straight after twist.

Max Torque Transmitted by a circular shaft

$$= \frac{\pi}{16} \tau \cdot D^3$$

Torque transmitted by a hollow circular shaft

$$= \frac{\pi}{16} \tau \left[\frac{D_o^4 - D_i^4}{D_o} \right]$$

Power transmitted by shafts

Power = $\frac{2\pi N T}{60}$ watts

N – rpm of shaft

T → Mean Torque N-m

ω – angular speed.

$$\therefore P = T \times \omega \quad \left(\because \frac{2\pi N}{60} = \omega \right)$$

Pb. 1

Two shafts of the same material & of same lengths are subjected to the same torque. if the first shaft is of a solid circular section the second shaft is of hollow circular section where internal dia is 2/3 of outside dia & the maximum shear stress developed in each shaft same, compare the weights of the shafts.

Torque transmitted by the solid shaft is given by $T = \frac{\pi}{16} \tau D^3$.

Torque transmitted by hollow shaft

$$T = \frac{\pi}{16} \tau \left[\frac{D_o^4 - D_i^4}{D_o} \right] \Rightarrow$$

$$= \frac{\pi}{16} \tau \left[\frac{D_o^4 - (2/3 \, D_o)^4}{D_o} \right].$$

$$= \frac{\pi}{16} \tau \left[\frac{D_o^4 - \frac{16}{81} D_o^4}{D_o} \right]$$

$$= \frac{\pi}{16} \tau \times \frac{65 \, D_o^4}{81 \times D_o}.$$

$$= \frac{\pi}{16} \tau \times \frac{65 \, D_o^3}{81}.$$

A torque transmitted by solid & hollow shafts are equal.

$$\frac{\pi}{16} \tau D^3 = \frac{\pi}{16} \tau \times \frac{65}{81} D_o^3.$$

$$\therefore D^3 = \frac{65}{81} D_o^3.$$

$$D = \left(\frac{65}{81} \right)^{1/3} (D_o^3)^{1/3}.$$

$$= \left(65/81 \right)^{1/3} \cdot D_o \Rightarrow \therefore D = 0.929 D_o.$$

Now, weight of solid shaft, W_s = Weight Density × Volume of solid shaft.

W_s = w × Area of Cross-Section × Length.

$$= w \times \frac{\pi}{4} D^2 \times L.$$

Weight of hollow shaft.

W_h = w × Area of hollow shaft × Length

$$= w \times \frac{\pi}{4} [D_o^2 - D_i^2] \times L$$

$$= w \times \frac{\pi}{4} [D_o^2 - (\tfrac{2}{3} D_o)^2] \times L$$

$$= w \times \frac{\pi}{4} [D_o^2 - \tfrac{4}{9} D_o^2] \times L$$

$$= w \times \frac{\pi}{4} \times \tfrac{5}{9} D_o^2 \times L$$

$$\frac{W_s}{W_h} = \frac{w \times \frac{\pi}{4} D^2 \times L}{w \times \frac{\pi}{4} \times \frac{5}{9} D_o^2 \times L}$$

$$= \frac{9 D^2}{5 D_o^2}$$

$$= \frac{9}{5} \frac{(0.929 D_o)^2}{D_o^2} = \frac{1.55}{1}$$

$$\frac{\text{Weight of solid shaft}}{\text{weight of hollow shaft}} = \frac{1.55}{1}$$

A solid steel shaft has to transmit 75 kw at ... rpm. Taking allowable shear stress as $70\ N/mm^2$, ... suitable dia for the shaft, if the max. torque transmitted at each revolution exceeds the mean by 30%.

Ans: Power transmitted, $P = 75\ kw = 75 \times 10^3\ w$.

$N = 200$ rpm.

$\tau = 70\ N/mm^2$.

T = Mean Torque Transmitted

T_{max} = Max. torque Transmitted $= 1.3T$

D = suitable diameter of the shaft.

Power is given by the relation.

$$P = \frac{2\pi N T}{60}$$

$$75 \times 10^3 = \frac{2\pi \times 200 \times T}{60}$$

$\therefore T = 3580.98\ N.m.$

$\Rightarrow 3580.98 \times 10^3\ N.mm.$

$T_{max} = T \times 1.3 \Rightarrow 1.3 \times 3580.98 \times 10^3$.

$= 4655274\ N.mm.$

Max. Torque transmitted by a solid shaft is given by

$$T_{max} = \frac{\pi}{16} \times \tau \times D^3$$

$$4655274 = \frac{\pi}{16} \times 70 \times D^3.$$

$$D = \left(\frac{16 \times 4655274}{\pi \times 70}\right)^{1/3}.$$

$$= 69.57 \text{ mm}$$

$$\boxed{D = 70 \text{ mm}}$$

— × —

Expression for Torque interms of Polar moment of Inertia

$$\frac{T}{J} = \frac{\tau}{R} = \frac{C\theta}{L}.$$

where, $C \rightarrow$ Modulus of rigidity.

$\theta \rightarrow$ Angle of twist in radiation.

$L \rightarrow$ Length of shaft.

Polar modulus

Polar modulus is defined as the ratio of the polar moment of Inertia to the radius of the shaft.

It is also called torsional section modulus.

It is denoted by Z_p.

$$Z_p = \frac{J}{R}$$

i) for solid shaft, $J = \frac{\pi}{32} D^4$

$$\therefore Z_p = \frac{\pi}{16} D^3$$

ii) for hollow shaft, $J = \frac{\pi}{32} (D_o^4 - D_i^4)$

$$\therefore Z_p = \frac{\pi}{16 D_o} \times (D_o^4 - D_i^4)$$

Strength of a shaft & torsional rigidity :-

The strength of a shaft means the max. torque or max. power the shaft can transmit.

Torsional rigidity or stiffness of the shaft is defined as the product of modulus of rigidity (C) & Polar moment of inertia of the shaft (J).

$\therefore$ Torsional rigidity $= C \times J$.

Torsional rigidity is also defined as the torque required to produce a twist of one radian / unit length of the shaft.

$\therefore$ Torsional rigidity $= \frac{T \times L}{\theta}$

If L = One metre & θ = one radian

Then Torsional rigidity = Torque.

Pro: Determine the dia of solid steel shaft which will transmit 90kW at 160 rpm. Also determine the length of the shaft if the twist must not exceed 80 N/mm² & twist should not be more than 1° in a shaft length of 2m. Take $C = 1 \times 10^5$ N/mm²

Soln:

$P = 300$ kW $= 300 \times 10^3$ W.

$N = 250$ rpm.

$\tau = 30$ N/mm².

$\theta = 1° \Rightarrow \frac{\pi}{180} = 0.01745$ radian.

Length of shaft $L = 2$m $= 2000$ mm.

$C = 1 \times 10^5$ N/mm².

$$P = \frac{2\pi NT}{60}$$

$$300 \times 10^3 = \frac{2\pi \times 250 \times T}{60}$$

$$T = \frac{300 \times 10^3 \times 60}{2\pi \times 250}$$

$$= 11459.1 \times 10^3 \ N.mm.$$

(i) Diameter of shaft when max. $\tau = 30 N/mm^2$.

Max. torque transmitted by a solid shaft is given by

$$T = \frac{\pi}{16} \times \tau \times D^3 .$$

$$11459100 = \frac{\pi}{16} \times 30 \times D^3 .$$

$$D = \left(\frac{16 \times 11459100}{\pi \times 30}\right)^{1/3} .$$

$$D = 124.5 \ mm$$

(ii) Diameter of the shaft when twist should not be more than 1°.

$$\therefore \frac{T}{J} = \frac{C\theta}{L} .$$

where $J \rightarrow$ Polar M.I.

$$\therefore = \frac{\pi}{32} \times D^4 .$$

$$\frac{11459100}{\frac{\pi}{32} D^4} = \frac{10^5 \times 0.01745}{2000}$$

$D = 107.5 \text{ mm}$

The suitable dia of the shaft is the greater of two values given by eqn.

Dia of shaft = 124.5 mm @ 125 mm.

If dia is taken smaller of the two values say 107.5mm, then from eqn. $T = \frac{\pi}{16} \tau D^3$

∴ The value of shear stress will be.

$$11459100 = \frac{\pi}{16} \tau \times (107.5)^3$$

$$\therefore \tau' = 46.978 \text{ N/mm}^2.$$

Which is more than given value of 30 N/mm^2,

Flanged Coupling:-

A flange coupling is used to connect two shafts.

$$n \times q \times \frac{\pi d^2 \times D^*}{8} = \frac{\pi}{16} \times \tau \times D^3$$

$\tau \rightarrow$ shear stress in the shaft

$q \rightarrow$ shear stress in the bolt

$d \rightarrow$ diameter of bolt

$D \rightarrow$ Diameter of shaft.

$D^* \rightarrow$ Diameter of bolt pitch circle

$n \rightarrow$ Number of bolts.

Strength of a shaft of varying sections:-

When a shaft is made up of different lengths & of different diameters, the torque transmitted by individual sections should be calculated first.

The strength of such a shaft is the minimum value of these torques.

Pb:- A shaft ABC of 500 mm length & 40 mm ext. dia. is bored, for a part of its length AB to a 20 mm dia. & for the remaining length BC to a 30 mm diameter bore. If the shear stress is not to exceed 80 N/mm^2. Find the max. power, the shaft can transmit at a speed of 200 r.p.m.

If the angle of twist in the length of 20mm dia bore is equal to that in the 30mm dia bore, find the length of the shaft that has been forced to 20mm & 30mm dia.

Given:

$L = 500mm, D = 40mm$

$d_1 = 20mm$

$d_2 = 30mm$

$\tau = 80 N/mm^2$

$N = 200 rpm$

$$T_1 = \frac{\pi}{16} \tau. \left[\frac{D^4 - d_1^4}{D}\right]$$

$= 942.500 N \cdot m$

$$T_2 = \frac{\pi}{16} \cdot \tau. \left[\frac{D^4 - d_2^4}{D}\right]$$

$= 687.2 Nm$

$$P = \frac{2\pi NT}{60} W$$

$= 14.39 kW$

$$\frac{T}{C} \cdot \frac{L_1}{J_1} = \frac{T}{C} \cdot \frac{L_2}{J_2}$$

$$\frac{L_1}{J_1} = \frac{L_2}{J_2}$$

$$\frac{L_1}{\frac{\pi}{32}(40^4 - 20^4)} = \frac{L_2}{\frac{\pi}{32}[40^4 - 30^4]}$$

$$L_1 = 289 \text{ mm}$$

$$L_2 = 211 \text{ mm}$$

Composite Shaft :-

A shaft made up of two or more diff. materials, & behaving as a single shaft is known as composite shaft.

Hence in a Composite shaft one type of shaft is rigidly seelved over another type of shaft.

Total torque transmitted by a composite shaft is the sum of the torques transmitted by each individual shaft.

But the angle of twist in each shaft will be equal.

Combined bending & Torsion:-

When a shaft is transmitting torque or power, it's subjected to shear stresses. At the same time the shaft is also subjected to bending moments due to gravity or inertia loads.

The Principal stresses & the max. shear stress when a shaft is subjected to bending & torsion.

$$\frac{q}{r} = \frac{T}{J}$$

$$q = \frac{T}{J} \times r.$$

$$\frac{M}{I} = \frac{\sigma}{y} \text{ (or) } \sigma = \frac{M \times y}{I}.$$

$$\tan 2\theta = \frac{2\tau}{\sigma}$$

Major Principal stress $= \frac{16}{\pi D^3}\left[M + \sqrt{M^2 + T^2}\right].$

minor Principal stress $= \frac{16}{\pi D^3}\left[M - \sqrt{M^2 + T^2}\right].$

Max τ = (Max. Principal stress - Minor Principal stress) / 2

$= \frac{16}{\pi D^3}\left(\sqrt{M^2+T^2}\right)$

In hollow shaft,

Major Principal stress $= \frac{16 D_o}{\pi [D_o^4 - D_i^4]}\left(M+\sqrt{M^2+T^2}\right)$.

Minor Principal stress $= \frac{16 D_o}{\pi [D_o^4 - D_i^4]}\left[M-\sqrt{M^2+T^2}\right]$.

Max shear stress $= \frac{16 D_o}{\pi [D_o^4 - D_i^4]}\left[\sqrt{M^2+T^2}\right]$.

Expression for strain energy stored in a body due to torsion :-

$= \frac{\tau^2}{4C} \cdot V$.

Total strain due to torsion

$= \frac{\tau^2}{4CD^2}(D^2+d^2) \times V$.

Springs:

Springs are the elastic bodies which absorb energy due to resilience.

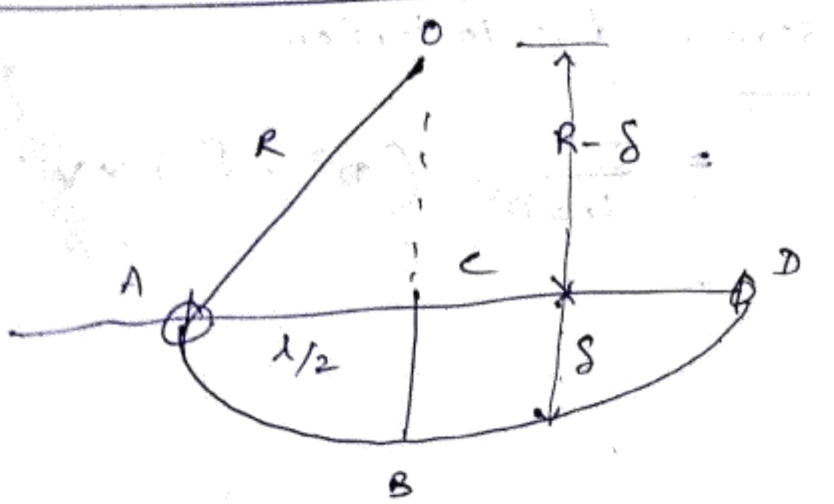
The absorbed energy may be released as & when required.

A spring which is capable of absorbing the greatest amount of energy for the given stress without permenantly distorted is known as leaf spring.

1. Laminated or Leaf Springs and

2. Helical springs.

Expression for Central deflection of the leaf Spring:



$$\delta = \frac{l^2 \times 2\sigma}{8E \times t} \Rightarrow \frac{\sigma \cdot l^2}{4Et}$$

...ical springs

Helical springs are the thick spring wire coiled into a helix.

1) Close-coiled helical springs &

2) Open-coiled helical springs.

Exp. for max. shear stress in wire

$$\therefore \tau = \frac{16 \times W}{\pi d^3} \times R$$

W → Axial load on spring.

R → Mean radius of coil.

d → diameter of wire

Exp. for deflection of spring.

$$\delta = \frac{64 W R^3 n}{C d^4}$$

n → no. of coils.

C → modulus of rigidity.

Exp. for stiffness of spring

$$s = \frac{C d^4}{64 \cdot R^3 \cdot n}$$

A closely coiled helical spring is to carry a load of 500N. It's mean coil dia. is to be 10 times that of the wire dia. Calculate these dia. if the max. shear stress in the mat. of the spring is to be 80 N/mm², if the stiffness is 20 N/mm deflection & C = 8.4 × 10⁴ N/mm² find no. of coils in the closely coiled helical spring.

Closely coiled helical spring:

$$\tau = \frac{16WR}{\pi d^3}$$

$$80 = \frac{16 \times 500 \times (D/2)}{\pi d^3}$$

$$= \frac{8000 \times \left(\frac{10d}{2}\right)}{\pi d^3}$$

$$d = 12.6mm.$$

$$D = 10 \times d = 12.6cm.$$

$$S = \frac{Load}{\delta}$$

$$20 = \frac{500}{\delta}$$

$$\delta = 500/20 = 25 mm$$

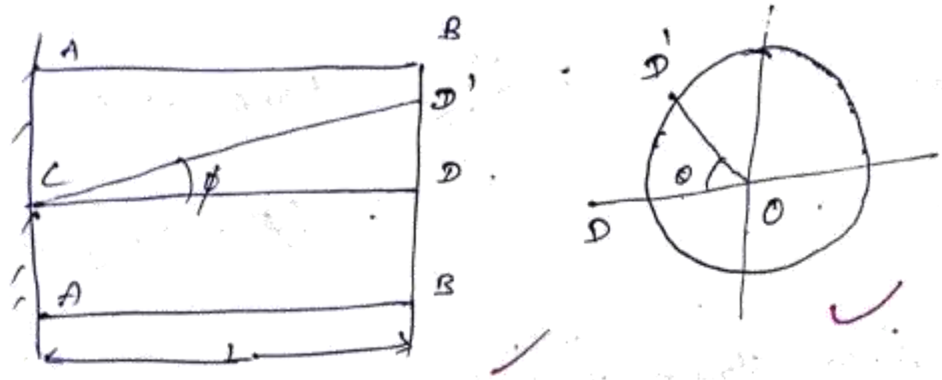
$$\delta = \frac{64 W R^3 \cdot n}{C \cdot d^4}$$

$$25 = \frac{64 \times 500 \times (63)^3 \times n}{8.4 \times 10^4 \times 12.6^4}$$

$$= 6.6 \simeq 7$$

$$\boxed{n = 7}$$

Derivation of Shear Stress Produced for a circular shaft subj. to torsion



When a circular shaft is subjected to torsion, shear stresses are set up in the matr. of the shaft. To determine the mag. of shear stress at any pt. on the shaft.

Consider a shaft fixed at one end AA & free end BB.

Let CD is any line on the outer surface of shaft.

Now, $\phi = \angle DCD'$. $\theta = \angle DOD'$

Now, distortion at outer surface due to Torque, T.

$= DD'$

Shear strain at outer surface

= Distortion / unit length

$= \frac{\text{Distortion at the outer surface}}{\text{Length of shaft}}$

$= \frac{DD'}{CD} = \tan\phi \Rightarrow \phi$.

if (ϕ is very small then $\tan\phi = \phi$)

Shear strain at outer surface

$\phi = DD'/L$

$DD' = OD \times \theta = R\theta$.

$\phi = \frac{R \times \theta}{L}$.

C : Shear stress induced / Shear strain Produced = Shear stress at Outer surface / Shear strain at Outer surface

$$= \frac{\tau}{(R\theta/L)} \Rightarrow \frac{\tau \times L}{R\theta} \Rightarrow \frac{C\theta}{L} = \frac{\tau}{R}$$

$$\tau = \frac{R \times C \times \theta}{L}$$

$$\tau \propto R \text{ (or) } \frac{\tau}{R} = \text{Constant}$$

$$\frac{\tau}{R} = \frac{C\theta}{L} \qquad \tau/R = q/r$$

$$\frac{\tau}{R} = \frac{C\theta}{L} = q/r$$

Cylindrical

A solid shaft is to transmit 300 kw at 100 rpm.

a) If shear stress is not to exceed $80 N/mm^2$. find d

a) If the τ is not to exceed $80 N/mm^2$, find dia

b) What % saving in weight, if this shaft is replaced by a hollow one, whose internal dia equ 0.6 of external dia, the length, the material & max shear stress being the same?

Power $P = \frac{2\pi N T}{60} \Rightarrow 300 \times 10^3 = \frac{2\pi \times 100 \times T}{60}$

$$T = 28647.8 \text{ N-m}$$

$$T = \frac{\pi}{16} \times \tau \times D^3 \Rightarrow 28647.8 = \frac{\pi}{16} \times 80 \times D^3$$

$$\therefore D = 121.8 \text{ mm} \simeq 122 \text{ mm}.$$

For hollow shaft

$$T = \frac{\pi}{16} \times \tau \times \left(\frac{D_o^4 - D_i^4}{D_o}\right)$$

$$= \frac{\pi}{16} \times 800 \times \left[\frac{D_o^4 - (0.6 D_o)^4}{D_o}\right].$$

$28647800 =$ " "

$$D_o = 127.6 \text{ mm} \simeq 128 \text{ mm}$$

Now % saving in weight

$$= \frac{W_s - W_h}{W_s} \times 100$$

$$= \frac{w \times \frac{\pi}{4} D^2 \times L - w \times \frac{\pi}{4}\left[D_o^2 - D_i^2\right] \times L}{w \times \frac{\pi}{4} D^2 \times L}$$

$$\Rightarrow \frac{D^2 - (D_o^2 - D_i^2)}{D^2} \times 100$$

$$\Rightarrow \frac{(122^2 - (128^2 - 76.8^2))}{122^2} \times 100$$

$$\Rightarrow 29.55\ \%.$$

——— X ———

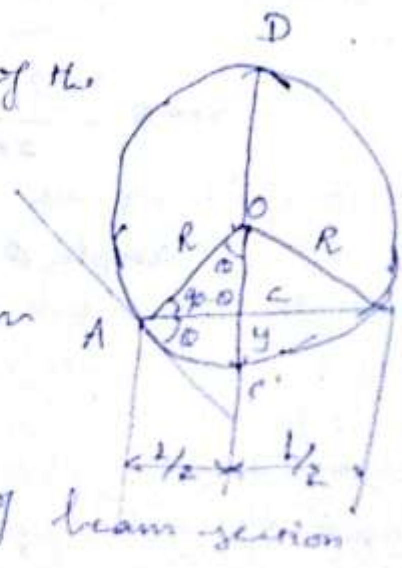
Unit - IV

Deflection of Beams

• If a beam carries uniformly distributed load or a point load, the beam is deflected from its original position.

* This chapter. We're going to study the amount by which a beam is deflected from its position.

Deflection & Slope of a Beam subjected to Uniform Bending moment



Let R → Radius of curvature of the deflected beam.

y → deflection of beam at the centre

I → Moment of Inertia of beam section

E → Young's modulus for the beam material

θ → slope of the beam at the end A.

Hence $\tan\theta = \theta$, where θ is in radians

$$\frac{dy}{dx} = \tan\theta = \theta$$

$$AC = BC = L/2$$

$$AC \times CB = DC \times CC'$$

$$L/2 \times L/2 = (2R - y) \times y$$

$$\frac{L^2}{4} = 2Ry - y^2$$ (neglecting y^2 because negligible)

$$\frac{L^2}{4} = 2Ry.$$

$$y = L^2/8R.$$

Bending Moment equation.

$$\frac{M}{I} = \frac{E}{R} \Rightarrow R = \frac{E \times I}{M}$$

$$\therefore y = \frac{L^2}{8 \times \frac{EI}{M}} \Rightarrow y = \frac{ML^2}{8EI}$$ Deflection eqn

$$\sin\theta = \frac{L}{2R}.$$

$\because$ angle θ is very small. $\sin\theta = \theta$

$$\therefore \theta = L/2R$$

$$= \frac{L}{2 \times \frac{EI}{M}}.$$

$$= \frac{M \times L}{2EI}$$

$\therefore$ eqn. slope for deflection ...

Relation b/w slope, deflection & radius of curvature

Deflection = y.

Slope = dy/dx

Bending moment : $EI \frac{d^2y}{dx^2}$

Shearing force : $EI \frac{d^3y}{dx^3}$.

The rate of loading = $EI \frac{d^4y}{dx^4}$.

Units In the above eqns. E is taken as N/mm^2

I is taken in mm^4 y is taken in mm.

M is taken in Nm & x is taken in m.

Methods of determining slope & deflection at a section in a loaded beam :-

The following are the important methods for finding the slope & deflection at a section in a loaded beam.

i) Double integration method

ii) Moment Area method and

iii) Macaulay's method.

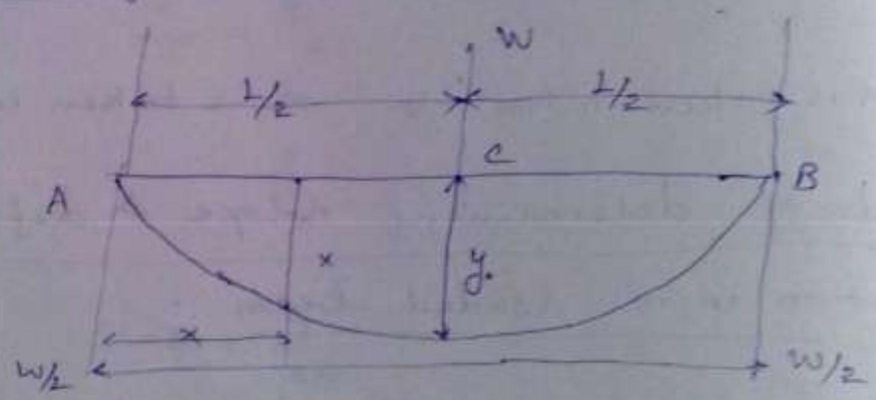
Incase of double integration method,

$$M = EI\frac{d^2y}{dx^2} \text{ (or) } \frac{d^2y}{dx^2} = \frac{M}{EI}$$

first integration of the above equ. gives the value of dy/dx (or) slope. The second integration gives the value of y or deflection

The first two methods are used for a single load whereas the third method is used for Several

Deflection of SSB carrying a Pt. load at the centre



$$R_A = R_B = W/2$$

Consider a section x at a distance x from A. The bending moment at this section is given by.

$$M_x = R_A \times x$$

$$= W/2 \times x$$

$M = EI\, d^2y/dx^2$

$$EI \frac{d^2y}{dx^2} = \frac{W}{2} \times x$$

On Integration, we get.

$$EI \cdot \frac{dy}{dx} = \frac{W}{2} \times \frac{x^2}{2} + C_1$$

where C_1 is the constant of Integration.

The boundary condition is that at $x = L/2$,

slope $\left(\frac{dy}{dx}\right) = 0$ [As the max. deflection is at the centre will be zero]. centre, hence slope at the

Subs. this boundary ~~eqn~~ condition in eqn we get

$$0 = \frac{W}{4} \times \left(\frac{L}{2}\right)^2 + C_1$$

$$C_1 = -\frac{WL^2}{16}$$

Substituting the value of C_1 in eqn.

$$EI \frac{dy}{dx} = \frac{Wx^2}{4} - \frac{WL^2}{16}$$

The above eqn. is slope eqn.

slope is maximum at A, At A, $x = 0$ & hence

slope at A, will be $EI\left(\frac{dy}{dx}\right)_{at\ A} = \frac{W}{4} \times 0 - \frac{WL^2}{16}$

$\left(\frac{dy}{dx}\right)_{at\ A}$ is represented by θ_A

$$EI \times \theta_A = -\frac{wL^2}{16}$$

$$\theta_A = -\frac{wL^2}{16EI} \qquad \theta_A = \theta_B$$

$$\therefore \theta_A = \theta_B = -\frac{wL^2}{16EI}$$

It gives the slope in radians.

<u>Deflection at any Pt :</u>

$$EI \times y = \frac{w}{4} \cdot \frac{x^3}{3} - \frac{wL^2}{16} x + C_2$$

$C_2 \rightarrow$ constant of integration.

$$\therefore EI \times 0 = 0 - 0 + C_2 \qquad \because (\text{At } A\ x = 0)$$

$$C_2 = 0$$

$$\therefore EI \times y = \frac{wx^3}{12} - \frac{wL^2 \cdot x}{16}$$

where $x = L/2$.

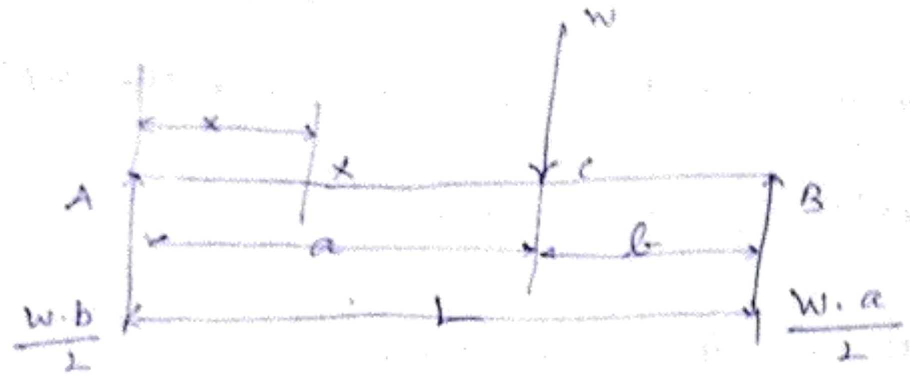
$$EI \times y_c = \frac{w}{12}\left(\frac{L}{2}\right)^3 - \frac{wL^2}{16} \cdot$$

$$= \frac{wL^3}{96} - \frac{wL^3}{32}$$

$$y_c = -\frac{wL^3}{48EI}$$

('-' sign shows that deflection is

Deflection of a SSB with an eccentric Pt load



$$\theta_A = -\frac{W.a.b}{6EI.L}(a+2b)$$

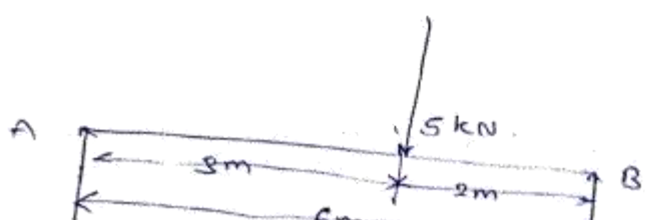
$$y_c = \frac{Wa^2.b^2}{3EIL}.$$

$$y_{max} = \frac{W.b}{9\sqrt{3}.EI.L}(a^2+2ab)^{3/2}.$$

Pb.2

Determine the slope at left support, deflection under the load & max. deflection of a SSB of l 5m, which is carrying a pt. load of 5kN at a dist of 3m from the left end. Take $E = 2\times10^5 N/mm^2$ & $I = 1\times10^8 mm^4$.

Soln:



Length $L = 5m = 5000 mm$

$W = 5kN = 5 \times 10^3 N$

$a = 3m = 3000 mm$

$b = L - a = 5 - 3 = 2m = 2000 mm$

$E = 2 \times 10^5 N/mm^2$

$I = 1 \times 10^8 mm^4$

$$\theta_A = -\frac{W.a.b}{6.E.I.L}(a + 2b)$$

$$= -\frac{(5000 \times 3000 \times 2000)}{6 \times 2 \times 10^5 \times 10^8 \times 5000} \times (3000 + 2 \times 2000)$$

$$\theta_A = -0.00035 \text{ radians}$$

$$y_c = \frac{W.a^2 b^2}{3EIL}$$

$$= \frac{5000 \times 3000^2 \times 2000^2}{3 \times 2 \times 10^5 \times 10^8 \times 5000}$$

$$= 0.6 mm$$

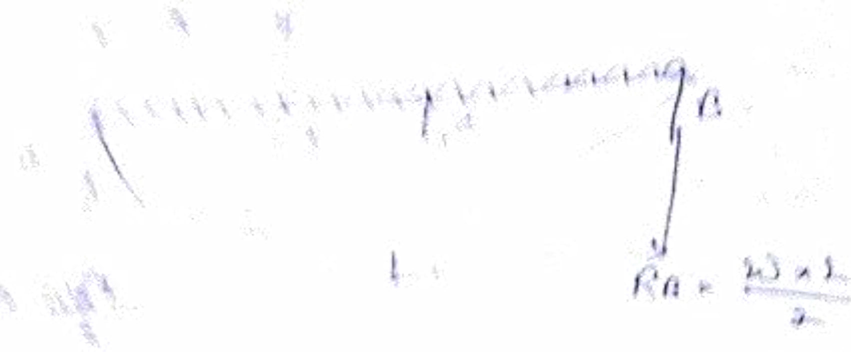
$$y_{max} = \frac{W.b}{9\sqrt{3} E.I.L}(a^2 + 2ab)^{3/2}$$

$$= \frac{5000 \times 2000 (3000^2 + 2 \times 3000 \times 2000)^{3/2}}{9 \times \sqrt{3} \times 2 \times 10^5 \times 10^8 \times 5000}$$

$$= 0.6173 mm$$

x

Deflection of beam with a uniformly distrib[illegible]



$$y_c = \frac{Wl^4}{384EI}$$

$$y_c = \frac{5}{384} \cdot \frac{Wl^3}{EI}$$

A beam of length 5m & of uniform rectangular section is supported at its ends. It carries a uniformly distributed load of 9 kN/m over the entire length. Calculate the width & depth of the beam if permissible bending stress is 7 N/mm² & central deflection is not to exceed 1cm. E = 1×10^4 N/mm²

Given

$l = 5m = 5000mm$

[illegible] $w = 9 kN/m$

Total load $W = w \times l = 9 \times 5 = 45 kN$

$\Rightarrow 45000 N.$

$y_c = 1cm = 10mm.$

[illegible]

[illegible]

$M = [illegible] = \frac{bd^3}{12}$ [illegible]

[illegible]

[illegible]

$bd^3 = [illegible]$

$= [illegible]$

$M = [illegible]$

$= [illegible]$

$\frac{M}{I} = \frac{f}{y}$

$\frac{[illegible]}{\left(\frac{bd^3}{12}\right)} = \frac{7}{(d/2)}$

$\frac{[illegible]}{bd^3} = \frac{14}{d}$

$$bd^2 = \frac{28125000 \times 12}{14}$$

$$= 24107142.85\ mm^3$$

$$d = \frac{838.906 \times 10^7}{24107142.85} = 364.58\ mm$$

$$b \times (364.58)^2 = 24107142.85$$

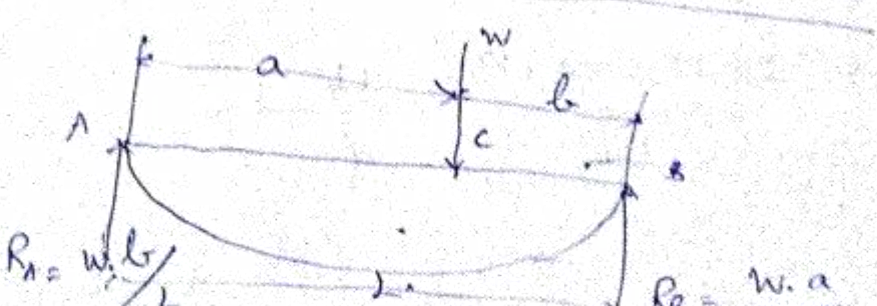
$$b = 181.36\ mm$$

Macaulay's Method:-

This method was devised by Mr. M. H. Macaulay & is known as Macaulay's method.

This method mainly consists in the special manner in which the bending moment at any section is expressed & in the manner in which the integrations are carried out.

Deflection of a SSB with an Eccentric Point load:



$R_A = \frac{W.b}{L}$ & $R_B = \frac{W.a}{L}$

The bending moment at any section AC at a distance x from A is given by

$$M_x = R_A \times x = \frac{W.b}{L} \times x$$

$$M_x = R_A . x - W \times (x - a)$$

$$= \frac{W.b}{L} . x - W(x - a)$$

$$M_x = \frac{W.b}{L} \Big| - W(x - a)$$

$$M = EI \, d^2y/dx^2$$

$$EI \, d^2y/dx^2 = \frac{W.b}{L} . x \Big| - W(x - a).$$

$$EI \, dy/dx = \frac{W.b}{L} . \frac{x^2}{2} + C_1 \Big| - \frac{W(x-a)^2}{2}.$$

$$\frac{(x-a)^2}{2} \text{ \& not } \frac{x^2}{2} - ax.$$

Integrating equ.

$$EIy = \frac{W.b}{2L} . \frac{x^3}{3} + C_1x + C_2 \Big| - \frac{W}{2} \frac{(x-a)^3}{3}.$$

$$\theta_A = -\frac{Wb}{6EIL}(l^2 - b^2)$$

$$y_c = -\frac{Wa^2 \cdot b^2}{3IEL}$$

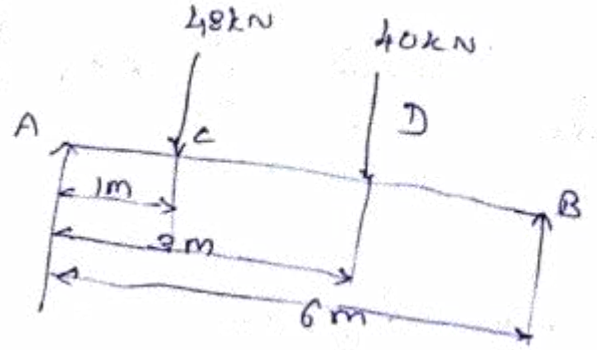
Pb.2 A beam of length 6m is simply supported at ends & carries two point loads of 48 kN & 40 kN at a distance of 1m & 3m respectively from the left support. Find (i) Deflection under each load

(ii) Max. deflection and

(iii) The point at which max. deflection occurs.

Take $E = 2 \times 10^5\ N/mm^2$ & $I = 85 \times 10^6\ mm^4$.

Soln:-



Given:-

$I = 85 \times 10^6\ mm^4$

$R_B \times 6 = 48 \times 1 + 40 \times 3$

$R_B = \frac{168}{6} = 28$

R_A = Total load $- R_B = (48 + 40) - 28 = 60$

$$EI\frac{d^2y}{dx^2} = R_A x \Big| - 48(x-1) \Big| - 40(x-3)$$

$$= 60x \Big| - 48(x-1) \Big| - 40(x-3)$$

Integrating above equation.

$$EI\frac{dy}{dx} = \frac{60x^2}{2} + C_1 \Big| -\frac{48(x-1)^2}{2} \Big| -\frac{40(x-3)^2}{2}$$

$$= 30x^2 + C_1 \Big| -24(x-1)^2 \Big| -20(x-3)^2.$$

$$EIy = 10x^3 + C_1x + C_2 \Big| -8(x-1)^3 \Big| -\frac{20}{3}(x-3)^3.$$

$$0 = 0 + 0 + C_2 \qquad \therefore C_2 = 0.$$

(ii) at $x = 6m$; $y = 0$.

$$0 = 10\times 6^3 + C_1\times 6 + 0 - 8(6-1)^3 - \frac{20}{3}(6-3)^3.$$

$$0 = 2160 + 6C_1 - 8\times 5^3 - \frac{20}{3}\times 3^3.$$

$$C_1 = -163.333.$$

$$EIy = 10x^3 - 163.33x \Big| -8(x-1)^3 \Big| -\frac{20}{3}(x-3)^3.$$

$$EI\cdot y_c = 10\times 1^3 - 163.33\times 1$$

$$= -153.33\ kN\,m^3$$

$$y_c = \frac{-153.33\times 10^{12}}{2\times 10^5\times 85\times 10^6}$$

$$= -9.019\ mm.$$

$-ve$ sign indicates load downwards.

Deflection under Second load

$$EI.y_2 = 10 \times 3^3 - 163.33 \times 3 - 8(3-1)$$

$$= -283.99 \times 10^{12} \ Nmm^3$$

$$y_d = \frac{-283.99 \times 10^{12}}{2 \times 10^5 \times 85 \times 10^6} = -16.7 \ mm$$

Maximum Deflection:

$$30x^2 + C_1 - 24(x-1)^2 = 0$$

$$6x^2 + 48x - 187.33 = 0$$

$$x = \frac{-48 \pm \sqrt{48^2 + 4 \times 6 \times 187.33}}{2 \times 6}$$

$$x = 2.87 \ m.$$

$$EI y_{max} = 10 \times 2.87^3 - 163.33 \times 2.87 - 8(2.8$$

$$= 284.67 \ kN m^3.$$

$$= 284.67 \times 10^{12} \ Nmm^3.$$

$$y_{max} = \frac{-287.67 \times 10^{12}}{2 \times 10^5 \times 85 \times 10^6}$$

$$= -16.745 \ mm.$$

——— x ———

Moment Area Method :-

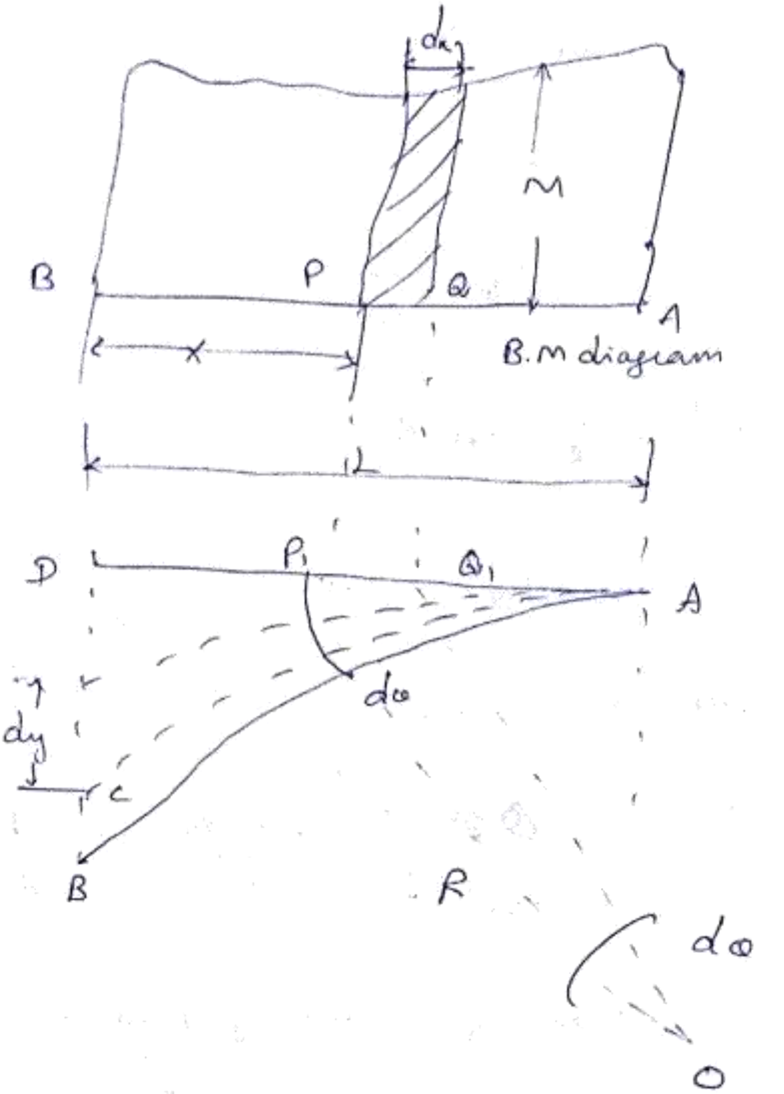


Fig. shows a AB carrying some type of loading & hence subjected to bending moment as shown in fig.

Let R = Radius of curvature of deflected part P_1Q_1

$d\theta$ = Angle subtended by the area P_1Q_1 at the centre O.

M = Bending moment P & Q.

P_1C = Tangent at point P_1.

Q_1D = Tangent at Pt Q_1.

$$P_1Q_1 = R \cdot d\theta$$

$$P_1Q_1 = dx$$

$$dx = R \cdot d\theta$$

$$d\theta = \frac{dx}{R}$$

$$\frac{M}{I} = \frac{E}{R} \quad \text{(or)} \quad R = \frac{EI}{M}$$

$$d\theta = \frac{dx}{\left(\frac{EI}{M}\right)} = \frac{M\,dx}{EI}$$

$$\theta = \int_0^L \frac{M \cdot dx}{EI} = \frac{1}{EI}\int_0^L M\,dx$$

$$\theta_B = \frac{\text{Area of B.M dia. b/w A \& B}}{EI}$$

$$\theta_B - \theta_A = \frac{\text{Area of B.M. b/w A \& B}}{E \cdot I}$$

$$dy = x \cdot d\theta$$

$$dy = x \cdot \frac{M \cdot dx}{EI}$$

$$y = \int_0^L x \cdot \frac{M\,dx}{EI}$$

$$= \frac{1}{EI}\int_0^L x \cdot M \cdot dx$$

$$y = \frac{1}{EI} \times A \times \bar{x} = \frac{A\bar{x}}{EI}$$

where A = Area of B.M dia. b/w A & B

$\bar{x}$ = Distance C.G of area A from B.

Mohr's Theorems:

i) The change of slope b/w any two pts is equal to the net area of the B.M dia. b/w these points divided by EI.

ii) The total deflection b/w any two pts is equal to the moment of the area of B.M diagram b/w the two pts about the last point.

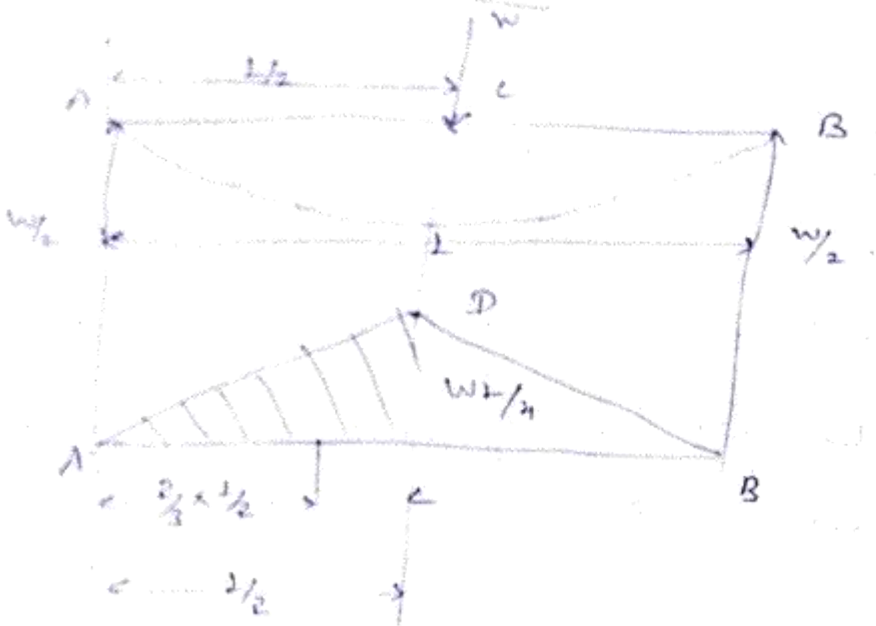
The Mohr's theorems is conveniently used for following cases.

1. Problems on Cantilevers

2. Simply supported beams carrying symmetrical loading.

3. Beams fixed at both ends.

slope & deflection of SSB carrying a Pt. load at the centre by Mohr's theorem.



$$A = \frac{\text{Area of B.M diagram between A \& C}}{EI}$$

$$= \frac{1}{2} \times \frac{L}{2} \times \frac{WL}{4} = \frac{WL^2}{16}.$$

$$\text{Slope at } A \text{ (or) } \theta_A = \frac{WL^2}{EI}.$$

$$y = \frac{A\bar{x}}{EI}$$

$$= \frac{WL^2}{16}$$

$$\bar{x} = \frac{2}{3} \times \frac{L}{2} = \frac{L}{3}.$$

$$y = \frac{\frac{WL^2}{16} \times \frac{L}{3}}{EI} = \frac{WL^3}{48EI}$$

$$= \frac{1287500}{2.7 \times 200\times10^6 \times 300\times10^{-4}} \times 10^3 = 7.94mm.$$

Deflection at D. y_D -

$$= R_B' \times 10 - \frac{1}{2} \times 10 \times \frac{1250}{2EI} \times 10/3$$

$$= 7.33mm$$

deflection at B, $y_B = 0$.

Maxwell's Theorem :-

The Maxwell's reciprocal theorem states under. The work done by the first system of loads due to displacements caused by a second system of loads equals the work done by the second system of loads due to displacements caused by the first system of loads.

Proof:-

Let Point forces P_i, $i = 1, 2, \ldots n$ acts on an elastic body constrained in a space. Then the strain energy due to this force system is given by.

$$V_A = \sum_{i=1}^{n} \frac{1}{2} P_i \delta_i$$

where δ_i are the corresponding deflections

Let Point forces P_j, $j = 1, 2, \dots m$ be the new set of point forces.

$$U_B = \sum_{j=1}^{m} \frac{1}{2} P_j \delta_j .$$

$$U_A = \frac{1}{2} \sum_{i=1}^{n} (P_i)_A (\delta_i)_A .$$

$$U_{A,B} = \sum_{i=1}^{n} (P_i)_A (\delta_i)_B$$

$$U_B = \frac{1}{2} \sum_{j=1}^{m} (P_j)_B (\delta_j)_B .$$

$$U = U_A + U_{A.B} + U_B .$$

$$U' = U_B + U_{B.A} + U_A .$$

$$U = U'$$

$$U_A + U_{A,B} + U_B = U_B + U_{B.A} + U_A$$

$$U_{A.B} = U_{B.A}$$

$$\sum_{i=1}^{n} (P_i)_A (\delta_i)_B = \sum_{j=1}^{m} (P_j)_B (\delta_j)_A$$

x ——————

THIN CYLINDERS, SPHERES AND THICK CYLINDERS

1) How does a thin cylinder fail due to internal fluid pressure?

Thin cylinder failure due to internal fluid pressure by the formation of circumferential stress and longitudinal stress.

2) Name the stress develops in the cylinder.

The stresses developed in the cylinders are:

1. Hoop or circumferential stresses.
2. Longitudinal stresses
3. Radial stresses

3) Define radial pressure in thin cylinder.

The internal pressure which is acting radially inside the thin cylinder is known as radial pressure in thin cylinder.

4)Differentiate between thin and thick cylinders

S.No	Thin	Thick
1	Ratio of wall thickness to the diagram of cylinder is less than 1/20.	Ratio of wall thickness to the diagram of cylinder is more than 1/20
2	Hoop stress is assumed to be constant throughout the wall thickness.	Hoop stress varies from inner to outer wall thickness.

5) Describe the lame's theorem: [MAY/JUNE 2016][NOV/DEC 2014] [MAY/JUNE 2017] (Apr/May 2018)

(Apr/May 2019)

Ratio stress, $\sigma_r = b/r^2 - a$

Hoop stress, $\sigma_c = b/r^2 + a$

6) State the expression for max shear stress in a cylinder shell

In a cylindrical shell, at any point on it circumference there is a set of two mutually perpendicular stresses $\sigma_c \sigma_\gamma$ which are principal stresses and as such the planes in which these act are the principal planes.

$$\tau_{max} = \frac{\sigma_c - \sigma_\gamma}{2} = \frac{\frac{pd}{2t} - \frac{pd}{4t}}{2} = \frac{pd}{8t}$$

$$\boxed{\tau_{max} = \frac{pd}{8t}}$$

7) Define-hoop stress & longitudinal stress

(i) Hoop stress: (σ_c)

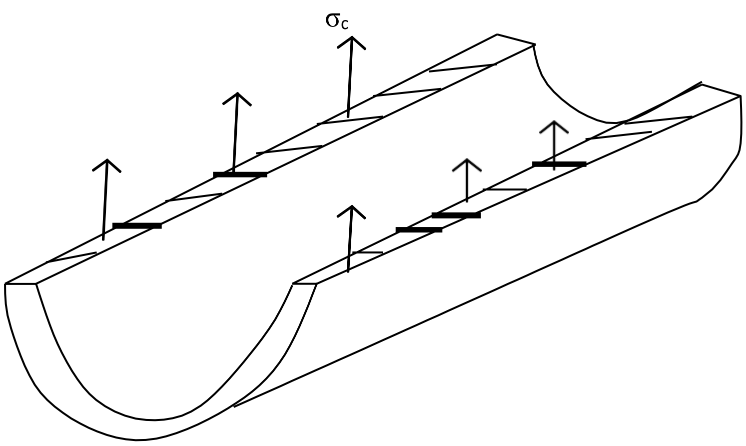
These act in a tangential dirn, to the circumference of the shell.

$$\sigma_c = \frac{pd}{2t}$$

(ii) Longitudinal stress: (σ_l)

The stress in the longitudinal direct due to tendency of busting the cylinder along the transverse place is called longitudinal stress

$$\sigma_l = \frac{pd}{4t}$$



8) State the assumption made in lame's theorem for thick cylinder analysis. [APR/MAY 2015] [NOV/DEC 2017] [NOV/DEC 2018]

1. The material is homogeneous and Isotropic.
2. The material is stressed within elastic limit.
3. All the fibers of the material are to expand (or) contract independently without being constrained by the adjacent fibers.
4. Plane section perpendicular to the longitudinal axis of the cylinder remain plane after the application of internal pressure.

9) What is meant by circumferential stress? [NOV/DEC 2014]

The stress in the circumferential direction in due to tendency of bursting the cylinder along the longitudinal axis is called circumferential stress (or) hoop stress.

$$\sigma_c = \frac{pd}{2t}$$

10) A storage tank of internal diameter 280 mm is subjected to an internal pressure of 2.56 MPa. Find the thickness of the tank. If the hoop & longtudital stress are 75 MPa and 45 MPa respectively

$\sigma_c = 75$ MPa, $\sigma_l = 45$ MPa, d = 280 mm, p = 2.5MPa

$\sigma_c > \sigma_l \Rightarrow$ use σ_c

$$\sigma_c = \frac{pd}{2t}$$

$$t = \frac{pd}{2\sigma_c} = \frac{2.5\times 280}{2\times 75}$$

$$\boxed{t = 4.66 \text{ mm}}$$

11) A spherical shell of 1m internal diameter undergoes a diameter strain of 10^{-4} due to internal pressure. What is the corresponding change in volume?

$$\delta V = e_v \times V$$

$$= 3\times e\times V = 3\times 10^{-4}\times \frac{\pi}{6}\times(1000)^3$$

$$\delta V = 157.079 \text{ mm}^3$$

12) A thin cylindrical closed at both ends is subjected to an internal pressure of 2 MPa. Internal diameter is 1m and the wall thickness is 10mm. What is the maximum shear stress in the cylinder material?

$p = 2\text{mPa} = \frac{2N}{mm^2}$ $\qquad$ $d = 1m = 100mm \quad t = 10mm$

$$\sigma_c = \frac{pd}{2t} = \frac{2\times 1000}{2\times 10} = 100\,N/mm^2$$

$$\sigma_l = \frac{pd}{4t} = \frac{2\times 1000}{4\times 10} = 50\,N/mm^2$$

$$\tau_{max} = \frac{\sigma_c - \sigma_l}{2} = \frac{100-50}{2} = \frac{50}{2}$$

$$\tau_{max} = 25\ N/mm^2$$

13) Find the thickness of the pipe due to an internal pressure of 10N/mm^2 if the permissible stress is 120 N/mm^2 and the diameter of the pipe is 750 mm

$p = 10\ N/mm^2,\ \sigma_c = 120\,N/mm^2,\ d = 750\,mm$

$$\sigma_c = \frac{pd}{2t}$$

$$t = \frac{pd}{2\sigma_c} = \frac{10\times 750}{2\times 120} = 31.25 \text{ mm}$$

14) A spherical shell of 1m diameter is subjected to an internal pressure 0.5 N/mm^2. Find the thickness if the allowable stress in the material of the shell is 75N/mm^2.

$d = 1m = 1000$ mm, $\qquad p = 0.5\,N/mm^2 \quad \sigma_c = 75N/mm^2$

$$\sigma_c = \frac{pd}{4t}$$

$$t = \frac{pd}{4\sigma_c}$$

$$= \frac{0.5\times 1000}{4\times 75} = \boxed{1.67\,mm}$$

15) Define thick cylinder

When the ratio of thickness (t) to internal diameter of cylinder is more than 1/20 then the cylinder is known as thick cylinder

16) In a thick cylinder will the radial stress is vary over the thickness of wall?

Yes, in thick cylinder radial stress is maximum at inner and minimum at the outer radius.

17) Define thin cylinder. (Nov/Dec 2017)

If the thickness of wall of the cylinder vessel is less than 1/15 to 1/20 of its internal diameter, the cylinder vessels is known as thin cylinder.

18) In a thin cylinder will the radial stress over the thickness of wall?

No, In the cylinder radial stress developed in its wall is assumed to be constant since the wall thickness is very small as compared to the diameter of cylinder

19) What is the ratio of circumference stress to longitudinal stress of a thin cylinder?

The ratio of circumferential stress to longitudinal stress of a thin cylinder is two.

20) Distinguish between cylinder shell and spherical shell.

S.No.	Cylindrical shell	Spherical shell
1.	Circumferencial stress is twice the longitudinal stress	Only hoop stress presents
2.	It withstands low pressure than spherical shell for the same diameter	It withstand more pressure than cylinder shell for the same diameter

21) What is the effect of riveting a thin cylinder shell?

Riveting reduce the area offering the resistance. Due to this, the circumferential and longitudinal stresses are more. It reduces the pressure carrying capacity of the shell.

PART-B

1) A cylindrical thin drum 80cm in diameter and 3m long has a shell thickness of 1cm. If the drum is subjected to an internal pressure of 2.5 N/mm^2, determine (i) change in diameter (ii) change in length and (iii) change in volume $E=2\times10^5 N/mm^2$ and poisons ratio=0.25 (Apr/May 2019)

d = 80cm

L = 3m = 300cm

t = 1cm

$p = 250 N/cm^2$

$E = 2 \times 10^7\ N/cm^2$

$\mu = 0.25$

Change in diameter (δd)

$$\delta d = \frac{pd^2}{2tE}\left[1-\frac{\mu}{2}\right]$$

$$= \frac{250\times80^2}{2\times1\times2\times10^7}\left[1-\frac{0.25}{2}\right]$$

$$\boxed{\delta d = 0.35cm}$$

Change in length (δℓ)

$$\delta \ell = \frac{pdL}{2tE}\left[\frac{1}{2}-\mu\right]$$

$$= \frac{250\times80\times300}{2\times1\times2\times10^7}\left[0.5-0.25\right]$$

$$\boxed{\delta \ell = 0.0375cm}$$

Change in volume (δv)

$$\frac{\delta V}{V} = 2\frac{\delta d}{d} + \frac{\delta l}{l}$$

$$\frac{\delta V}{V} = 2\frac{0.035}{80} + \frac{0.0375}{300} = 0.001$$

$$\text{original volume, } V = \frac{\pi}{4}d^2 \times \ell = \frac{\pi}{4}\times80^2\times300$$

$$V = 1507964.473 cm^3$$

$\delta V = 0.001 \times V = 0.001 \times 1507964.473 = 1507.96\ cm^3$

2) A spherical shell of internal diameter 0.9m and of thickness 10mm is subjected to an internal pressure of 1.4N/mm^2. Determine the increase in diameter and increase in volume. $E=2\times10^5 N/mm^2$ and poissons ratio=1/3 (Apr/May 2019)

$d = 0.9m = 900mm$

$t = 10mm$

$p = 1.4 N/mm^2$

$E = 2x10^5 N/mm^2$

$\mu = \frac{1}{3}$

Change in diameter: (✿d)

$$\delta d = \frac{pd^2}{4tE}\left[1-\frac{1}{m}\right]$$

$$= \frac{1.4\times 900^2}{4\times 10\times 2\times 10^5}\left[1-\frac{1}{3}\right]$$

$$\boxed{\delta d = 0.0945mm}$$

Change in volume (δv)

$$e_V = 3x\frac{\delta d}{d} = 3x\frac{0.0945}{900} = 315x10^{-6}$$

$$\frac{\delta V}{V} = 315x10^{-6}$$

$$V = \left(\frac{\pi}{6}\right)xd^3 = \left(\frac{\pi}{-}\right)x900^3$$

$$\delta V = 12028.5mm^3$$

3) A boiler shell is to be made of 15mm thick plate having tensile stress of 120 N/mm² If the efficiencies of the longitudinal and circumferential joints are 70% and 30%. Determine the maximum permissible diameter of the shell for an internal pressure of 2 N/mm² (Nov/Dec 2018)

Maximum diameter of circumference stress

$$\sigma_c = \frac{pd}{2t\eta_l}$$

$$120 = \frac{2\times d}{2\times 15\times 0.7}$$

$$d = \frac{120\times 2\times 15\times 0.7}{2}$$

$$d = 1260mm$$

Maximum diameter for longitudinal stress

$$\sigma_l = \frac{pd}{4t\times\eta_c}$$

$$120 = \frac{2\times d}{4\times 15\times 0.3}$$

$$d = \frac{120\times 4\times 15\times 0.3}{2}$$

$$d = 1080\,mm$$

4) A thin cylindrical shell with following dimensions is filled with a liquid at atmospheric pressure. Length=1.2m, external diameter=20cm, thickness of metal=8mm, Find the value of the pressure exerted by the liquid on the walls of the cylinder and the hoop stress induced if an additional volume of 25cm^3 of liquid is pumped into the cylinder. Take E=2.1×10^{5}N/mm^2 and poisons ratio=0.33 (Nov/Dec 2018)

$$L = 1.2m = 1200mm$$
$$D = 20cm = 200mm$$
$$t = 8mm$$
$$d = D - 2t = 184mm$$
$$\delta V = 25cm^3 = 25000mm^3$$
$$E = 2.1x10^5 N/mm^2$$
$$\mu = 0.33$$

$$\text{Volume, } V = \frac{\pi}{4} \times d^2 \times \ell$$
$$= \frac{3.14}{4} \times 184^2 \times 1200$$
$$= 31908528mm^3$$
$$\delta V = V \times \frac{pd}{2tE}\left(\frac{5}{2} - \frac{2}{m}\right)$$
$$25000 = 31908528 \times \frac{p \times 184}{2 \times 8 \times 2.1 \times 10^5}\left[\frac{5}{2} - 2(0.33)\right]$$
$$p = 7.7N/mm^2$$
$$\sigma_c = \frac{pd}{2t} = \frac{7.7x184}{2x8} = 89.42N/mm^2$$

5) A cylindrical shell 3m long which is closed at the ends has an internal diameter of 1.5m and a wall thickness of 20mm. Calculate the circumferential and longitudinal stresses induced and also change in the dimensions of the steel. If it is subjected to an internal pressure of 1.5 N/mm^2 Take E=2×10^{5}N/mm^2 and poisons ratio=0.3 (Apr/May 2018)

$$l = 3m = 3000mm$$
$$t = 20mm$$
$$d = 1.5m = 1500mm$$
$$p = 1.5N/mm^2$$
$$E = 2x10^5 N/mm^2$$
$$\mu = 0.3$$

Hoop stress, $\sigma_c = \frac{pd}{2t} = \frac{1.5 \times 1500}{2 \times 20} = 56.25$

$$\sigma_c = 56.25N/mm^2$$

Longitudinal stress, $\sigma_{\ell} = \frac{pd}{4t} = \frac{1.5\times1500}{4\times20} = 28.125$

$\sigma_{\ell} = 28.125 N/mm^2$

Change in diameter (δd)

$$\delta d = \frac{pd^2}{2tE}\left[1-\frac{\mu}{2}\right]$$

$$= \frac{1.5\times1500^2}{2\times20\times200\times10^3}\left[1-\frac{0.3}{2}\right]$$

$$\boxed{\delta d = 0.7225mm}$$

Change in length (δℓ)

$$\delta\ell = \frac{pdL}{2tE}\left[\frac{1}{2}-\mu\right]$$

$$= \frac{1.5\times1500\times3000}{2\times20\times200\times10^3}\left[0.5-0.3\right]$$

$$\boxed{\delta\ell = 0.16875mm}$$

Change in volume (δv)

$$\frac{\delta V}{V} = \frac{pd}{2tE}\left[\frac{5}{2}-\frac{2}{m}\right]$$

$$\text{original volume, } V = \frac{\pi}{4}d^2\times\ell = \frac{\pi}{4}\times1500^2\times3000$$

$$V = 5301437603mm^3$$

$$\delta V = \frac{1.5\times1500x5301437603}{2\times20\times200\times10^3}\left[\frac{5}{2}-2\times0.3\right]$$

$$\boxed{\delta V = 2832955.72mm^3}$$

6) A compound cylinder formed by shrinking one tube to another is subjected to an internal pressure of $90MN/m^2$. Before the fluid is admitted, the internal and external diameter of the compound cylinders are 180mm and 300mm respectively and the diameter at the junction is 240mm. If after shrinking on, the radial pressure at the common surface is $12MN/m^2$. Determine the final stresses developed in the compound cylinder (Apr/May 2018)

Solution. Internal pressure in the cylinder,

$$p_1 = 90 \text{ MN/m}^2$$

Internal radius of the cylinder, $r_1 = \frac{180}{2} = 90 \text{ mm} = 0{\cdot}09 \text{ m}$

External radius of the cylinder, $r_3 = \frac{300}{2} = 150 \text{ mm} = 0{\cdot}15 \text{ m}$

Radius at the junction, $r_2 = \frac{240}{2} = 120 \text{ mm} = 0{\cdot}12 \text{ m}$

Radial pressure at the common surface after shrinking on,

$$p = 12 \text{ MN/m}^2$$

Final stresses developed:

Let the *Lame's equations* be:

For inner tube: $\sigma_r = \frac{b}{r^2} - a$

and, $\sigma_c = \frac{b}{r^2} + a$

For outer tube: $\sigma_r = \frac{b'}{r^2} - a'$

and, $\sigma_c = \frac{b'}{r^2} + a'$

(a) *Before the fluid is admitted:*

Inner tube:

At,
$$r = r_1 = 0{\cdot}09 \text{ m}, \sigma_r = 0$$

$$\frac{b}{0{\cdot}0081} - a = 0$$

$\therefore$
$$123{\cdot}456\, b - a = 0 \qquad ...(i)$$

or,

At,
$$r = r_2 = 0{\cdot}12 \text{ m},$$
$$\sigma_r = 12 \text{ MN/m}^2$$

$\therefore$
$$\frac{b}{0{\cdot}0144} - a = 12$$

or,
$$69{\cdot}44\, b - a = 12 \qquad ...(ii)$$

From eqns. (*i*) and (*ii*), we get

$$b = -\,0{\cdot}222 \text{ and } a = -\,27{\cdot}41$$

Hence circumferential stress at any point in the *inner tube* will be given by

$$\sigma_c = -\frac{0{\cdot}222}{r^2} - 27{\cdot}41$$

The *minus sign indicates that the stress will be wholly compressive.*

At,
$$r = r_1 = 0{\cdot}09 \text{ m},$$

$$\sigma_{c(0{\cdot}09)} = -\frac{0{\cdot}222}{0{\cdot}09^2} - 27{\cdot}41 = 54{\cdot}82 \text{ MN/m}^2 \text{ (comp.)}$$

At,
$$r = 0{\cdot}12 \text{ m},$$

$$\sigma_{c(0{\cdot}12)} = -\frac{0{\cdot}222}{0{\cdot}12^2} - 27{\cdot}41 = 42{\cdot}82 \text{ MN/m}^2 \text{ (comp.)}$$

Outer tube:

At,
$$r = 0{\cdot}15 \text{ m}, \sigma_r = 0$$

$\therefore$
$$\frac{b'}{0{\cdot}15^2} - a' = 0$$

or,
$$44{\cdot}44\, b' - a' = 0 \qquad ...(iii)$$

At,
$$r = 0{\cdot}12 \text{ m}, \sigma_r = 12 \text{ MN/m}^2$$

$\therefore$
$$\frac{b'}{0{\cdot}12^2} - a' = 0$$

or,
$$69{\cdot}44\, b' - a' = 12 \qquad ...(iv)$$

From eqns. (*iii*) and (*iv*), we get

$$b' = +\,0{\cdot}48, \text{ and } a' = +21{\cdot}33$$

Hence the circumferential stress at any point in the *outer tube* will be given by

$$\sigma_c = \frac{0{\cdot}48}{r^2} + 21{\cdot}33$$

At,
$$r = 0{\cdot}12 \text{ m},$$

$$\sigma_{c(0{\cdot}12)} = \frac{0{\cdot}48}{0{\cdot}12^2} + 21{\cdot}33 = 54{\cdot}66 \text{ MN/m}^2 \text{ (tensile)}$$

At,
$$r = 0{\cdot}15 \text{ m},$$

$$\sigma_{c(0{\cdot}15)} = \frac{0{\cdot}48}{0{\cdot}15^2} + 21{\cdot}33 = 42{\cdot}66 \text{ MN/m}^2 \text{ (tensile)}$$

(b) *After the fluid is admitted:*

Let the *Lame's equation* be:

$$\sigma_r = \frac{b}{r^2} - a$$

At, $r = 0{\cdot}09$ m, $\sigma_r = 90$ MN/m²

$$\therefore \quad 90 = \frac{b}{0{\cdot}09^2} - a$$

or, $90 = 123{\cdot}45\, b - a$ ...(v)

At, $r = 0{\cdot}15$ m, $\sigma_r = 0$

$$\therefore \quad 0 = \frac{b}{0{\cdot}15^2} - a$$

or $0 = 44{\cdot}44\, b - a$ ...(vi)

From eqns. (*v*) and (*vi*), we get

$b = 1{\cdot}139$ and $a = 50{\cdot}61$

Hence, the circumferential stress at any point in the compound tube is given by,

$$\sigma_c = \frac{b}{r^2} + a$$

At, $r = 0{\cdot}09$ m, $\sigma_{c(0\cdot09)} = \frac{1{\cdot}139}{0{\cdot}09^2} + 50{\cdot}61 = 191{\cdot}23$ MN/m² (tensile)

$r = 0{\cdot}12$ m, $\sigma_{c(0\cdot12)} = \frac{1{\cdot}139}{0{\cdot}12^2} + 50{\cdot}61 = 129{\cdot}71$ MN/m² (tensile)

$r = 0{\cdot}15$ m, $\sigma_{c(0\cdot15)} = \frac{1{\cdot}139}{0{\cdot}15^2} + 50{\cdot}61 = 101{\cdot}23$ MN/m² (tensile)

The *final circumferential stresses* at different points are tabulated below:

Tensile stress........ +

Compressive stress..... −

Circumferential (or hoop) stress (MN/m²)	*Inner tube*		*Outer tube*	
	$r = 0{\cdot}09$ m	$r = 0{\cdot}12$ m	$r = 0{\cdot}12$ m	$r = 0{\cdot}15$ m
(*i*) Initially	− 54·82	− 42·82	+ 54·66	+ 42·66
(*ii*) Due to fluid pressure	+ 191·23	+ 129·71	+ 129·71	+ 101·23
Final	+ 136·41	+ 86·89	+ 184·31	+ 143·89

Hence the *final circumferential stresses are:*

Inner tube: $\sigma_{c\,(0\cdot09)}$ **= 136·41 MN/m²** (*tensile*)

$\sigma_{c\,(0\cdot12)}$ **= 86·89 MN/m²** (*tensile*) **(Ans.)**

Outer tube: $\sigma_{c\,(0.12)}$ **= 184·31 MN/m²** (*tensile*)

$\sigma_{c\,(0\cdot15)}$ **= 143·89 MN/m²** (*tensile*) **(Ans.)**

7) Determine the maximum and minimum hoop stress across the section of a pipe of 400 mm internal diameter and 100 mm thick, when the pipe contains a fluid at a pressure of 8 N/mm^2. Also sketch the radial pressure distribution and hoop stress distribution across the section. (May 2017) (Nov/Dec 2017)

Solution,

Given:

Internal dia = 400 mm

∴ Internal radius, $r_1 = \frac{400}{2} = 200\,\text{mm}$

Thickness = 100 mm

∴ External radius $r_2 = \frac{600}{2} = 300\,\text{mm}$

Fluid pressure, p_0 = 8N/mm^2

or at $x = r_1$, $p_x = p_0 = 8\text{N/mm}^2$

The radial pressure (p_x) is given by equation (18.1) as

$$p_x = \frac{b}{x^2} a$$

Now apply the boundary conditions to the above equation. The boundary conditions are:

1. At $x = r_1 = 200$ mm, $p_x = 8$ N/mm^2

2. At $x = r_2 = 300$ mm, $p_x = 0$

Substituting these boundary conditions in equation(i), we get

$$8 = \frac{b}{200^2} - a = \frac{b}{40000} - a \qquad \ldots(\text{ii})$$

and

$$0 = \frac{b}{300^2} - a = \frac{b}{90000} - a \qquad \ldots(\text{iii})$$

subtracting equation (iii) from equation (ii), we get

$$8 = \frac{b}{40000} - \frac{b}{90000} = \frac{9b - 4b}{360000} = \frac{5b}{360000}$$

$$b = \frac{360000 \times 8}{5} = 5760000$$

Substituting this value in equation (iii), we get

$$0 = \frac{5760000}{90000} - a \quad \text{or} \quad a = \frac{5760000}{90000} = 6.4$$

The values of 'a' and 'b' are substituted in the hoop stress.

Now hoop stress at any radius x is given by equation (18.2) as

$$\sigma_x = \frac{b}{x^2} + a = \frac{576000}{x^2} + 6.4$$

At x = 200 mm, $\sigma_{200} = \frac{576000}{200^2} + 6.4 = 14.4 + 6.4 = 20.8 N/mm^2$. Ans.

At x = 300 mm, $\sigma_{300} = \frac{576000}{300^2} + 6.4 = 6.4 + 6.4 = 12.8 N/mm^2$.Ans.

Fig.15 Shows the radial pressure distribution and hoop stress distribution across the section. AB is taken a horizontal line. AC = 8N/mm^2. The variation between B and C is parabolic. The curve BC shows the variation of radial pressure across AB.

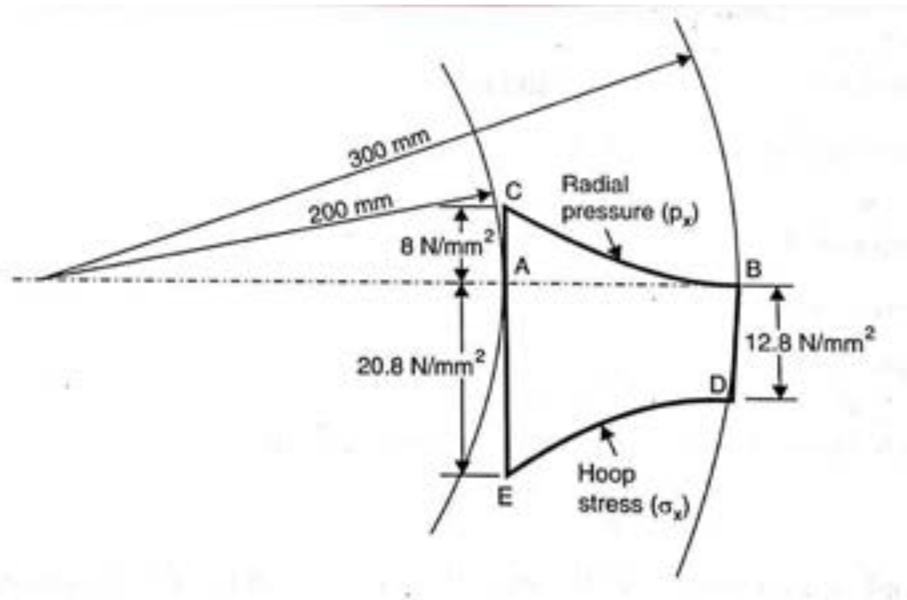


Fig. 15

The curve DE which is also parabolic, shows the variation of hoop stress across AB. Value BD = 12.8 N/mm^2 and AE = 20.8 N/mm^2 . The radial pressure is compressive whereas the hoop stress is tensile.

8) A cylindrical vessel is 2m diameter and 5m long is closed at ends by rigid plates. It is subjected to an internal pressure of 4N/mm^2 of the maximum principal stress is not to exceed 210N/mm^2. Find the thickness of the shell. Assume E=2×10^{5}N/mm^2 and poisons ratio=0.3, find the change in diameter, length and volume of the shell. [MAY/JUNE 2016-8 marks]

Given data:

Diameter, d=2m=2000mm

Length, l=5m=5000mm

Initial pressure, p=4N/mm^2

Maximum principal stress means the circumferential stress=σ_c=210N/mm^2

Young modulus =E=2×10^{5}N/mm^2

Poisons ratio=u=0.3

To find:

1.) Thickness of the shell (t)

2.) Change in diameter (∫d)

3.) Change in length and (∫ℓ)

4.) Change in volume (∫v)

Solution:

$$\sigma_c = \frac{pd}{zt}$$

$$t = \frac{pd}{2\times\sigma_c} = \frac{4\times 2000}{2\times 210} = 19.047\text{mm}$$

Change in diameter (∫d)

$$\int d = \frac{pd^2}{2tE}\left[1-\frac{1}{2}\times\mu\right]$$

$$= \frac{4\times 2000^2}{2\times 19.047\times 2\times 10^5}\left[1-0.5\times 0.3\right]$$

$$\int d = 1.785\text{mm}$$

Change in length (∫ℓ)

$$\int \ell = \frac{pd\ell}{2tE}\left[\frac{1}{2}-\mu\right]$$

$$= \frac{4\times 2000\times 5000}{2\times 19.047\times 2\times 10^5}\left[\frac{1}{2}-0.3\right]$$

$$\int \ell = 1.050\text{mm}$$

Change in volume (∫v)

$$\frac{\int v}{v} = \frac{pd}{2tE}\left[\frac{5}{2}-2\times\mu\right] = \frac{4\times 2000}{2\times 19.047\times 2\times 10^5}\left[\frac{5}{2}-2\times 0.3\right]$$

$$\boxed{\int v / v = 1.995\times 10^{-3}\,\text{mm}^3} \qquad \left[V = \frac{\pi}{4}\times d^2\times L\right]$$

$$\int v = 1.995\times 10^{-3}\times\frac{\pi}{4}\times 2000^2\times 5000$$

$$\boxed{\int v = 313121500\text{mm}^3}$$

9) A spherical sheet of 1.50m internal diameter and 12mm shell thickness is subjected to pressure of 2N/mm^2. Determine the stress induced in the material of the shell [APR/-MAY/JUNE 2016-8marks]

Given data:

Internal diameter, d=1.5m=1500mm

Shell thickness, t=12mm

Pressure, P=2N/mm^2

To find:

(1) Stress induced in the material of shell

$$\sigma_1 = \frac{p}{4t}$$
$$= \frac{2 \times 1500}{4 \times 12}$$
$$= 62.5 N/mm^2$$

10) A spherical shell of internal diameter 1.2m and of thickness 12mm is subjected to an internal pressure of 4N/mm^2. Determine the increase in diameter and increase in volume. Take E=2×10^{5}N/mm^2 and μ=0.33. [APR.MAY/JUNE 2016] 8marks

Given data:

Internal diameter of spherical shell, d=1.2m=1200mm

Thickness of spherical shell, t=12mm

Internal pressure, P=4N/mm^2

Young's modulus, E=2×10^{5}N/mm^2

Poisons ratio =μ= $\frac{1}{m} = 0.33$

To find:

(i) Increase in diameter, δd

(ii) Increase in volume, δv.

Change in diameter: (✿d)

$$\delta d = \frac{pd^2}{4tE}\left[1 - \frac{1}{m}\right]$$
$$= \frac{4 \times 1200^2}{4 \times 12 \times 2 \times 10^5}\left[1 - 0.33\right]$$
$$\boxed{\delta d = 0.402mm}$$

Change in volume (δv)

$$\delta v = v \times ev$$
$$= v \times \frac{3pd}{4tE}\left[1-\frac{1}{m}\right]$$
$$= \frac{\pi d^2}{6} \times \frac{3pd}{}\left[\quad -\frac{1}{m}\right]$$
$$= \frac{\pi pd^4}{8tE}[1-0.33]$$
$$= \frac{3.14 \times 4 \times 1200^4}{8 \times 12 \times 2 \times 10^5}[1-0.33]$$
$$\boxed{\delta = 908{,}841.6 mm^3}$$

Result:

1) Change in diameter =δd=0.402mm

2.) Change in volume =δv =908841.6mm

11) A steel cylinder of 300mm external diameter is to be shrunk to another steal cylinder of 150mm internal diameter. After shrinking the diameter at the function is 250mm and radial pressure at the common function is 28N/mm^2. Find the original difference in radial function. Take E=2×10^{5}N/mm^2 [Apr/May 2016-8 marks]

Given:

External diameter of outer cylinder =300mm

Radius of outer cylinder = r_2=150mm

Internal diameter of inner cylinder =150mm

Radius of inner cylinder =r_1=75mm

Diameter at the function =250mm

∴radius at the function =r^*=125mm

Radial pressure at the function, P^*=N/mm^2

Young modulus =E=2×10^{5}N/mm^2

Original difference of radius at the function = $\frac{2r^*}{E}(a_1 - a_2) - - - (1)$

Find the values of a_1 and a_2 using the lame's equation.

For outer cylinder

$$P_x = \frac{b_1}{x_1^2} - a_1$$

(i) At function x=r^* =125mm and P^*=28N/mm^2

(ii) At x=150mm, P_x=0

Substitute in above equation, we get

$$28 = \frac{b_1}{125^2} - a_1 = \frac{b_1}{15625} - a_1 ----(2)$$

$$0 = \frac{b_1}{150} - a_1 = \frac{b_1}{22500} - a_1 -----(3)$$

solving equation (2)×(3) weget

$$b_1 = 1432000 \qquad a_1 = 63.6$$

For inner cylinder

$$P_x = \frac{b_2}{x^2} - a_2$$

(i) At function x=r* =125m $P_x = P^* = 28N/mm^2$

(ii) At x=75mm, $P_x = 0$

Substitute these two condition ion above equation

$$28 = \frac{62}{75^2} - a_2 = \frac{b_2}{15625} - a_2 -----(4)$$

$$0 = \frac{b_2}{75^2} - a_2 = \frac{b_2}{15625} - a_2 ------(5)$$

solving equation (4)& (3) weget

$b_2 = -246100$

$a_2 = -43.75$

substitute the valuies of a_2 & a_1 in equation

$$= \frac{2r^*}{E}(a_1 - a_2)$$

$$= \frac{2\times125}{2\times105} \qquad [63.6-(-43.75)]$$

$$= \frac{125}{105}\times107.35$$

$$= 0.13mm$$

12) Calculate (i) the change in diameter (ii) Change in length and (iii) Change in volume of a thin cylindrical shell 100cm diameter, 1cm thick and 5m long, when subjected to internal pressure of $3N/mm^2$. Take the value of $E=2\times10^5 N/mm^2$ and poison's ratio, μ=0.3 (Nov/Dec 2017)[Nov/Dec 2016][13 marks] [Nov/Dec 2015]

Given data:

Diameter of cylindrical shell, (d) =100cm =1000mm

Thickness of shell (t) =1cm=10mm

Length of the shell (ℓ) =5m=5000mm

Internal pressure =$P=3N/mm^2$

Young modular=$E=2\times10^5 N/mm^2$

Poison's ratio =μ=0.3

Solution:

Longitudinal stress, $\sigma_l = \frac{pd}{4t} = \frac{3\times1000}{4\times10} = 75$

$$\boxed{\sigma_l = 75N/mm^2}$$

Hoop stress, $\sigma_c = \frac{pd}{2t} = \frac{3\times1000}{2\times10} = 150$

$$\boxed{\sigma_c = 150N/mm^2}$$

(i) Change in diameter

$$\delta d = \frac{pd^2}{2tE}(1-\frac{1}{2m})$$

$$= \frac{3\times1000^2}{2\times10\times2\times10^5}\left[1-\frac{1}{2}\times0.3\right]$$

$$\boxed{\delta d = 0.637mm}$$

(ii) Change in length (✿ℓ)

$$\delta\ell = \frac{pdL}{2tE}\left[\frac{1}{2}-\mu\right]$$

$$= \frac{3\times1000\times5000}{2\times10\times200\times10^3}\quad[0.5-0.3]$$

$$\boxed{\delta\ell = 0.75mm}$$

(iii) Change in volume ,

$$\delta v = v\times\frac{pd}{2tE}\left(\frac{5}{2}-\frac{2}{m}\right)$$

$$\text{Volume, } v = \frac{\pi}{4}\times d^2\times\ell$$

$$= \frac{3.14}{4}\times1000^2\times5000$$

$$= 39.25\times10^8\ mm^3$$

$$\delta v = 39.25\times10^8\times\frac{3\times1000}{2\times10\times2\times\times10^5}\left[\frac{5}{2}-2(0.3)\right]$$

$$\delta v = 5593125mm^3$$

Result:

(i) Change in diameter (δd) =0.637mm

(ii) Change in length ($\delta\ell$) =0.75mm

(iii) Change in length (δv) =5593125mm^3

13) Calculate the thickness of metal necessary for a cylindrical shell of internal diameter 16mm ton with slant of internal pressure of 25mN/m$_2$. If maximum permissible shell stress is 125MN/m$_2$. [NOV/DEC-2016]

Given data:

Internal diameter, d=160mm.

Internal pressure, P=25MN/m^2 =25N/Mm^2

Maximum permissible shell stress =125MN/m^2=125N/mm^2

To find:

Thickness (t)

Solution:

$$\sigma_{max} = \frac{pd}{8t}$$
$$125 = \frac{25\times160}{8\times t}$$
$$t = \frac{25\times160}{125\times8}$$
$$t = 4mm$$

Thicknessof cylinderricalshellis 4mm

14) A boiler is subjected to an internal steam pressure of 2N/mm^2. The thickness of boiler plate is 2.6cm and permissible tensile stress is 120N/mm^2. Find the maximum diameter, when efficiency of longitudinal joint is 90% and that of circumference joint is 40%. [NOV/DEC 2015 , 16marks]

Given data:

Internal steam pressure, P=2N/mm^2

Thickness boiler plate, t=2.6cm & 26mm

Permissible tensile stress (σ) =120N/mm^2

Efficiency of longitudinal joint, η_l =90% =0.90

Efficiency of circumferences joint, η_c =40% =0.40

In case of joint the permissible stress may be longitudinal (or) circumferential stress.

To find:

Maximum diameter (d)

Solution:

Maximum diameter of circumference stress

$$\sigma_c = \frac{pd}{2t\eta_l}$$
$$120 = \frac{2\times d}{2\times0.90\times2.6}$$
$$d = \frac{120\times2\times0.90\times26}{2}$$
$$d = 2808mm$$

Maximum diameter for longitudinal stress

$$\sigma_2 = \frac{pd}{4t \times \eta_c}$$

$$120 = \frac{2 \times d}{4 \times 26 \times 0.40}$$

$$d = \frac{120 \times 4 \times 0.40 \times 26}{2}$$

$$d = 2496\,\text{mm}$$

The longitudinal (or) circumferential stresses induced in the material directly proportional to diameter (d). Hence the stress induced will be less if the value of 'd' is less. Hence take the minimum value of diameter.

Hence, diameter (d) =249.6cm

15) A thin cylindrical shell 2.5 long has 700 mm internal diameters and 8mm thickness, if the shell is subjects to an internal pressure of 1Mpa, find

(i) The hoop and longitudinal stresses developed

(ii) Maximum shell stress induced and

(iii) The change in diameter, length and volume. Take modulus of elasticity of the wall material as 200Gpa and poison's ratio as 0.3 [AP/MAY 2015- 16 marks]

Given data:

Length of cylindrical shell, ℓ=2.5m=2500mm

Internal diameter ‡ d, =700mm

Thickness of shell, t=8mm

Internal pressure, P=1mpa=1N/mm^2

Modulus of elasticity = E=200Gpa=200×10^{3}N/mm^2

Poison's ratio = μ=0.3

To find:

1.) Hoop stress and longitudinal stress

2.) Maximum shell stress induced.

3.) Change in diameter, (δd)

4.) Change in volume,(δv)

5.) Change in length (δ ℓ)

Solution:

Hoop stress, $\sigma_c = \frac{pd}{2t} = \frac{1 \times 700}{2 \times 8} = 43.75$

$$\sigma_c = 43.75\,\text{N/mm}^2$$

Longitudinal stress, $\sigma_\ell = \frac{pd}{4t} = \frac{1\times 700}{4\times 8} = 21.87$

$\sigma_\ell = 21.875 N/mm^2$

Change in diameter (δd)

$$\delta d = \frac{pd^2}{2tE}\left[1-\frac{\mu}{2}\right]$$

$$= \frac{1\times 700^2}{2\times 8\times 200\times 0^3}\left[1-\frac{0.3}{2}\right]$$

$$\boxed{\delta d = 0.130 mm}$$

Change in length (δℓ)

$$\delta\ell = \frac{pdL}{2tE}\left[\frac{1}{2}-\mu\right]$$

$$= \frac{1\times 700\times 2500}{2\times 8\times 200\times 10^3}\left[0.5-0.3\right]$$

$$\boxed{\delta\ell = 0.109 mm}$$

Change in volume (δv)

$$\delta v = \frac{pdv}{2tE}\left[\frac{5}{2}-\frac{2}{m}\right]$$

original volume, $V = \frac{\pi}{4}d^2\times\ell = \frac{\pi}{4}\times 700^2\times 2500$

$$V = 961625000 mm^3 = 96.16\times 10^7 mm^3$$

$$\delta_V = \frac{1\times 700\times 96.16\times 10^7}{2\times 8\times 200\times 10^3}\left[\frac{5}{2}-2\times 0.3\right]$$

$$\boxed{\delta v = 399665 mm^3}$$

Maximum shell stress induced (σmax)

$$\sigma_{max} = \frac{pd}{t} = \frac{1\times 700}{8\times 8} = 10.937 N/mm^2$$

$$\sigma_{max} = 10.937 N/mm^2$$

Result:

1.) Hoop stress σ_c =43.75N/mm^2

2.) Longitudinal stress, σ_ℓ=21.875N/mm^2

3.) Maximum shell stress, σ_{max} =10.937N/mm^2

4.) Change in diameter, δd=0.130mm

5.) Change in length, δℓ=0.109mm

6.) Change in length, δv=399665mm^3

16) A thick cylinder with external diameter 320mm and internal diameter 160mm is subjected to an internal pressure of 8N/mm^2. Draw the variation of radial and hoop stresses in the cylinder wall. Also determine the maximum shell stress in the cylinder wall. [APR/MAY- 2015 -16marks]

Given data:

Internal diameter, d_1 =160mm

External diameter, d_2 =320mm

Internal radius, r_1 =80mm

External radius, r_2 =160mm

Internal pressure, P_1 =[8N/mm^2

To find:

1.) To draw variation of radial and hoop stress.

2.) The maximum shell stress in the cylinder.

Solution: we know that by lame's equation

$$\sigma_r = \frac{b}{r^2} - a -----(1)$$

$$\sigma_c = \frac{b}{r^2} + a -----(2)$$

At, $r=r_1$ =80, and $\sigma_r = P_1$ =8N/mm^2

$R=r_2$ =160mm and $\sigma_r = P_2 = 0$

Substitute in equation (1)

$$8 = \frac{b}{(80)^2} - a \Rightarrow 8 = 1.562\times10^{-4} b - a ----(3)$$

$$0 = \frac{b}{(160)^2} - a \Rightarrow 0 = 3.9\times10^{-5} b - a -----(4)$$

Equation (3) and (4) becomes

$$a - 1.562\times10^{-4} b = -8 ----(5)$$

$$a - 3.9\times10^{-4} b = 0 -----(6)$$

Solving equation (5) and (6)

A=13.34

B=34217.27

Substitute values of a and b in equation (2)

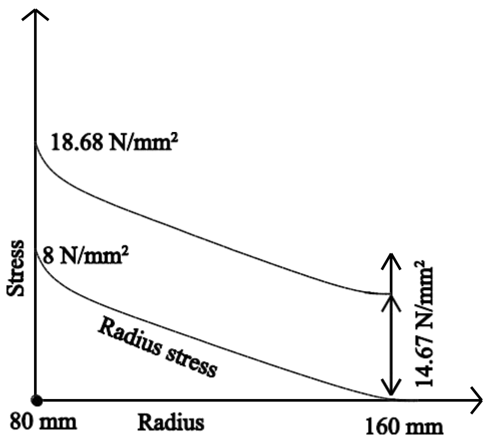
$$\sigma_c = \frac{b}{(80)^2} + a \Rightarrow = \frac{34217.27}{80^2} + 13.34$$

$\sigma_c = 18.686 N / mm^2$

At $r = r_2 = 160mm$

$$\sigma_c = \frac{b}{(160)^2} + a \Rightarrow \frac{34217.27}{(160)^2} + 13.34$$

$\sigma_c = 14.67 N / mm^2$



17) Desire relations for change in dimensions and change in volume of a thin cylinder subjected to internal pressure P. (May / June 2017) [NOV/DEC 2014]-16marks

Due to Internal pressure, the cylindrical shells are subjected to lateral and linear strain. Thus the change in dimensions such as length, diameter may increases.

We know that

$$e_c = \frac{\delta d}{d} = \frac{\sigma_c}{E} - \frac{\sigma_a}{mE}$$

Where , δd – change in diameter

$\frac{1}{m}$ = poison's ratio

E – young's Modulus

Circumferential stress,

$$e_c = \frac{pd}{2tE} - \frac{pd}{\mu t mE}$$

$$\boxed{e_c = \frac{pd}{2tE}\left[1 - \frac{1}{2m}\right]}$$

Change in diameter,

$$\delta d = e_c \times d$$

$$\delta d = \frac{pd^2}{2tE}\left[1 - \frac{1}{2m}\right]$$

$$e_a = \frac{\delta \ell}{\ell} = \frac{\sigma_a}{E} - \frac{\sigma_c}{mE}$$

Longitudinal strain, $= \frac{pd}{4tE} - \frac{pd}{2tmE}$

$$\boxed{e_a = \frac{pd}{2tE}\left[\frac{1}{2} - \frac{1}{m}\right]}$$

Change in length,

$$\delta \ell = e_a \times \ell$$
$$\delta \ell = \frac{pd\ell}{2tE}\left[\frac{1}{2} - \frac{1}{m}\right]$$

Volume strain,

$$e_v = \frac{\text{final volume} - \text{initial volume}}{\text{initial volume}}$$
$$= \frac{\frac{\pi}{4}(d + \delta d^2)(\ell + \delta \ell) - \frac{\pi}{4} d^2 \ell}{\frac{\pi}{4} d^2 \ell}$$

By neglecting higher order terms of $\delta \ell$ and δd

$$e_v = \frac{2\delta d}{d} + \frac{\delta \ell}{\ell}$$
$$= 2e_c + e_a$$
$$= \frac{2pd}{2tE}\left(1 - \frac{1}{2m}\right) + \frac{pd}{2tE}\left(\frac{1}{2} - \frac{1}{m}\right)$$
$$= \frac{pd}{2tE}\left[2 - \frac{2}{2m} + \frac{1}{2} - \frac{1}{m}\right]$$
$$= \frac{pd}{2tE}\left[2 + \frac{1}{2} - \frac{2}{m}\right]$$
$$e_v = \frac{pd}{2tE}\left[\frac{5}{2} - \frac{2}{m}\right]$$

Change in volume,

$$\delta v = e_v \times v$$
$$= \frac{pdv}{2tE}\left[\frac{5}{2} - \frac{2}{m}\right]$$
$$\delta v = v \times \frac{\sigma_c}{E}\left(\frac{5}{2} - \frac{2}{m}\right)$$

18) Find the thickness of metal necessary for a thick cylindrical shell of internal diameter 160mm to withstand an internal pressure on 8N/mm^2. The maximum hoop stress in section is not to exceed 35N/mm^2. [NOV/DEC- 2014 -] [16 marks]

Given data:

Internal diameter, d_1 =160mm

Internal radius = $r_1 = \frac{d_1}{2} = \frac{160}{2} = 80mm$

Internal pressure,=P_1 =8N/mm^2

Maximum hoop stress =σ_c =35N/mm^2

To find:

Thickness of metal (t)

Solution:

Thelame's equation 's are

$$\sigma_r = \frac{b}{r^2} - a -----(1)$$

$$\sigma_c = \frac{b}{r^2} + a -----(2)$$

At $r = r_i = 80mm$ and $\sigma_r = P_1 = 8N/mm^2$

$$(\sigma_c)_{max} = 35N/mm^2$$

substituting in equation (1) and (2), weget

$$8 = \frac{b}{(80)^2} - a \Rightarrow 8 = 1.56\times10^{-4} b - a ----(3)$$

$$35 = \frac{b}{(80)^2} + a \Rightarrow 35 = 1.56\times10^{-4} b + a ----(4)$$

Equation (3) and (4) becomes

$$a - 1.56\times10^{-4} b = -8 -----(5)$$

$$-a - 1.56\times10^{-4} b = -35 ----(6)$$

Solving equation (5) and (6), we get

$$(5)\times1 \qquad -a + 1.56\times10^{-4} b = -8$$

$$(6)\times1 \qquad -a - 1.56\times10^{-4} b = -35$$

$$-2a = -27$$

$$\boxed{a = 13.5}$$

Substitute (a) value in equation (5)

$$13.5 - 1.56 \times 10^{-4} b = -8$$

$$-1.56 \times 10^{-4} b = -8 - 13.5$$

$$-1.56 \times 10^{-4} b = -21.5$$

$$b = \frac{21.5}{1.56 \times 10^{-4}}$$

$$\boxed{b = 137.82}$$

19) A cylindrical shell in diameter and 3m length is subjected to an internal pressure of 2MPa. Calculate the maximum thickness if the stress should not exceed 50MPa. Find the change in diameter and volume of shell. Assume poisson's ratio of 0.3 and young's modulus of 200kN/mm^2. [MAY/JUNE -2014-16marks]

Given data:

Diameter of cylindrical shell, d=1m=1000mm

Length of cylindrical shell, ℓ=3, m=3000mm

Internal pressure, P=2Mpa = 2N/mm^2

Maximum stress,. σ_c =50Mpa =50N/mm^2

Young's modulus =E =200KN/mm^2 =2×10^{5}N/mm^2

Poison's ratio, $\frac{1}{m} = 0.3$

To find:

(i) Change in diameter, ✿d

(ii) Change in volume, ✿v.

Solution:

$$\sigma_c = \frac{pd}{2t} = \frac{2 \times 1000}{2 \times t}$$

Hoop stress, $50 = \frac{2 \times 1000}{2 \times t}$

$$t = 20\text{mm}$$

Change in diameter, δd

$$\delta d = \frac{Pd^2}{2tE}\left[1 - \frac{1}{2m}\right]$$

$$= \frac{2 \times (1000)^2}{2 \times 20 \times 2 \times 10^5}\left[1 - \frac{1}{2} \times 0.3\right]$$

$$\delta d = 0.2125\text{mm}$$

Change in volume,

$$\delta v = \frac{pdv}{2tE}\left[\frac{5}{2} - \frac{2}{m}\right]$$

Volume of cylinder, $V = \frac{\pi}{4} d^2 \times \square$

$$= \frac{\pi}{4}(1000)^2 \times 3000$$

$$= 2.355 \times 10^9 \text{ mm}^3$$

$$\delta v = \frac{Pdv}{2tE}\left[\frac{5}{2} - \frac{2}{m}\right]$$

$$= \frac{2 \times 1000 \times 2.35 \times 10^9}{2 \times 20 \times 2 \times 10^5} \quad [2.5 - 0.6]$$

$$\boxed{\delta v = 118625 \text{mm}^3}$$

Result:

(i) Thickness of cylinder. t=20mm

(ii) Change in diameter. δd=0.2125mm

(iii) Change in volume, δv=1118625mm^3.