STELLA MARY'S COLLEGE OF ENGINEERING

(Accredited by NAAC, Approved by AICTE - New Delhi, Affiliated to Anna University Chennai)

Aruthenganvilai, Azhikal Post, Kanyalumari District, Tamilnadu - 629202.

CE8394 FLUID MECHANICS AND MACHINERY

(Anna University: R2017)

Prepared By

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DEPARTMENT OF MECHANICAL ENGINEERING

COURSE MATERIAL

REGULATION	**2017**
YEAR	**II**
SEMESTER	**03**
COURSE NAME	**FLUID MECHANICS AND MACHINERY**
COURSE CODE	**CE8394**
NAME OF THE COURSE INSTRUCTOR	**Mr. S.R. RAJKUMAR**

SYLLABUS:

UNIT I FLUID PROPERTIES AND FLOWCHARACTERISTICS 12

Units and dimensions- Properties of fluids- mass density, specific weight, specific volume, specific gravity, viscosity, compressibility, vapor pressure, surface tension and capillarity. Flow characteristics – concept of control volume - application of continuity equation, energy equation and momentum equation.

UNIT II FLOWTHROUGHCIRCULAR CONDUITS 12

Hydraulic and energy gradient - Laminar flow through circular conduits and circular annuli- Boundary layer concepts – types of boundary layer thickness – Darcy Weisbach equation –friction factor- Moody diagram- commercial pipes- minor losses – Flowthrough pipes in series and parallel.

UNIT III DIMENSIONAL ANALYSIS 12

Need for dimensional analysis – methods of dimensional analysis – Similitude –types of similitude - Dimensionless parameters- application of dimensionless parameters – Model analysis.

UNIT IV PUMPS 12

Impact of jets - Euler’s equation - Theory of roto-dynamic machines – various efficiencies– velocity components at entry and exit of the rotor- velocity triangles - Centrifugal pumps– working principle - work done by the impeller - performance curves - Reciprocating pump- working principle – Rotary pumps –classification.

UNIT V TURBINES **12**

Classification of turbines – heads and efficiencies – velocity triangles. Axial, radial and mixed flow turbines. Pelton wheel, Francis turbine and Kaplan turbines- working principles - work done by water on the runner – draft tube. Specific speed - unit quantities – performance curves for turbines – governing of turbines.

TEXT BOOKS :

1. Modi P.N. and Seth, S.M. "Hydraulics and Fluid Mechanics", Standard Book House, NewDelhi 2013.

REFERENCES:

1. Graebel.W.P, "Engineering Fluid Mechanics", Taylor & Francis, Indian Reprint, 2011
2. Kumar K. L., "Engineering Fluid Mechanics", Eurasia Publishing House(p) Ltd., New Delhi 2016
3. RobertW.Fox, AlanT. McDonald, Philip J.Pritchard, "Fluid Mechanics and Machinery", 2011.
4. Streeter, V. L. andWylieE. B., "Fluid Mechanics", McGrawHill Publishing Co. 2010

Course Outcome Articulation Matrix

	Program Outcome												***PSO***		
Course Code / CO No	***1***	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***	***10***	***11***	***12***	***1***	***2***	***3***
CE8394 / C203.1	3	3	3	1	0	1	2	1	0	0	0	2	2	3	2
CE8394 / C203.2	3	2	3	2	0	2	2	1	0	0	0	2	2	3	2
CE8394 / C203.3	3	3	3	3	0	2	0	1	0	0	0	2	2	3	2
CE8394 / C203.4	3	3	3	1	0	2	2	1	0	0	0	3	3	3	2
CE8394 / C203.5	3	3	3	1	0	2	2	1	0	0	0	2	2	3	2
Average	3	3	3	2	0	2	2	1	0	0	0	2	2	3	2

UNIT - I

PROPERTIES OF FLUIDS

Fluid mechanics:

It is the branch of science which deals which behaviour of fluids at rest as well as in motion.

Fluid statics: fluid is a substance which is capable of flowing fluid

Study of fluids at rest.

Fluid kinematics:

Study of fluids in motion where pressure forces are not considered.

Fluid dynamics:

Study of fluids in motion where pressure forces are considered.

Density (or) Mass density:

It is defined as ratio of mass of a fluid to its volume.

Unit kg/m^3.

$$\rho = \frac{\text{Mass of fluid}}{\text{Volume of fluid}}$$

Specific weight (or) weight density:

It is ratio between weight of a fluid to its volume.

$$w = \frac{\text{weight of fluid}}{\text{Volume of fluid}} = \frac{\text{mass of fluid} \times g}{V} = \rho g$$

Specific volume:

$$\text{specific volume} = \frac{\text{volume of fluid}}{\text{mass of fluid}} = V/m$$

$$= \frac{1}{\rho}$$

Specific gravity (or) relative density:

$$S = \frac{\text{Density of fluid}}{\text{Density of water}} = \frac{\rho_{liq}}{\rho_{water}}$$

Density of water is 1000 kg/m³.

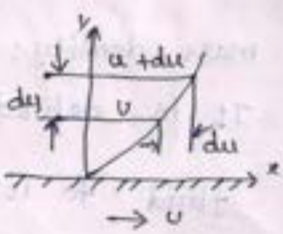
Specific gravity of mercury is 13.6

$S_m = \frac{\rho_m}{1000}$ → density of mercury

Viscosity:

Viscosity is the property of a fluid which offers resistance to movement of one layer of fluid over another adjacent layer of fluid.

Shear stress is directly proportional to viscosity with respect to y.



Unit = NS/m^2 Dynamic

1 pascal = N/m^2

= pascal × sec.

$$\tau \propto \frac{du}{dy}$$

$$\tau = \mu \cdot \frac{du}{dy} , \quad \frac{\tau}{du/dy} = \mu \qquad \sigma = P/A$$

$$\frac{F/A}{\frac{1}{t}\times\frac{1}{t}} = \frac{N/m^2}{m/s \times \frac{1}{m}} = NS/m^2$$

1 Paise = $\frac{1}{10}$ NS/m^2

Temperature increases → viscosity decrease

Temperature decrease → Viscosity increase

Kinomatic viscosity:

$$\text{Kinematic viscosity} = \frac{\text{Dynamic viscosity}}{\text{Density}} = m^2/s$$

$$= \frac{\mu}{\rho}$$

Objectives:

The application of conversation to flow through pipes and hydraulic machines are studied.

To understand importance of dimensional analysis.

To understand importance of various types of flow in pumps & turbine.

FLUID PROPERTIES & FLOW CHARACTERISTIC

Units and dimensions - Properties of fluids mass density, specific weight, specific volume, specific gravity, viscosity, Compressibility, vapour Pressure, Surface tension & capillarity. Flow characteristic concept of control volume - application of continuity equation, energy equation & momentum equation.

Compressibility & bulk modulus:

$$\text{Bulk modulus} = \frac{\text{Increase of Pressure}}{\text{Volumetric strain}}$$

$$\text{compressibility} = 1/K$$

Cohesion:

'cohesion' means intermolecular attraction between molecular of same liquid.

Adhesion:

"Adhesion" means attraction between the molecules of a liquid & the molecular liquid. The Property enables a liquid to stick to another body.

Problem:

$m = 5$ kg

$V = 3\ m^3$

$\rho = 1.67\ kg/m^3$

Specific $= 0.6\ m^3/kg$

Surface tension:

It is defined as tensile force acting on surface of a liquid in contact with a gas (or) on the surface between two immisible liquids such as that are contact surface behaves like a membrance under tension.

Surface tension on liquid droplet:

σ = surface tension of liquid

p = Pressure intensity inside droplets

d = diameter of droplet

Pressure force = Surface tension

$$p \times \frac{\pi}{4} d^2 = \sigma \times \pi d$$

$d \rightarrow$ diameter decrease pressure intensity increase

Hollow surface:

$$P = \frac{8\sigma}{d}$$

Liquid jet:

$$P = \frac{\sigma \times 2l}{L \times d}$$

capillarity:

It is defined as phenomenon of rise (or) fall of a liquid surface in a small tube relative to adjacent general level of when the tube is held vertically. The rise of liquid surface is capillary rise. which fall is defined as capillary depression.

$$S = \frac{\rho_{mercury}}{\rho_{water}}$$

$$13.6 = \frac{\rho_{mer}}{1000}$$

$$\rho_{mercury} = 13600 \text{ kg/cm}^3$$

Expression for capillary rise:

σ = Surface tension of liquid

θ = angle of contact between liquid & glass tube

The weight of liquid of height h in tube,

$$= \frac{\pi d^2}{4} \times \rho \times g \times h \quad \text{—} \; ①$$

Surface Tension force = σ x circumference x angle

$$= \sigma \times \pi d \times \cos\theta \quad \text{—} \; ②$$

eqn ① = ②

$$\frac{\pi d^2}{4} \times \rho \times g \times h = \sigma \times \pi d \times \cos\theta$$

$$h = \frac{\sigma \times \pi d \times \cos\theta \times 4}{\pi d^2 \times \rho \times g \times h}$$

$$= \frac{4\sigma \times \cancel{\pi} \times \cancel{d} \cdot \cos\theta}{\cancel{\pi} d^{\cancel{2}} \times \rho \times g}$$

$$\boxed{h = \frac{4\sigma \cdot \cos\theta}{\rho g d}}$$

Expression for capillary fall:

$$h = \frac{4\sigma\cos\theta}{\rho g d} \quad (\theta = 128^\circ \text{ for mercury})$$

The weight of liquid of height tube

$$= \frac{\pi d^2}{4} \times \rho \times g \times h$$

Surface force = c x angle of contact

$$= \sigma \times \pi d \times \cos\theta$$

$$\boxed{h = \frac{4\sigma \cdot \cos\theta}{\rho g d}}$$

Problems :

1) Calculate capillary rise in a glass tube of 2.5 mm dia when immersed vertically in a) water & b) mercury. Take Surface tension $\sigma = 0.0725\ N/m^2$ for water & $\sigma = 0.52\ N/m^2$ for mercury in contact with air. The specific gravity for mercury is 13.6 and angle of contact $130°$.

Data :

$d = 2.5\ mm$

$\Rightarrow 2.5 \times 10^{-3}\ mm$

$\sigma = 0.0725\ N/m^2$

$\sigma_{water} = 0.0725\ N/m^2$

$\sigma_{mercury} = 0.52\ N/m^2$

Specific gravity for mercury $= 13.6$

Angle of contact $\theta_m = 130°$

Find :

i) $h_1 = ?$ for water

ii) $h = ?$ for mercury

Soln :

For water,

$$h = \frac{4\sigma \cos\theta}{\rho g d} = \frac{4 \times 0.0725 \times 1}{1000 \times 9.81 \times 2.5 \times 10^{-3}} = \boxed{0.01182\ m}$$

For mercury,

$$h = \frac{4\sigma \cos\theta}{\rho g d} = \frac{4 \times 0.52 \times \cos 130}{13.600 \times 13.6 \times 2.5 \times 10^{-3}} = \boxed{-0.0028\ m}$$

vapour pressure & cavitation:

A change from liquid State to gaseous state is known as "Vaporisation". when vaporisation takes place 20°c & Pressure molecular and top of the vessel. This Pressure iscalled "vapour Pressure".

Problem:

1) A flat plate area of $1.5 \times 10^6\ mm^2$ is pulled with a speed of $0.4\ m/s$ relative to another Plate located at a distance of $0.15\ mm$ from it. Find & force and power required to maintain speed If fluid seperating them is having viscosity as 1 poise.

Data:

$$A = 1.5 \times 10^6\ mm^2$$

$$du = 0.4\ m/s$$

$$dy = 0.15\ mm \Rightarrow 0.15 \times 10^{-3}\ m$$

$$u = 1\ \text{Poise} \Rightarrow 1/10\ N.s/m^2$$

Find:

i) Force

(ii) Power

Sol:

$$\tau = \frac{\text{shear force}}{\text{area}}$$

$$\tau \times \text{area} = \text{shear force}$$

$$\tau = u . du/dy = 1/10 \times \frac{0.4}{0.15 \times 10^{-3}}$$

$$\tau = 266.66\ N/m^2$$

Shear force $= \tau \times$ area

$= 266.66 \times 1.25$

Shear force $= 400\,N$

(i) Power.

Power $= F \times u$

$= 400 \times 0.4$

Power $= 160\,W$

8) Calculate the specific weight, density and specific gravity of 1 litre of a liquid which weight 7N.

Data:

Volume $= 1$ lit $\Rightarrow \frac{1}{1000}\,m^3$

Weight $= 7\,N$.

1 lit $= \frac{1}{1000}\,m^3$

(or)

1 lit $= 1000\,cm^3$

Find:

i) Specific weight

ii) Density

iii) Specific gravity

Sol:

i)

$$\text{Specific weight} = \frac{\text{Weight}}{\text{Volume}} = \frac{7}{1/1000} = 7000\ N/m^3$$

ii)

$$\text{Density } \rho = \frac{w}{g} = \frac{7000}{9.81} = 713.5\ kg/m^3$$

iii)

$$\text{Specific Gravity} = \frac{\text{Density of liquid}}{\text{Density of water}}$$

$$= \frac{713.5}{1000} = 0.7135$$

$\because$ Density of water $= 1000\ kg/m^3$

KINEMATICS OF FLOW & IDEAL FLOW

CHAPTER -5

Kinematics :

Kinematics is a branch of science deals with motion of particular without considering force.

Two methods :

1) Lagrangian method - single fluid particle

2) Eulerian method - At a point.

Types of fluid flow :

i) Steady and Unsteady flow

ii) Uniform and non-uniform flow

iii) Laminar and tubulant flow

iv) Compressible and incompressible flow

v) Rotational and irrotational flow

vi) One, two and three - dimensional flows

i) steady & unsteady flow :

steady → do not change with respect to time (Pressure, density & volume)

Unsteady → change with respect to time (Pressure, density & volume)

ii) Uniform & non-uniform flow :

Uniform → do not change with respect to space to the (Pressure, volume, density).

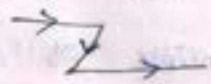
non-uniform → change with respect to space to the (Pressure, volume & density)

(iii) Laminar & Turbulant flow:

Laminar → Steam lines are straight & Parallel flow.

 fluid flow

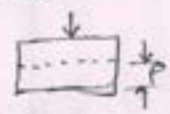
Turbulant → Fluid particulars move the zig-zag way.

 fluid flow

(iv) Compressible & Incompressible flow:

Compressible → Change in density

$\rho \neq$ constant.



Incompressible → Does not change in density. ρ = constant

v) Rotational & Irrotational flow:

Rotational → rotate about their own axis

Irrotational → do not rotate about their own axis.

Flow of discharge:

Rate of flow (or) discharge

$$Q = A+V$$

$$Q = m^3/s$$

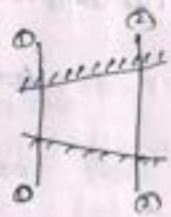
Continuity equation:

The equation based on the Principle of conservation of mass is called "continuity equation".

$$Q_1 = \rho_1 A_1 V_1$$

$$Q_2 = \rho_2 A_2 V_2$$

$$Q_1 = Q_2$$



The fluid of water $\rho_2 = \rho_1$

$$A_1 V_1 = A_2 V_2$$

where,

ρ_1 = Density at section 1-1

A_1 = Area of pipe at section 1-1

V_1 = Average velocity at cross section 1-1

Q = discharge

Problems :

1) The diameter of a pipe at the section 1-2 are 10cm & 15cm respectively. Find the discharge the pipe if velocity of water flowing through a pipe at section 1 as 5 m/s. Determine also the velocity at section 2.

Data :

$D_1 = 10\ cm$ $\Rightarrow 1cm = 100m$

$\Rightarrow 0.1\ m$

$D_2 = 15\ cm$

$\Rightarrow 0.15\ m$

$V_1 = 5\ m/s$

Find :

i) $V_2 = ?$

ii) $Q_1 = ?$, $Q_2 = ?$

Solution :

$$A_1 = \pi/4 (d_1^2) = \pi/4 (0.1)^2 = 7.83 \times 10^{-3}\ m^2$$

$$A_2 = \pi/4 (D_2)^2 = \pi/4 (0.15)^2 = 0.01767\ m^2$$

$$A_1 V_1 = A_2 V_2$$

$$V_2 = \frac{A_1 V_1}{A_2} = \frac{7.83 \times 10^{-3} \times 5}{0.01767} = 2\ m/s$$

$$Q_1 = A_1 V_1$$

$$= 7.83 \times 10^{-3} \times 5$$

$$Q_1 = 0.03915\ m^3/s$$

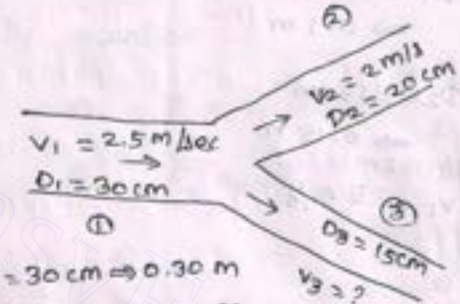
$$Q_2 = A_2 V_2$$

$$= 0.01767 \times 2$$

$$Q_2 = 0.03534\ m^3/s$$

8) A 30cm diameter pipe, conveying water, branches into two pipes of diameters 20 cm and 15cm respectively. If the average velocity in the 30 cm diameter pipe is 2.5 m/s. Find the discharge in this pipe. Also determine the velocity in 15cm pipe if the average velocity in 20cm diameter pipe is 2 m/s.

Data:



$D_1 = 30$ cm $\Rightarrow 0.30$ m

$D_2 = 20$ cm $\Rightarrow 0.20$ m

$D_3 = 15$ cm $\Rightarrow 0.15$ m

Find:

$V_3 = ?$

Solution:

$$A_1 = \pi/4 (d_1)^2$$

$$= \pi/4 (0.30)^2$$

$$A_1 = 0.07068 \ m^2$$

$$A_2 = \pi/4 (d_2)^2$$

$$= \pi/4 (0.20)^2$$

$$A_2 = 0.0314 \ m^2$$

$$A_3 = \pi/4 (d_3^2)$$

$$= \pi/4 (0.15)^2$$

$$A_3 = 0.01767 \ m^2$$

Discharge,

$$Q_1 = Q_2 + Q_3$$

$$Q_1 = A_1 \times V_1$$

$$= 0.07068 \times 2.5$$

$$\boxed{Q_1 = 0.1767 \ m^3/s}$$

$$Q_2 = A_2 \times V_2$$

$$= 0.0314 \times 2$$

$$\boxed{Q_2 = 0.0628 \ m^3/s}$$

$$\therefore Q_1 = Q_2 + Q_3$$

$$Q_3 = Q_1 - Q_2$$

$$= 0.1767 - 0.0628$$

$$\boxed{Q_3 = 0.1139 \ m^3/s}$$

$$Q_3 = A_3 \times V_3$$

$$V_3 = \frac{Q_3}{A_3}$$

$$= \frac{0.1139}{0.01767}$$

$$\boxed{V_3 = 6.44 \ m/s}$$

3) Calculate the density, spefic weight & weight of one litre of Petrol of specific gravity 0.7

Data:

$$\text{volume} = 1 \text{ lit} \Rightarrow \frac{1}{1000} = 0.01 \, m^3$$

$$\text{specific gravity} = 0.7$$

Find:

i) Density

ii) Specific weight

iii) weight

sol:

i) $$\text{Specific Gravity} = \frac{\text{Density of liquid}}{\text{Density of water}}$$

$$\text{Density of liquid} = S_g \times \rho_w$$

$$= 0.7 \times 1000$$

$$\boxed{\rho_{liq} = 700 \text{ kg/m}^3}$$

ii)

Specific weight γ

$$w = \rho \times g$$

$$= 700 \times 9.81$$

$$w = 6867 \text{ N/m}^3$$

iii) weight,

$$w = \frac{\text{weight}}{\text{volume}}$$

$$\text{weight} = w \times \text{volume}$$

$$= 6867 \times 0.01$$

$$\text{weight} = 6.867 \text{ N}$$

4) Calculate specific weight, density, specific volume & specific gravity.

$$mass = 5\ kg \quad , \quad V = 3\ m^3$$

$$Density = \frac{m}{V} = 5/3 = 1.67\ kg/m^3$$

i) $$specific\ weight = \rho \times g$$

$$= 1.67 \times 9.81$$

$$Specific\ weight = 16.3827\ N/m^3$$

ii) $$Specific\ volume = \frac{V}{m} = 3/5 = 0.6\ m^3/kg$$

iii) Specific Gravity,

$$specific\ gravity = \frac{\rho_{liq}}{\rho_{water}}$$

$$= \frac{1.67}{1000}$$

$$Specific\ gravity = 1.67 \times 10^{-3}\ kg/m^3$$

Then,

$$S_g = 0.5 \quad , \quad \rho_{liq} = ?$$

$$specific\ gravity = \frac{\rho_{liq}}{\rho_{water}}$$

$$\rho_{liq} = Specific \times \rho_{water}$$

$$= 0.5 \times 1000$$

$$\rho_{liq} = 500\ kg/m^3$$

Model Question paper : 2010

1) A tube is made of 2 capillaries of diameter 1 mm and 1.5 mm respectively. The tube is kept vertically & partially filled with water of surface Tension is 0.0736 N/m and zero is contact angle. Calculate the difference in the levels of the Menisa caused by capillarity.

Data :

Diameter, $D_1 = 1 mm \Rightarrow 0.001 m$

$D_2 = 1.5 mm \Rightarrow 0.0015 m$

Surface Tension,
$\sigma = 0.0736 N/m$

$\theta = 0$

Find :

$h = ?$

Sol :

$$h_1 = \frac{4 . \sigma . \cos\theta}{\rho g . d_1}$$

$$= \frac{4 \times 0.0736 \times \cos(0)}{1000 \times 9.81 \times 0.001}$$

$h_1 = 0.030 m$

$$h_2 = \frac{4 . \sigma . \cos\theta}{\rho g d_2}$$

$$= \frac{4 \times 0.0736 \times \cos(0)}{1000 \times 9.81 \times 0.0015}$$

$h_2 = 0.020 m$

Difference,

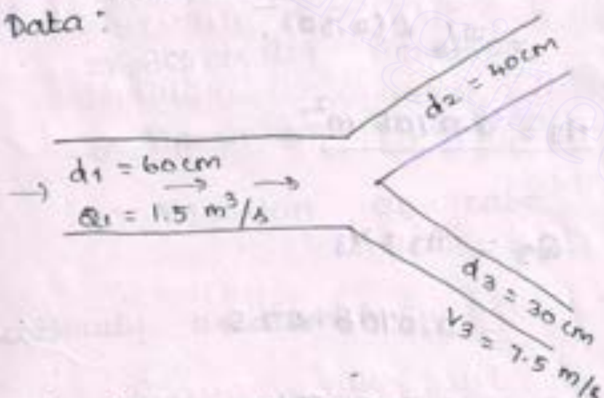
$$h_1 - h_2 = 0.030 - 0.020$$

$$h_1 - h_2 = 0.01 \text{ m}$$

$$\boxed{h_1 - h_2 = 10 \text{ mm}}$$

8) A pipe line 60cm in diameter bifurcates at a Y junction into 2 branches 40 cm and 30 cm in diameter. If the rate of flow in the main pipe is $1.5 \text{ m}^3/\text{s}$ and the mean velocity of flow in 30cm pipe is 7.5 m/s. Determine the rate of flow in 40 cm pipe.

Data:



$d_1 = 60 \text{ cm} \Rightarrow 0.60 \text{ m}$

$Q_1 = 1.5 \text{ m}^3/\text{s}$

$d_2 = 40 \text{ cm} \Rightarrow 0.40 \text{ m}$

$d_3 = 30 \text{ cm} \Rightarrow 0.30 \text{ m}$

$V_3 = 7.5 \text{ m/s}$

Rate of f

Find:

$Q_2 = ?$

Sol:

Area, $A_1 = \pi/4 (d_1^2)$

$= \pi/4 (0.6)^2$

$\underline{A_1 = 0.2874 \, m^2}$

$Q_1 = A_1 \times V_1$

$V_1 = \frac{Q_1}{A_1}$

$= \frac{1.5}{0.2874}$

$\underline{V_1 = 5.219 \, m/s}$

$A_3 = \pi/4 \, (d_3)^2$

$= \pi/4 \times (0.30)^2$

$\underline{A_3 = 0.0706 \, m^2}$

$Q_3 = A_3 \times V_3$

$= 0.0706 \times 7.5$

$\underline{Q_3 = 0.5301 \, m/s}$

$A_2 = \pi/4 \, (d_2)^2$

$= \pi/4 \, (0.4)^2$

$\underline{A_2 = 0.1256 \, m^2}$

$Q_1 = Q_2 + Q_3$

$Q_1 - Q_3 = Q_2$

$Q_2 = 1.5 - 0.5301$

$\underline{Q_2 = 0.9699 \ m^3/s}$

$\therefore \ Q_2 = A_2 \times V_2$

$V_2 = \frac{Q_2}{A_2}$

$= \frac{0.9699}{0.1256}$

$\boxed{V_2 = 7.72 \ m/s}$

Basic Equations of compressible Flow:

1) Continuity equation

2) Bernoullis equation

3) Momentum equation

4) Equation of state.

1) Continuity equation:

Equation of ~~~~ MOTION:

$\boxed{F = m.a}$

Euler's Equation of motion:

$= \rho d.A.ds \times a$

$\boxed{\frac{dp}{\rho} + g\,dz + v.dv = 0}$

Bernoulli's Equation: or Energy Equation:

In an ideal incompressible fluid when the flow is steady and continuous, the sum of pressure energy, kinetic energy and potential energy is constat along steam line.

Euler's Equation of motion,

$$\frac{dp}{\rho} + g\,dz + v.dv = 0$$

Assumptions:

i) The fluid is ideal
ii) The flow is steady
iii) The flow is incompressible
iv) The flow is irrotational.

Applications:

i) venturimeter
ii) Orifice meter
iii) Pitot-tube

Derivation of Bernoulli's Equation:

$$\int \frac{dp}{\rho} + \int g\,dz + \int v.dv = 0$$

$$\frac{p}{\rho} + g.z + \frac{v^2}{2} = 0$$

$$\frac{p}{\rho g} + \frac{gz}{g} + \frac{v^2}{2g} = 0$$

$$\frac{p}{\rho g} + z + \frac{v^2}{2g} = 0$$

$$\therefore \boxed{\frac{p}{\rho g} + z + \frac{v^2}{2g} = 0}$$

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energy,

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Problems :

1) Water is flowing through a pipe of 5cm diameter under a pressure of 29.43 N/cm^2 (gauge) and with mean velocity of 2.0 m/s. Find the total head (or) total energy per unit weight of the water at a cross-section, which is 5 m above the datum line.

Data :

Diameter, $D = 5\,cm \Rightarrow 0.05\,m$

Pressure, $P = 29.43\,N/cm^2 \Rightarrow 29.43 \times 10^4\,N/m^2$

velocity, $v = 2\,m/s$

Datum line, $z = 5\,m$

Find :

Total head = ?

sol :

$$\text{Total head} = \frac{P}{\rho g} + \frac{v^2}{2g} + z$$

$$= \frac{29.43 \times 10^4}{1000 \times 9.81} + \frac{2^2}{2 \times 9.81} + 5$$

$$= 30 + 0.204 + 5$$

Total head = 35.204 m

P.NO : 298

Control Volume :

A control volume is fixed region in space which does not move (or) change shape. The fluid flows into and out of this fixed region. It's closed boundaries are called "control surface".

Actually the control surface may be in motion through space relative to an absolute frame of reference.

P.NO : 702

Momentum Equation :

P.NO : 288

The momentum per second of a flowing fluid (or) momentum flux) is equal to the product of mass per second and the velocity of the flow.

$$s = \rho AV (V_2 - V_1)$$

where,

ρAV – Mass per second

V_1 = Initial velocity in the direction of s

V_2 = Final velocity in the direction of s.

P.No : 298 **Moment of Momentum Equation :**

Moment of momentum equation is derived from moment of momentum principle which states that the resulting torque acting on a rotating fluid is equal to the rate of change of moment of momentum.

$F = m \times a$

$= m$

~~Problems:~~

1)

P.No : 288 **Impulse momentum Equation ;**

The force acting on a fluid mass 'm' is given by the Newton's second law of motion. which is known as impulse - momentum equation.

$$F = m \times a$$

But ,

$$a = \frac{dv}{dt}$$

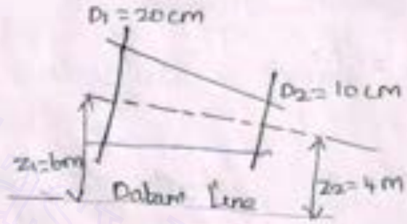
$$F = m\frac{dv}{dt}$$

$$= \frac{d(mv)}{dt} \quad \{ m \text{ is constant})$$

$$\boxed{F = \frac{d(mv)}{dt}}$$

* Problems :

1) Water is flowing through a pipe having diameters 20 cm and 10 cm at section ① and ② respectively. The rate of flow through the pipe is 35 lit/s. Section 1 is 6m above the datum and section 2 is 4m above datum. If the pressure at section 1 is 39.24 N/cm². Find the intensity of pressure at section 2.



Data:

$d_1 = 20 cm \rightarrow 0.2 m$

$d_2 = 10 cm \rightarrow 0.1 m$

$Q = 35 \; lit/s \rightarrow 0.035 \; m^3/s$

$z_1 = 6 \; m$

$z_2 = 4 \; m$

$P_1 = 39.24 \; N/cm^2 \rightarrow 39.24 \times 10^4 \; N/m^2$

Find:

$P_2 = ?$

Sol:

$A_1 = \pi/4 (d_1)^2 = \pi/4 (0.2)^2 = 0.0314 \; m^2$

$A_2 = \pi/4 (d_2)^2 = \pi/4 (0.1)^2 = 0.0078 \; m^2$

$Q = A_1 V_1 = A_2 V_2$

$Q = A_1 V_1$

$V_1 = \frac{Q}{A_1} = \frac{0.035}{0.0314} = 1.114 \; m/s$

$$V_2 = \frac{Q}{A_2} = \frac{0.035}{0.0078} = 4.487 \ m/s$$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$\frac{39.24 \times 10^4}{1000 \times 9.81} + \frac{(1.114)^2}{2 \times 9.81} + 6 = \frac{P_2}{1000 \times 9.81} + \frac{(4.487)^2}{2 \times 9.81} + 4$$

$$40 + 0.06325 + 6 = \frac{P_2}{9810} + 1.0261 + 4$$

$$46.06325 = \frac{P_2}{9810} + 5.026$$

$$46.06325 - 5.026 = \frac{P_2}{9810}$$

$$41.03725 = \frac{P_2}{9810}$$

$$P_2 = 41.03725 \times 9810$$

$$\boxed{P_2 = 402575.42 \ N/m^2}$$

$$\therefore \boxed{P_2 = 40.25 \ N/cm^2}$$

17-07-14

UNIT - II

FLOW THROUGH CIRCULAR CONDUITS

Hydraulic and energy gradient - Laminar flow through circular conduits and circular annuli - Boundary layer concept - Types of boundary layer thickness - Darcy waisbach equation - friction factor - Moody diagram - commercial pipes - mirror losses - flow through pipe in series and parallel.

P.NO : 387

Flow of viscous fluid through circular pipe :

The flow of viscous fluid through circular pipe, the velocity distribution across a section, the ratio of maximum velocity to average velocity, the shear stress distribution and drop of Pressure for a given length is to be determined.

$$Re = \frac{\rho V D}{\mu}$$ $\boxed{\bar{u} = V}$ → equal.

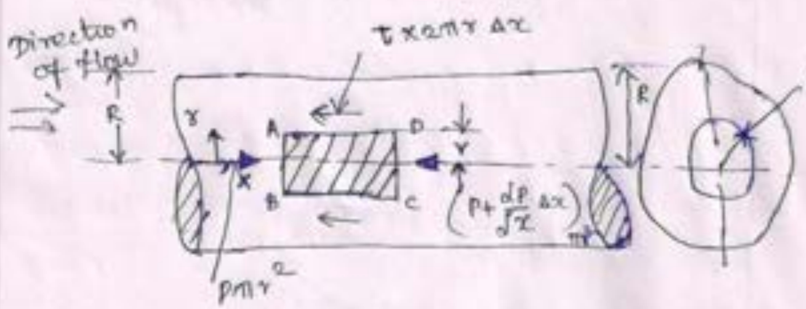
where,

ρ = Density of fluid flowing through pipe

V = Average velocity of fluid

D = Diameter of pipe

μ = viscosity of fluid.



Less than 2000 ⟹ Laminar flow

More than 4000 ⟶ Turbulent flow

↓

Reynold number

Hagen Poiseuille Formula :

$$\boxed{(P_1 - P_2) = \frac{32 \mu \bar{u} L}{D^2}}$$

where,

$P_1 - P_2$ is the drop of pressure

$\therefore$ Loss of Pressure head $= \frac{P_1 - P_2}{\rho g}$

$$\therefore \boxed{\frac{P_1 - P_2}{\rho g} = h_f = \frac{32 \mu \bar{u} L}{\rho g D^2}}$$

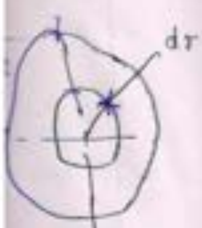
Hydraulic Gradient & Total Energy line :

Hydraulic Gradient line :

It is defined as the line which gives the sum of pressure head $\left(\frac{P}{w}\right)$ and datum head (z).

Total Energy line :

It is defined as the line which gives the sum of pressure head, datum head & kinetic head.



1) An oil of viscosity $0.1\ NS/m^2$ and relative density 0.9 is flowing though a circular pipe of diameter 50 mm and length of 3000 m. The rate of flow of fluid through the pipe is $3.5\ lit/sec$. Find the pressure drop in a length of 3000m.

Data :

Viscosity $\mu = 0.1\ NS/m^2$

Relative density $= 0.9$ $\quad \rho_{relative} = \frac{\rho_{oil}}{\rho_{water}}$

Density of oil $= 0.9 \times 1000$ $\quad \rho_{oil} = \rho_{water} \times \rho_{relative}$

$= 900\ kg/m^3$ $\quad = 1000 \times 0.9$

$\rho_{oil} = 900\ kg/m^3$

Diameter of pipe, $D = 50\ mm \Rightarrow 0.05\ m$

$L = 3000\ m$

$Q = 3.5\ lit/s$

$\Rightarrow 0.0035\ m^3/s$

Find :

i) Pressure drop $(P_1 - P_2)$

Sol :

Pressure drop,

$$(P_1 - P_2) = \frac{32 \mu \bar{U} L}{D^2}$$

Area, $A = \pi/4\ D^2$

$$= \pi/4 \times (0.05)^2$$

$$A = 1.96 \times 10^{-3}\ m^2$$

~~$A = 0.1963\ m^2$~~ ~~$A = 0.00$~~

$$Q = A \times V$$

$$V = \frac{Q}{A} = \frac{0.0035}{1.96 \times 10^{-3}} = 1.78 \ m/s$$

$$V = \overline{u} = 1.78 \ m/s$$

Pressure drop,

$$(P_1 - P_2) = \frac{32 \mu \bar{u} L}{D^2}$$

$$= \frac{32 \times 0.1 \times 1.78 \times 3000}{(0.05)^2}$$

$$(P_1 - P_2) = 6835200 \ N/m^2$$

$$\boxed{(P_1 - P_2) = 68.35 \ N/cm^2}$$

P.No. 465

CHAPTER - 11

FLOW THROUGH PIPES

Laminar Flow :

Reynolds number is less than 2000 for pipe flow, the flow known as "laminar flow".

Turbulant Flow :

Reynolds number is more than 4000 for pipe flow, the flow known as "turbulant flow".

Loss of Energy in pipes :

When a fluid is flowing through a pipe, the fluid experiences some resistance due to which some of the energy of fluid is lost.

Energy losses

Major Energy	Minor energy
This is due to friction calculated by the following formulae.	a) Sudden expansion of pipe
a) Darcy - weisbach formula	b) Sudden contraction of pipe
b) Chezy's formulae	c) Bend in pipe
	d) Pipe fittings etc.
	e) An obstruction in pipe

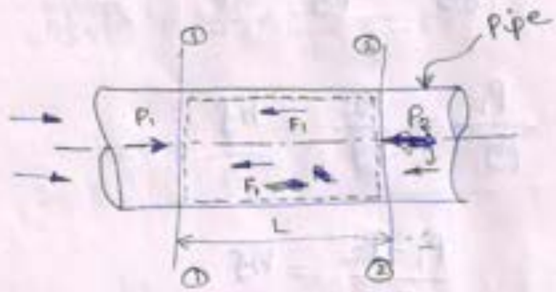
434 (X)

Darcy-weisbach Equation:

$P_1 \longrightarrow (+)$

$P_2 \longleftarrow (-)$

16 mark → 434 - 436



let,

P_1 = Pressure intensity at section 1-1,

V_1 = Velocity of flow at section 1-1,

L = length of the pipe b/w sections 1-1 and 2-2

d = diameter of pipe

f' = frictional resistance per unit wetted area per unit velocity

h_f = loss of head due to friction

P_2 = Pressure intensity at section 2-2

V_2 = velocity of flow at section 2-2

Energy equation,

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_f$$

Pipe in horizontal, $Z_1 = Z_2$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + h_f$$

Pipe is uniform velocity ... $V_1 = V_2$

$$\frac{P_1}{\rho g} = \frac{P_2}{\rho g} + h_f$$

$$\frac{P_1}{\rho g} - \frac{P_2}{\rho g} = h_f$$

$$\frac{P_1 - P_2}{\rho g} = h_f$$

$$\boxed{P_1 - P_2 = h_f \cdot \rho g}$$

Frictional resistance } $\cdot$ frictional resistance per unit wetted area per unit velocity $\times (\text{velocity})^2$

$$F_1 = f' \times \pi d L \times V^2 \quad \left[\because \text{wetted area} = \pi d \times L \quad \text{velocity} = V = V_1 = V_2 \right]$$

$$F_1 = f' \times P \times L \times V^2 \quad [\pi d = \text{Perimeter} = P]$$

$$P_1 A - P_2 A - F = 0$$

$$(P_1 - P_2) A - F = 0$$

$$(P_1 - P_2) A = F$$

$$P_1 - P_2 = \frac{F}{A}$$

$$P_1 - P_2 = \frac{f' \times P \times L \times V^2}{A}$$

$$h_f \cdot \rho g = \frac{f' \times P \times L \times V^2}{A}$$

$$\frac{P}{A} = \frac{\pi d}{\pi d^2/4} = \frac{4}{d}$$

$$\Rightarrow \quad h_f \cdot \rho g = f' \times \frac{4}{d} \times L \times V^2$$

$$h_f = \frac{f'}{\rho g} \times \frac{4}{d} \times L \times V^2$$

$$h_f = \frac{f'}{\rho g} \times \frac{4LV^2}{d}$$

Putting, $\frac{f'}{\rho} = \frac{f}{2}$

where, $f \rightarrow$ is known as co-efficient of friction

$$h_f = \frac{f}{2g} \times \frac{4LV^2}{d}$$

$$\boxed{h_f = \frac{4fLV^2}{2gd}} \rightarrow \text{Darcy - equation}$$

$$f = \frac{16}{Re}$$

for $Re < 2000$ (viscous flow)

$$f = \frac{0.079}{Re^{1/4}}$$ (varying from 4000 to 10^6)

$f \rightarrow$ co-efficient of friction

$$Re = \frac{\overset{\rightarrow \text{velocity}}{V} \times d}{\underset{\rightarrow \text{viscosity}}{\nu}}$$

P.NO. 466

5. Chezy's Formulae for loss of head due to friction in pipes:

$$h_f = \frac{f'}{\rho g} \times \frac{P}{A} \times L \times V^2$$

$$\frac{A}{P} = \frac{\text{Area of flow}}{\text{Perimeter}}$$ is called "Hydraulic" mean depth (or) hydraulic radius and is denoted by 'm',

Hydraulic mean depth, $m = \frac{A}{P}$

$$m = \frac{\pi d^2/4}{\pi d} = \frac{d}{4}$$

$$\frac{A}{P} = m \quad \text{(or)} \quad \frac{P}{A} = \frac{1}{m}$$

$$\Rightarrow h_f = \frac{f'}{\rho g} \times \frac{1}{m} \times L \times V^2$$

$$V^2 = h_f \times \frac{\rho g}{f'} \times m \times \frac{1}{L}$$

$$V = \sqrt{\frac{\rho g}{f'} \times m \times \frac{h_f}{L}}$$

$$\sqrt{\frac{\rho g}{f'}} = C \qquad \therefore \boxed{V = C\sqrt{mi}}$$

where, C = constant

$$\frac{h_f}{L} = i$$

where, i = loss of head per unit length of pipe due

Problems :

1) Find the head lost due to friction in a pipe of diameter 300 mm & length 50 m, through which water is flowing at a velocity of 3 m/s using i) Darcy formula, ii) Chezy's formula for which c = 60.

Take v for water = 0.01 stoke

Data:

Dia. of pipe , d = 300 mm $\Rightarrow \frac{300}{1000} = 0.30$ m

length of pipe , L = 50 m

velocity of flow, v = 3 m/s

chezy's constant , c = 60

kinematic viscosity, $\nu = 0.01$ stoke

$\Rightarrow 0.01\ cm^2/s$

$\nu \Rightarrow 0.01 \times 10^{-4}\ m^2/s$

Find :

i) Darcy formula

ii) chezy's formula

sol :

i) Darcy Formulae,

$$h_f = \frac{4.f.L.v^2}{2gd}$$

$$f = \quad Re = \frac{\text{velocity} \times \text{diameter}}{\text{viscosity}}$$

$$= \frac{3 \times 0.30}{0.01 \times 10^{-4}}$$

$$Re = 9 \times 10^5$$

length Pipe

Re → is more than 4000 . so its formula

$$f = \frac{0.079}{Re^{1/4}}$$

$$= \frac{0.079}{(9\times10^5)^{1/4}}$$

$$f = 0.00256$$

∴ Head lost , $hf = \dfrac{4f.Lv^2}{2gd}$

$$= \frac{4\times0.00256\times50\times(3)^2}{2\times9.81\times0.30}$$

$$\boxed{hf = 0.7825\ m}$$

ii) chezy formulae,

$$V = c\sqrt{mi}$$

$$m = \frac{d}{4} = \frac{0.30}{4} = 0.075\,m$$

$$3 = 60\sqrt{0.075\times i}$$

$$\frac{3}{60} = \sqrt{0.075\times i}$$

$$0.05 = \sqrt{0.075}\times\sqrt{i}$$

$$\frac{0.05}{\sqrt{0.075}} = \sqrt{i}$$

$$0.1825 = \sqrt{i}$$

$$(0.1825)^2 = (\sqrt{i})^2$$

$i = 0.03333$

$i = \frac{h_f}{L}$

$h_f = i \times L$

$= 0.0333 \times 50$

$\boxed{h_f = 1.665\ m}$

2) Find the diameter of a pipe of length 2000 m when the rate of flow of water through the pipe is 200 litres/s and the head lost due to friction is 4 m. Take the value of c = 50 in chezy's formulae.

Data:

Length, $L = 2000\ m$

Discharge, $Q = 200\ litres/s \Rightarrow 0.2\ m^3/s$

Head lost due to friction, $h_f = 4\ m$

Value of Chezy's constant, $C = 50$

Find:

Diameter of the pipe, $d = ?$

Sol:

velocity of flow

$$V = \frac{Discharge}{Area}$$

$$= \frac{Q}{\pi/4\, d^2} = \frac{0.2}{\pi/4 \times d^2}$$

$$V = \frac{0.2 \times 4}{\pi d^2}$$

Hydraulic mean depth, $m = \frac{d}{4}$

Loss of head per unit length, $i = \frac{h_f}{L}$

$$= \frac{4}{2000}$$

$$i = 0.002$$

Chezy's formulae.

$$V = C\sqrt{mi}$$

$$\frac{0.2 \times 4}{\pi d^2} = 50\sqrt{\frac{d}{4} \times 0.002}$$

$$\frac{0.2 \times 4}{\pi d^2 \times 50} = \sqrt{\frac{d}{4} \times 0.002}$$

$$\frac{0.00509}{d^2} = \sqrt{\frac{d}{4} \times 0.002}$$

Squaring both sides,

$$\frac{(0.00509)^2}{d^4} = \frac{d}{4} \times 0.002$$

$$\frac{0.0000259}{d^4} = \frac{d}{4} \times 0.002$$

$$d^5 = \frac{0.0000259 \times 4}{0.002}$$

$$d^5 = 0.0518$$

$$d = (0.518)^{1/5}$$

$$d = 0.553\ m$$

Diameter of Pipe, $\boxed{d = 553\ mm}$

3) A crude oil of kinematic viscosity 0.4 stoke is flowing through a pipe of diameter 300 mm at the rate of 300 litres/sec. Find the head lost due to friction for a length of 50 m of the pipe.

Data:

Kinematic viscosity, $\nu = 0.4$ stoke $\Rightarrow 0.4\,cm^2/s$

$$\nu \Rightarrow 0.4 \times 10^{-4}\,m^2/s$$

Diameter of pipe, $d = 300\,mm \Rightarrow 0.30\,m$

Discharge, $Q = 300$ litres/sec $\Rightarrow 0.3\,m^3/s$

Length of pipe, $L = 50\,m$

Find:

Head lost due to friction, $h_f = ?$

Sol:

Velocity of flow,

$$V = \frac{\text{Discharge}}{\text{Area}} = \frac{Q}{A}$$

$$= \frac{Q}{\pi/4\, d^2}$$

$$= \frac{Q \times 4}{\pi \times d^2} = \frac{0.3 \times 4}{\pi \times 0.3^2}$$

$$V = 4.24\,m/s$$

$\therefore$ Reynolds Number, $R_e = \dfrac{V \times d}{\nu}$

$$= \frac{4.24 \times 0.3}{0.4 \times 10^{-4}}$$

$$R_e = 3.18 \times 10^4$$

Re is more than 4000, so the formulae f is given by,

$$f = \frac{0.079}{Re^{1/4}}$$

$$= \frac{0.079}{(3.18\times10^{4})^{1/4}}$$

$$f = 0.00591$$

∴ Head lost due to friction,

$$h_f = \frac{4 . f . L . v^2}{2 g d}$$

$$= \frac{4\times0.00591\times50\times(4.24)^2}{2\times9.81\times0.3}$$

$$\boxed{h_f = 3.61\,m}$$

4) An oil of specific gravity 0.7 is flowing through a pipe of diameter 30 cm at the rate of 500 litres/sec. Find the head lost due to friction and power required to maintain the flow for a length of 1000 m. Take kinematic viscosity of oil 0.29 stokes.

Data :

$S = 0.7$

$d = 30\,cm \Rightarrow 300\,mm \rightarrow 0.3\,m$

$Q = 500\,lit/sec \Rightarrow 0.5\,m^3/s$

$\nu = 0.29\,stokes \Rightarrow 0.29\,cm^2/s \rightarrow 0.29\times10^{-4}\,m^2/s$

$L = 1000\,m$

rate
friction
for
y
$\times 10^{-4}\ m^2/s$

Find :

i) Heat lost due to friction, hf

ii) Power, P

Sol :

i) Heat lost due to friction, hf

$$h_f = \frac{4 \times f \times L \times V^2}{2 \times g \times d}$$

$$Q = A \times V$$

$$V = \frac{Q}{Area}$$

$$= \frac{0.5}{\pi/4 \times (0.3)^2}$$

$$V = 7.07\ m/s$$

$$Re = \frac{V.d}{\nu}$$

$$= \frac{7.07 \times 0.3}{0.29 \times 10^{-4}}$$

$$Re = 73137.93$$

Re is morethan 4000

$$f = \frac{0.079}{Re^{1/4}}$$

$$= \frac{0.079}{(73137.93)^{1/4}}$$

$$f = 0.0048$$

∴ Head lost due to friction,

$$h_f = \frac{4fLV^2}{2gd}$$

$$= \frac{4 \times 0.0048 \times 1000 \times (7.07)^2}{2 \times 9.81 \times 0.5}$$

$$h_f = 163.04\,m$$

ii) Power required,

$$P = \frac{\rho \cdot g \cdot Q \cdot h_f}{1000}$$

Specific Gravity, $S = \frac{\rho_{oil}}{\rho_{water}}$

$$\rho_{oil} = S \times \rho_{water}$$

$$= 0.7 \times 1000$$

$$\rho_{oil} = 700\ kg/m^3$$

$$\therefore \text{Power}, P = \frac{700 \times 9.81 \times 0.5 \times 163.04}{1000}$$

$$P = 559.79\ kW$$

P.NO : 581 Reynold's Number :

It is defined as the ratio of inertia force of a flowing fluid & the viscous force of the fluid.

$$Re = \frac{V \times d}{\nu}$$

(or)

$$Re = \frac{\rho V d}{\mu}$$

P.NO : 470

5) An oil of specific Gravity 0.9 & viscosity 0.06 poise is flowing through a pipe of diameter 200 mm at the rate of 60 lit/sec. Find the head lost due to friction for a 500 m length of pipe. Find the power required to maintain this flow.

Data :

Sp. gr of oil = 0.9

viscosity, μ = 0.06 poise = $\frac{0.06}{10}$ Ns/m^2

Dia. of pipe, d = 200 mm $\rightarrow$ 0.2 m

Discharge, Q = 60 lit/s $\rightarrow$ 0.06 m^3/s

Length, L = 500 m

Density, ρ = 0.9 × 1000 $\Rightarrow$ 900 kg/m^3

Find :

Power required.

Sol:

Reynold's number $Re = \frac{\rho V d}{\mu} = 900 \times \frac{V \times 0.2}{0.06/10}$

$$V = \frac{Q}{A} = \frac{0.06}{\frac{\pi}{4}(0.2)^2} = 1.909 \text{ m/s}$$

$$Re = \frac{900 \times 1.91 \times 0.2 \times 10}{0.06} = 57300$$

If Re lies between 4000 and 10^5, the value of co-efficient of friction, f

$$f = \frac{0.079}{Re^{0.25}} = \frac{0.079}{(57300)^{0.25}} = 0.0051$$

$\therefore$ Head lost due to friction,

$$h_f = \frac{4 \times f \times L \times V^2}{d \times 2g}$$

$$= \frac{4 \times 0.0051 \times 500 \times 1.91^2}{0.2 \times 2 \times 9.81}$$

$$h_f = 9.48 \text{ m}$$

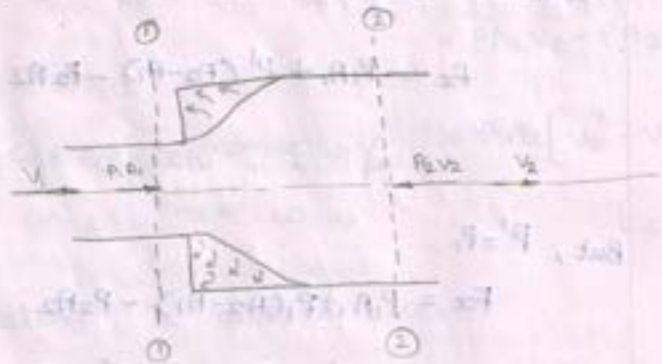
$\therefore$ Power required,

$$P = \frac{\rho g . Q . h_f}{1000}$$

$$= \frac{900 \times 9.81 \times 0.06 \times 9.48}{1000}$$

$$P = 5.02 \text{ kW}$$

Loss of Head Due to sudden Enlargement:



let,

$P_1 \Rightarrow$ Pressure intensity at section 1-1,

$V_1 \rightarrow$ velocity of flow at section 1-1,

$A_1 \rightarrow$ Area of pipe at section 1-1,

$P_2 \Rightarrow$ Pressure intensity at section 2-2,

$V_2 \rightarrow$ velocity of flow at section 2-2,

$A_2 \rightarrow$ Area of pipe at section 2-2.

Applying Bernoullis equation at section 1-1 & 2-2.

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_e$$

$h_e \Rightarrow$ loss of head due to sudden Enlargement

But, pipe is horizontal, $Z_1 = Z_2$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + h_e$$

$$h_e = \left(\frac{P_1}{\rho g} - \frac{P_2}{\rho g}\right) + \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right) \rightarrow I\text{-eqn}$$

consider the control volume of liquid between sections 1-1 and 2-2.

$$F_x = P_1A_1 + P'(A_2 - A_1) - P_2A_2$$

$P' \rightarrow$ Pressure intensity of the liquid eddies on the area $(A_2 - A_1)$

But, $P' = P_1$

$$F_x = P_1A_1 + P_1(A_2 - A_1) - P_2A_2$$

$$= \cancel{P_1A_1} + P_1A_2 - \cancel{P_1A_1} - P_2A_2$$

$$F_x = P_1A_2 - P_2A_2$$

$$F_x = A_2(P_1 - P_2) \longrightarrow \underline{\text{II}} - \text{eqn}$$

Momentum of liquid/sec at section 1-1,

$$= \text{mass} \times \text{velocity}$$

$$= \rho A_1V_1 \times V_1$$

$$= \rho A_1V_1^2$$

momentum of liquid/sec at section 2-2,

$$= \rho A_2V_2 \times V_2$$

$$= \rho A_2V_2^2$$

$\therefore$ change of momentum $= \rho A_2V_2^2 - \rho A_1V_1^2$

But, from continuity equation,

$$A_1V_1 = A_2V_2$$

$$A_1 = \frac{A_2V_2}{V_1}$$

$\therefore$ change of momentum/sec $= \rho A_2 V_2^2 - \rho \times \frac{A_2 V_2}{V_1} \times V_1^2$

$= \rho A_2 V_2^2 - \rho A_2 V_1 V_2$

change of momentum/sec $= \rho A_2 \left[V_2^2 - V_1 V_2 \right]$

$\rightarrow$ III - eqn

Equating II and III - eqn.

$$\cancel{A_2} (P_1 - P_2) = \rho \cancel{A_2} \left[V_2^2 - V_1 V_2 \right]$$

$$\frac{P_1 - P_2}{\rho} = V_2^2 - V_1 V_2$$

Dividing by g on both sides,

$$\frac{P_1 - P_2}{\rho g} = \frac{V_2^2 - V_1 V_2}{g}$$

$$\frac{P_1}{\rho g} - \frac{P_2}{\rho g} = \frac{V_2^2 - V_1 V_2}{g}$$

sub. the value of $\frac{P_1}{\rho g} - \frac{P_2}{\rho g}$ in eqn - I

$$h_e = \frac{V_2^2 - V_1 V_2}{g} + \frac{V_1^2}{2g} - \frac{V_2^2}{2g}$$

$$= \frac{2V^2 - 2V_1 V_2 + V_1^2 - V_2^2}{2g}$$

$$= \frac{V_2^2 - V_1^2 - 2V_1 V_2}{2g}$$

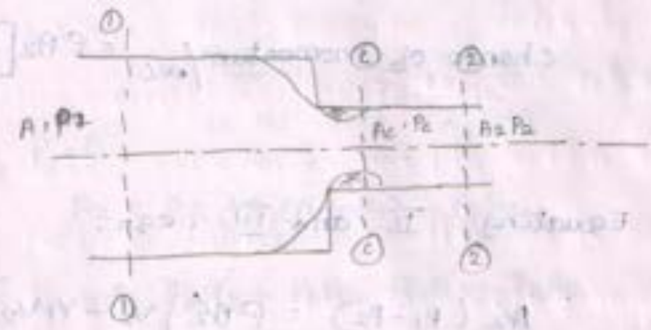
$$= \left[\frac{V_1 - V_2}{2g} \right]^2$$

$$\boxed{h_e = \frac{(V_1 - V_2)^2}{2g}}$$

$\therefore$ This loss of head due sudden Enlargement is verified.

P.No: 473

Loss of Head due to sudden construction:-



Let,

$A_c \Rightarrow$ Area of velocity at section c-c

$V_c \Rightarrow$ velocity of flow at section c-c

$A_2 \Rightarrow$ Area of flow at section 2-2

$V_2 \Rightarrow$ velocity of flow at section 2-2

$A_1 \Rightarrow$ Area of flow at section 1-1

$V_1 \Rightarrow$ velocity of flow at section 1-1.

$h_c \Rightarrow$ Loss of head due to sudden construction

From sudden Enlargement,

$$= \frac{(V_c - V_2)^2}{2g}$$

$$= \frac{1}{2g}\left(V_c \times \frac{V_2}{V_2} - V_2 \times \frac{V_2}{V_2}\right)^2$$

$$= \frac{1}{2g}\left[V_2\left(\frac{V_c}{V_2} - 1\right)\right]^2$$

$$= \frac{1}{2g} V_2^2 \left[\frac{V_c}{V_2} - 1\right]^2$$

$$= \frac{V_2^2}{2g}\left[\frac{V_c}{V_2} - 1\right]^2 \longrightarrow \text{I - eqn}$$

From continuity Equation,

$$A_c V_c = A_2 V_2$$

$$[\text{or}] \quad \frac{V_c}{V_2} = \frac{A_2}{A_c} = \frac{1}{A_c/A_2} = \frac{1}{C_c}$$

$$\left[\because C_c = \frac{A_c}{A_2} \right]$$

substituting the value $\frac{V_c}{V_2}$

$$h_c = \frac{V_2^2}{2g} \left[\frac{1}{C_c} - 1 \right]^2$$

$$h_c = \frac{k V_2^2}{2g}$$

where, $k \rightarrow \left[\frac{1}{C_c} - 1 \right]^2$

If the value of C_c is assumed to be equal to 0.62, then

$$k = \left[\frac{1}{0.62} - 1 \right]^2$$

$$k = 0.375$$

Then, h_c becomes,

$$h_c = \frac{k V_2^2}{2g}$$

$$= 0.375 \times \frac{V_2^2}{2g}$$

The value of C_c is not given then the head loss due to contraction is taken as,

$$h_c = 0.5 \frac{V_2^2}{2g}$$

P.NO: 475 (3) The rate of flow of water through a horizontal pipe is $0.25\ m^3/s$. The diameter of the pipe which is 200 mm suddenly enlarged to 400 mm. The pressure intensity in the smaller pipe is $11.772\ N/cm^2$. Determine

i) Loss of head due to sudden Enlargement

ii) Pressure intensity in the large pipe

iii) Power lost due to enlargement.

Data:

Discharge, $Q = 0.25\ m^3/s$

$d_1 = 200\ mm \Rightarrow 0.2\ m$

$d_2 = 400\ mm \Rightarrow 0.4\ m$

Pressure Intensity in the smaller pipe $= 11.772\ N/cm^2$

$\Rightarrow 11.772 \times 10^4\ N/m^2$

Find:

i) Loss of head due to sudden Enlargement

ii) Pressure intensity in the large pipe

iii) Power lost due to enlargement.

Sol:

Area, A_1

$$A_1 = \frac{\pi d_1^2}{4} = \frac{\pi \times (0.2)^2}{4} = 0.03141\ m^2$$

Area, A_2

$$A_2 = \frac{\pi d_2^2}{4} = \frac{\pi \times (0.4)^2}{4} = 0.125\ m^2$$

From continuity equation,

$$Q = A_1 V_1 = A_2 V_2$$

$Q = A_1 V_1$

$$V_1 = \frac{Q}{A_1} = \frac{0.25}{0.03141} = 7.96 \text{ m/s}$$

$Q = A_2 V_2$

$$V_2 = \frac{Q}{A_2} = \frac{0.25}{0.125} = 1.99 \text{ m/s}$$

i) Loss of head due to sudden Enlargement

$$h_e = \frac{(V_1 - V_2)^2}{2g}$$

$$= \frac{(7.96 - 1.99)^2}{2 \times 9.81}$$

$$\boxed{h_e = 1.816 \text{ m}}$$

ii) Pressure Intensity in the large pipe,

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_e$$

For horizontal pipe,

$$Z_1 = Z_2$$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + h_e$$

$$\frac{P_2}{\rho g} = \frac{P_1}{\rho g} + \frac{V_1^2}{2g} - \frac{V_2^2}{2g} - h_e$$

$$= \frac{11.772 \times 10^4}{1000 \times 9.81} + \frac{(7.96)^2}{2 \times 9.81} - \frac{(1.99)^2}{2 \times 9.81} - 1.816$$

$$= 12.0 + 3.229 - 0.2018 - 1.816$$

$$\frac{P_2}{\rho g} = 15.229 - 2.0178$$

$$\frac{P_2}{\rho g} = 13.21 \text{ m of water}$$

$$P_2 = 13.21 \times \rho \times g$$

$$= 13.21 \times 1000 \times 9.81$$

$$\boxed{P_2 = 12.96 \text{ N/cm}^2}$$

iii) Power lost due to sudden Enlargement,

$$P = \frac{\rho g \cdot Q \cdot he}{1000}$$

$$= \frac{1000 \times 9.81 \times 0.25 \times 1.816}{1000}$$

$$\boxed{P = 4.453 \text{ kW}}$$

P.No. 482

Loss of Head due to an obstruction in a Pipe :

Let,

a → Maximum area of obstruction
V → velocity of liquid pipe
A → Area of Pipe

Loss of Head due to obstruction } = loss of head due to enlargement from vena-contracta to section 2-2

$$= \frac{(V_c - V)^2}{2g} \rightarrow (i)$$

From continuity equation, → (ii)

$$a_c \times V_c = A \times V$$

where,

a_c = area of cross section at vena-contracta

C_c = co-efficient of construction

$$C_c = \frac{\text{area at vena-contracta}}{(A - a)} = \frac{a_c}{(A-a)}$$

$$a_c = C_c \times (A - a)$$

∴ substituting this value (ii)

$$c_c \times (A-a) \times V_c = A \times V$$

$$\therefore \quad V_c = \frac{A \times V}{c_c(A-a)}$$

sub V_c in eqn (i).

$$\text{Head loss due to obstruction} = \frac{(V_c - V)^2}{2g}$$

$$= \frac{\left(\frac{A \times V}{C_c(A-a)} - V\right)^2}{2g}$$

$$\left.\begin{matrix}\text{Head loss due to} \\ \text{Obstruction}\end{matrix}\right\} = \frac{V^2}{2g}\left[\frac{A}{C_c(A-a)} - 1\right]^2$$

Loss of Head due to Bend in Pipe:

$$h_b = \frac{KV^2}{2g}$$

$K \rightarrow$ Angle of bend & Radius of curvature of bend

Loss of Head in various pipe Fittings:

$$\left.\begin{matrix}\text{loss of Head in} \\ \text{various pipe fittings}\end{matrix}\right\} = \frac{KV^2}{2g}$$

where,

$V \Rightarrow$ velocity of flow

$K \Rightarrow$ co-efficient of pipe fitting

Flow through pipes in series (or)

Flow through compound pipes:

Let,

L_1, L_2 & L_3 → Length of pipes 1, 2 & 3 respectively.

d_1, d_2 & d_3 → Diameter of pipes 1, 2, & 3 respectively,

V_1, V_2, V_3 → velocity of flow through pipes 1, 2 & 3.

f_1, f_2 & f_3 → co-efficient of frictions for pipes 1, 2 & 3

H → Difference of water level in two tanks

The discharge passing through each pipe is same,

$$Q = A_1 V_1 = A_2 V_2 = A_3 V_3$$

The difference in liquid surface levels is equal to the sum of the total head loss in the pipes.

$$H = \frac{0.5 V_1^2}{2g} + \frac{4 f_1 L_1 V_1^2}{d_1 \times 2g} + \frac{0.5 V_2^2}{2g} + \frac{4 f_2 L_2 V_2^2}{d_2 \times 2g} + \frac{(V_2 - V_3)^2}{2g} + \frac{4 f_3 L_3 V_3^2}{d_3 \times 2g} + \frac{V_3^2}{2g}$$

The minor loss are neglected,

$$H = \frac{4 f_1 L_1 V_1^2}{d_1 \times 2g} + \frac{4 f_2 L_2 V_2^2}{d_2 \times 2g} + \frac{4 f_3 L_3 V_3^2}{d_3 \times 2g}$$

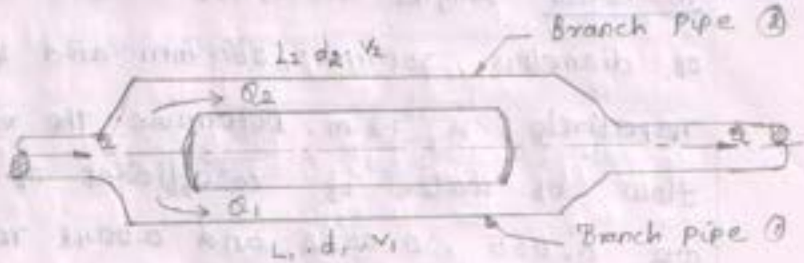
The co-efficient of friction is same for all pipes.

$$f_1 = f_2 = f_3 = f$$

$$\therefore \quad H = \frac{4 f L_1 V_1^2}{d_1 \times 2g} + \frac{4 f L_2 V_2^2}{d_2 \times 2g} + \frac{4 f L_3 V_3^2}{d_3 \times 2g}$$

$$H = \frac{4f}{2g}\left[\frac{L_1 V_1^2}{d_1} + \frac{L_2 V_2^2}{d_2} + \frac{L_3 V_3^2}{d_3}\right]$$

P.NO: 508 Flow through parallel pipes:



The rate of flow in the main pipe is equal to the sum of the rate of flow through branch pipes.

$$Q = Q_1 + Q_2$$

The loss of head for each branch pipe is same.

Loss of Head for Branch pipe 1 = loss of Head for Branch pipe 2

$$\frac{4f_1 L_1 V_1^2}{d_1 \times 2g} = \frac{4f_2 L_2 V_2^2}{d_2 \times 2g}$$

$$\because f_1 = f_2$$

$$\therefore \boxed{\frac{L_1 V_1^2}{d_1 \times 2g} = \frac{L_2 V_2^2}{d_2 \times 2g}}$$

Problems :

P.No. 503 (x) The difference in water surface levels in two tanks, which are connected by three pipes in series lengths 300 m, 170 m and 210 m and of diameters 300 mm, 200 mm and 400 mm respectively, is 12 m. Determine the rate of flow of water if co-efficient of friction are 0.005, 0.0052 and 0.0048 respectively - Considering i) Minor losses also ii) Neglecting minor losses.

Data :

$L_1 = 300\ m$

$L_2 = 170\ m$

$L_3 = 210\ m$

Diameter,

$d_1 = 300\ mm \Rightarrow 0.3\ m$

$d_2 = 200\ mm \Rightarrow 0.2\ m$

$d_3 = 400\ mm \Rightarrow \frac{400}{1000} \Rightarrow 0.4\ m$

Friction,

$f_1 = 0.005$

$f_2 = 0.0052$

$f_3 = 0.0048$

$H = 12\ m$

Find:

i) Minor losses also ii) Neglecting minor losses

Sol: i) Minor losses also

$Q = A_1V_1 = A_2V_2 = A_3V_3$

$A_1V_1 = A_2V_2$

$$V_2 = \frac{A_1V_1}{A_2} = \frac{\pi/4\, d_1^2}{\pi/4\, d_2^2} V_1 = \frac{d_1^2}{d_2^2} V_1$$

$$= \left(\frac{0.3}{0.2}\right)^2 \times V_1$$

$$\underline{V_2 = 2.25 V_1}$$

$$V_3 = \frac{A_1V_1}{A_3} = \frac{d_1^2}{d_3^2} = \left(\frac{0.3}{0.4}\right)^2 \times V_1$$

$$\underline{V_3 = 0.5625 V_1}$$

Now using equation,

$$H = \frac{0.5V_1^2}{2g} + \frac{4f_1L_1V_1^2}{d_1 \times 2g} + \frac{0.5V_2^2}{2g} + \frac{4f_2L_2V_2^2}{d_2 \times 2g} + \frac{(V_2-V_3)^2}{2g} + \frac{4f_3L_3V_3^2}{d_3 \times 2g} + \frac{V_3^2}{2g}$$

substituting V_2 and V_3

$$12.0 = \frac{0.5V_1^2}{2g} + \frac{4 \times 0.005 \times 300 \times V_1^2}{0.3 \times 2g} + \frac{0.5 \times (2.25V_1)^2}{2g} + 4 \times 0.0052 \times 170 \times \frac{(2.25V_1)^2}{0.2 \times 2g} + \frac{(2.25V_1 - 0.5625 \times V_1)^2}{2g} + \frac{4 \times 0.0048 \times 210 \times (0.5625V_1)^2}{0.4 \times 2g} + \frac{(0.5625V_1)^2}{2g}$$

$$12.0 = \frac{V_1^2}{2g}\left[0.5 + 20.0 + 2.53 + 89.505 + 2.847 + 3.189 + 0.316\right]$$

$$12.0 = \frac{V_1^2}{2g}\left[118.887\right]$$

$$V_1 = \sqrt{\frac{12 \times 2 \times 9.81}{118.887}}$$

$$\boxed{V_1 = 1.407 \text{ m/s}}$$

Rate of flow Q = Area $\times$ velocity

$$= A_1 \times V_1$$

$$Q = \pi/4 \, d_1^2 \times V_1$$

$$= \pi/4 \times (0.3)^2 \times 1.407$$

$$\boxed{Q = 99.45 \text{ lit/s}}$$

(ii) Neglecting minor losses,

using equation,

$$H = \frac{4f_1L_1V_1^2}{d_1 \times 2g} + \frac{4f_2L_2V_2^2}{d_2 \times 2g} + \frac{4f_3L_3V_3^2}{d_3 \times 2g}$$

$$12.0 = \frac{V_1^2}{2g}\left[\frac{4 \times 0.005 \times 300}{0.3} + \frac{4 \times 0.0052 \times 170 \times (2.25)^2}{0.2} + \frac{4 \times 0.0048 \times 210 \times (0.5625)^2}{0.4}\right]$$

$$12.0 = \frac{V_1^2}{2g}\left[20.0 + 89.505 + 3.189\right]$$

$$12.0 = \frac{V_1^2}{2g} \times 112.694$$

$$V_1 = \sqrt{\frac{2 \times 9.81 \times 12.0}{112.694}}$$

$$\boxed{V_1 = 1.445 \text{ m/s}}$$

Discharge $Q = A_1 \times V_1 = \pi/4 \times (0.3)^2 \times 1.445$

$$\boxed{Q = 102.1 \text{ lit/s}}$$

p.508 Three pipes of lengths 800 m, 500 m and 400 m and of diameters 500 mm, 400 mm and 300 mm respectively are connected in series. These pipes are to be replaced by a single pipe of length 1700 m. Find diameter of the single pipe.

Data:

Lengths,

$L_1 = 800\,m$

$L_2 = 500\,m$

$L_3 = 400\,m$

Diameters,

$d_1 = 500\,mm \Rightarrow 0.5\,m$

$d_2 = 400\,mm \Rightarrow 0.4\,m$

$d_3 = 300\,mm \Rightarrow 0.3\,m$

Single pipe Length, $L = 1700\,m$

Find:

Diameter of single pipe, $d = ?$

Sol:

$$\frac{L}{d^5} = \frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5}$$

$$\frac{1700}{d^5} = \frac{800}{0.5^5} + \frac{500}{0.4^5} + \frac{400}{0.3^5}$$

$$\frac{1700}{d^5} = 25600 + 48828.125 + 164609$$

$$\frac{1700}{d^5} = 239037$$

$$d^5 \times 239037 = 1700$$

$$d^5 = \frac{1700}{239037} = 0.007118$$

$$d = 0.3718\,m \Rightarrow \boxed{d = 371.8\,mm}$$

A main pipes divides into two parallel pipes which again forms one pipe as shown in fig. The length & diameter for first parallel pipe are 2000 m and 1.0 m respectively. While the length and diameter of second parallel pipe are 2000 m and 0.8 m. Find the rate of flow in each parallel pipe if total flow in main is 3.0 m^3/s. The co-efficient of friction for each parallel pipe is same & equal to 0.005.

Data:

$L_1 = 2000$ m

$d_1 = 1.0$ m

$L_2 = 2000$ m

$d_2 = 0.8$ m

Discharge $Q = 3.0\ m^3/s$

co-efficient of friction $f_1 = f_2 = f = 0.005$.

Find:

~~$Q = Q_1 + Q_2 = ?$~~

i) $Q_1 = ?$

ii) $Q_2 = ?$

Sol:

$$\frac{4fL_1V_1^2}{2gd_1} = \frac{4fL_2V_2^2}{2gd_2}$$

$$\frac{\cancel{4} \times \cancel{0.005} \times \cancel{2000} \times V_1^2}{\cancel{2} \times \cancel{9.81} \times 1.0} = \frac{\cancel{4} \times \cancel{0.005} \times \cancel{2000} \times V_2^2}{\cancel{2} \times \cancel{9.81} \times 0.8}$$

$$\frac{V_1^2}{1.0} = \frac{V_2^2}{0.8}$$

$$V_1^2 = \frac{V_2^2}{0.8} \times 1.0$$

$$V_1^2 = \frac{V_2^2}{0.8}$$

$$V_1 = \frac{V_2}{\sqrt{0.8}} = \frac{V_2}{0.894}$$

$$Q_1 = \pi/4\, d_1^2 \times V_1 = \pi/4\,(1)^2 \times \frac{V_2}{.894}$$

$$Q_2 = \pi/4\, d_2^2 \times V_2 = \pi/4\,(0.8)^2 \times V_2 = \pi/4 \times 0.64 \times V_2$$

Sub Q_1 & Q_2 value in I-eqn

$$\pi/4 \times \frac{V_2}{0.894} + \pi/4 \times 0.64\, V_2 = 3.0$$

$$0.8785\, V_2 + 0.5026\, V_2 = 3.0$$

$$V_2 \left[0.8785 + 0.5026\right] = 3.0$$

$$V_2 \times 1.3811 = 3.0$$

$$V_2 = \frac{3.0}{1.3811}$$

$$V_2 = \underline{2.17\ m/s}$$

$$V_1 = \frac{V_2}{0.894} = \frac{2.17}{0.894} = \underline{2.427}\ m/s$$

Hence,

$$Q_1 = \pi/4\, d_1^2 \times V_1$$

$$= \pi/4\, 1^2 \times 2.427$$

$$\boxed{Q_1 = 1.906\ m^3/s}$$

$$Q_2 = Q - Q_1$$

$$= 3.0 - 1.906$$

$$\boxed{Q_2 = 1.094\ m^3/s}$$

pg-611

Boundary Layer Flow:

A very thin layer of the fluid, is called the "boundary layer".

$\nu \rightarrow$ kinematic viscosity

$U \rightarrow$ Free Stream velocity of fluid

Reynolds Numer $(Re)_x = \frac{Ux}{\nu}$

Moody Diagram:

Reference

Moody diagrams present various values of friction factor (f) Reynolds number Re and relative roughness.

[or]

Moody diagram gives the value of Friction factor of any pipe with its relative roughness (k/D) and Reynolds number of flow Re known,

k = Absolute roughness

D = Diameter of pipe

Hence Moody's diagram is the most common source of reference for obtaining value of f.

$Re = \frac{Ux}{\nu}$

P.NO: 613

Turbulant Boundary Layer :

If the length of the plate is more than the distance x; the thickness of boundary layer will go on increasing in the down-stream direction. Then the laminar boundary layer becomes unstable & motion of fluid within it, is distributed and irregular which leads to a transition from laminar to boundary layer. The short length over which the boundary layer flow changes from laminar to turbulant is called "transition zone". Further downstream the transition zone, the boundary layer is turbulant & continuous to grow in thickness. This layer of boundary is called "turbulant boundary layer".

Laminar sub-layer :

The thickness of boundary sub-layer will is go very small. $\tau_0 = \mu \left(\frac{\partial u}{\partial y}\right)_{y=0} = \mu \frac{u}{y}$

Laminar Boundary Layer :

Near the leading edge of the surface of the plate, when the thickness is small, the flow in the boundary layer is laminar through the main flow is turbulant. This layer of the fluid is said to be laminar boundary layer.

$$(Re)_x = \frac{U \times x}{\nu}$$

$x \rightarrow$ Distance from leading edge

ν – Kinematic viscosity

U – Free steam velocity of fluid

P.NO - 613

Boundary Layer Thickness :

It is defined as the distance from boundary of the solid body measured in the y-direction to the point, where the velocity of the fluid is approximately equal to 0.99 times the free stream velocity (V) of the fluid.

1. $\delta_{lam.}$ = Thickness of laminar boundary layer
2. $\delta_{tur.}$ = Thickness of turbulant boundary layer
3. δ' = Thickness of laminar sub layer.

Displacement Thickness (δ^*):

It is defined as the distance, measured perpendicular to the boundary of the solid body, by which the boundary should be displaced to compensate for the reduction in flow rate on account of boundary layer formation.

$$\delta^* = \int_0^{\delta} \left(1 - \frac{u}{V}\right) dy$$

UNIT - III

DIMENSIONAL & MODEL ANALYSIS

Dimensional Analysis :

It is a mathematical technique used in research work for design & for conducting model tests.

Fixed Dimensions :

Length L
Mass M
Time T

Fundamental dimensions :

Fixed dimensions are called "fundamental dimensions" (or) "fundamental quantity".

Secondary (or) Derived Quantities :

Secondary (or) Derived quantities are those quantities which posses more than one fundamental dimensions.

S.No	Physical Quantity	Symbol	Dimensions
	(a) Fundamental		
1.	Length	L	L
2.	Mass	M	M
3.	Time	T	T
	(b) Geometric		
1.	Area	A	L^2
2.	Volume	V	L^3
	(c) Kinematic Quantities		
1.	Velocity	v	LT^{-1}
2.	Angular velocity	ω	T^{-1}
3.	Acceleration	a	LT^{-2}
4.	Discharge	Q	L^3T^{-1}
5.	Acceleration Due to Gravity	g	LT^{-2}
6.	Kinematic Viscosity	ν	L^2T^{-1}
	(d) Dynamic Quantities		
1.	Force	F	MLT^{-2}
2.	Weight	W	MLT^{-2}
3.	Density	ρ	ML^{-3}
4.	Specific weight	w	$ML^{-2}T^{-2}$

S.NO	Physical Quantity	Symbol	Dimensions
5.	Dynamic viscosity	μ	$ML^{-1}T^{-1}$
6.	Pressure Intensity	P	$ML^{-1}T^{-2}$
7.	Modulus of Elasticity	$\begin{cases} K \\ E \end{cases}$	$ML^{-1}T^{-2}$
8.	Surface Tension	σ	MT^{-2}
9.	Shear stress	τ	$ML^{-1}T^{-2}$
10.	Work, Energy	W (or) E	$ML^{2}T^{-2}$
11.	Power	P	$ML^{2}T^{-3}$
12.	Torque	T	$ML^{2}T^{-2}$
13.	Momentum	M	MLT^{-1}

Problems :

1) Determine the dimension of the quantities given below.

i) Angular velocity ii) Angular Acceleration

iii) Discharge iv) kinematic viscosity v) Force

vi) specific weight vii) Dynamic viscosity

Sol:

i) Angular velocity $= \dfrac{\text{Angle covered in radians}}{\text{Time}}$

$$= \frac{1}{T}$$

Angular velocity $= T^{-1}$

(ii) Angular Acceleration $= rad/sec^2$

$$= \frac{rad}{T^2} = \frac{1}{T^2}$$

$$\text{Angular Acceleration} = T^{-2}$$

iii) Discharge, $=$ Area $\times$ velocity

$$= L^2 \times \frac{L}{T} = \frac{L^3}{T}$$

$$\text{Discharge} = L^3 T^{-1}$$

iv) Kinematic viscosity $= \frac{\mu}{\rho}$

where μ is given by $\tau = \mu \frac{du}{dy}$

$$\mu = \frac{\tau}{\frac{du}{dy}} = \frac{\text{Shear Stress}}{\frac{\cancel{L}}{T} \times \frac{1}{\cancel{L}}}$$

$$= \frac{\text{Force / Area}}{\frac{1}{T}}$$

$$= \frac{\text{Mass} \times \text{Acceleration}}{\text{Area} \times \text{Time}} = \frac{M \times \frac{L}{T^2}}{L^2 \times \frac{1}{T}}$$

$$= \frac{M\cancel{L}}{L^{\cancel{2}} T^{\cancel{2}} \times \frac{1}{\cancel{T}}} = \frac{M}{LT}$$

$$\mu = ML^{-1}T^{-1}$$

$$\rho = \frac{\text{mass}}{\text{volume}} = \frac{M}{L^3} = ML^{-3}$$

$$\text{Kinematic viscosity} = \frac{\mu}{\rho} = \frac{\cancel{M}L^{-1}T^{-1}}{\cancel{M}L^{-3}} = \frac{T^{-1}}{L^{-2}}$$

$$\text{Kinematic viscosity} = L^2 T^{-1}$$

(v) Force = Mass × Acceleration

$$= M \times \frac{\text{Length}}{(\text{Time})^2} = \frac{ML}{T^2}$$

Force $= MLT^{-2}$

vi) specific weight $= \dfrac{\text{weight}}{\text{volume}}$

$$= \frac{\text{Force}}{\text{volume}} = \frac{MLT^{-2}}{L^3}$$

specific weight $= ML^{-2}T^{-2}$

vii) Dynamic viscosity, μ is derived

$$\mu = ML^{-1}T^{-1}.$$

P.No: 569 Dimensional Homogeneity :

Dimensional homogeneity means the dimension of each terms in an equation on both sides are equal. Thus if the dimensions of each term on both sides of an equation are the same the equation is known as "dimensionally homogeneous equation".

Let us consider,

$$V = \sqrt{2gH}$$

Dimensions of L.H.S $= V = \frac{L}{T} = LT^{-1}$

Dimensions of R.H.S $= \sqrt{2gH} = \sqrt{\frac{L}{T^2} \times L} = \sqrt{\frac{L^2}{T^2}}$

$$= \frac{L}{T} = LT^{-1}$$

$\frac{T^{-1}}{L^{-2}}$

Dimensions of L.H.S = Dimension of R.H.S $= LT^{-1}$

∴ Equation $V = \sqrt{2gH}$ is dimensionally homogeneous. so it can be used in any system of units.

P.No: 56

Methods of Dimensional Analysis :

1) Rayleigh's method

2) Buckingham's π-thorem

1) Rayleigh's method :

This method is used for determining the expression for a variable which depends upon three (or) four variable only. If the number of independent variables becomes more than four, then it is very difficult to find the expression for the dependent variables.

This can also written as

$$X = K x_1^a \cdot x_2^b \cdot x_3^c$$

where,

K – constant

a, b & c – arbitrary powers.

x – variable $[x_1, x_2 \; \& \; x_3]$

2) Buckingham's π-Theorem :

The Rayleigh's method of dimensional analysis becomes more laborious if the variables are more than the number of fundamental dimensions (M, L, T). This difficulty is overcome by using "Buckinghan's π-theorem". If there are n variables [independant & dependent variables] in a physical phenomenon & if these variables contain m fundamental dimension (M, L, T), Then the variables are arranged into $(n-m)$ dimensionless terms. Each term is called "π - term".

$$\pi_1 = \phi \; \pi_2, \pi_3 \cdots \cdots \pi_{n-m}$$

$$\pi_2 = \phi_1 \; \pi_1, \pi_3, \cdots \cdots \pi_{n-m}$$

8) The time period (t) of a pendulum depends upon the length (L) of the pendulum & acceleration due to gravity (g). Derive an expression for the time period.

Sol:

Time period t is a function of (i) L and (ii) g

$$t = K L^a . g^b$$

where,

K is a constant

Substituting the dimensions on both sides

$$T^1 = K L^a . (L T^{-2})^b$$

Equating the powers of M, L and T on both sides

Power of T, $1 = -2b \quad \therefore b = -\frac{1}{2}$

Power of L, $0 = a + b \quad \therefore a = -b = -\left(-\frac{1}{2}\right)$

$$a = \frac{1}{2}$$

Substituting the values of a and b in equation,

$$t = kL^{1/2} \cdot g^{-1/2}$$

$$t = k\sqrt{\frac{L}{g}}$$

The value of k is determined from experiments which given as,

$$k = 2\pi$$

$$\boxed{t = 2\pi\sqrt{\frac{L}{g}}}$$

P.No: 566

Method of Selecting Repeating variables:

The number of repeating variables are equal to the number of fundamental dimensions of the problem.

1. As far as possible, the dependant variable should not be selected as repeating variable.
2. Variable with
   i) Geometric property
      a) Length, l b) d c) Height, H
   ii) flow property
      a) velocity, v b) Acceleration
   iii) fluid property
      a) μ b) ρ c) w
3. The repeating variables selected should not form a dimensionless group.
4. The same number of fundamental dimensions.
5. No two repeating variable should have the same dimensions.

P.NO : 578

Model :

"Model" is the small scale replica of the actual structure (or) machine.

Prototype : [application : To Eliminate the defects, Improve performance]

The actual structure (or) machine is called "Prototype".

Similitude :

"Similitude" is defined as the similarity between the model & its prototype.

Types :

1) Geometric similarity
2) Kinematic similarity
3) Dynamic similarity

P.NO : 579

1) Geometric similarity :

$$\frac{\forall_p}{\forall_m} = \left(\frac{L_p}{L_m}\right)^3 = \left(\frac{b_p}{b_m}\right)^3 = \left(\frac{D_p}{D_m}\right)^3$$

where,

~~let~~,

L_m = Length of model

D_m = Diameter of model

b_m = Breadth of model

A_m = Area of model

$\forall_m$ = Volume of model

$L_p, b_p, D_p, \forall_p$ = Corresponding values of the prototype.

P.NO: 579 2) Kinematic similarity:

$$\frac{a_{p_1}}{a_{m_1}} = \frac{a_{p_2}}{a_{m_2}} = a_r$$

where,

V_{p_1} = velocity of fluid at point 1 in pro

V_{p_2} = " " " point 2 "

a_{p_1} = Acceleration of fluid at point 1 in

a_{p_2} = " " " point 2

$V_{m_1}, V_{m_2}, a_{m_1}, a_{m_2}$ = Corresponding values at corresponding points of fluid velocity & accel of the model

P.NO: 580 3) Dynamic similarity:

$$\frac{(F_i)_p}{(F_i)_m} = \frac{(F_v)_p}{(F_v)_m} = \frac{(F_g)_p}{(F_g)_m} = F_r$$

where,

$(F_i)_p$ = Inertia force at a point in Prototyp

$(F_v)_p$ = Viscous force " " " "

$(F_g)_p$ = Gravity force " " " "

$(F_i)_m$ $(F_v)_m$ $(F_g)_m$ = Corresponding values of at the corresponding in model.

Types of forces acting in moving fluid:

1) Inertia force, F_i
2) Viscous force, F_v
3) Gravity force, F_g
4) Pressure force, F_P
5) Surface Tension force, F_s
6) Elastic force, F_e

1. Inertia force (F_i):

It is equal to the product of mass & acceleration of the flowing fluid and acts in the direction opposite to the direction of acceleration.

2) Viscous force (F_v):

It is equal to the product of shear stress (τ) due to viscosity and surface area of the flow.

3) Gravity force (F_g):

It is equal to the product of mass & acceleration due to gravity of the flowing fluid.

4) Pressure force (F_P):

It is equal to the product of pressure intensity & cross-sectional area of the flowing fluid.

5) Surface Tension Force (Fs) :

It is equal to the Product of surface tension & length of surface of the flowing fluid.

6) Elastic force (Fe) :

It is equal to the Product of elastic stress & area of the flowing fluid.

PNO: 581 Dimensionless Numbers :

Dimensionless numbers are those which are obtained by dividing the inertia force by viscous force (or) gravity force (or) Pressure force (or) Surface tension force (or) Elastic force.

1) Reynold's number
2) Froude's number
3) Euler's number
4) Weber's number
5) Mach's number

1) Reynolds number :

It is defined as the ratio of inertia force of a flowing fluid & the viscous force of the fluid.

$$Re = \frac{V \times d}{\nu} \text{ (or) } \frac{\rho V d}{\mu}$$

2) Froude's number : (Fe)

The Froude's number is defined as the square root of the ratio of inertia force of a flowing fluid to the gravity force.

$$Fe = \frac{V}{\sqrt{Lg}} \quad (or) \quad Fe = \sqrt{\frac{Fi}{Fg}}$$

3) Euler's number : (Eu)

It is defined as the square root of the ratio of inertia force to the pressure force of flowing fluid.

$$Eu = \sqrt{\frac{Fi}{Fp}}$$

Calculation,

$$Eu = \frac{V}{\sqrt{P/\rho}}$$

4) Weber's number : (We)

It is defined as the square root of the ratio ~~of~~ to the surface tension force of the flowing fluid.

$$We = \sqrt{\frac{Fi}{Fs}}$$

Calculation,

$$We = \frac{V}{\sqrt{\sigma / \rho L}}$$

5) Mach's number : (M)

It is defined as the square root of the ratio of the inertia force to the elastic force of a flowing fluid

$$M = \sqrt{\frac{F_i}{F_e}}$$

Calculation,

$$M = \frac{V}{C}$$

P.No: 583 Model Laws (or) Similarity Laws :

The dynamic similarity between the model & the prototype, the ratio of the corresponding forces acting at the corresponding points in the model and prototype should be equal.

1. Reynold's model law - 583
2. Froude model law - 587
3. Euler model law - 595
4. Weber model law - 596
5. Mach mode law - 596

1) Reynold's model law :

Reynold's model law is the law in which models are based on Reynold's number. Models based on Reynold's number includes :

i) Pipe flow

ii) Resistance experienced by sub-marines, airplanes, fully immersed bodies etc.

$$[Re]_m = [Re]_p \quad (or) \quad \frac{\rho_m V_m L_m}{\mu_m} = \frac{\rho_p V_p L_p}{\mu_p}$$

where,

V_m = velocity of fluid in model

ρ_m = Density of fluid in model

L_m = Length (or) Linear dimension of the model

μ_m = viscosity (or) fluid in model

V_p, ρ_p, L_p and μ_p = Corresponding values of velocity, density, linear & viscosity of fluid in prototype.

2) Froude model Law :

Froude model law is the law in which the models are based on Froude number which means for dynamic similarity between the model & prototype, the Froude number for both of them should be equal.

$$[Fe]_{model} = [Fe]_{Prototype}$$

[or]

$$\frac{V_m}{\sqrt{g_m L_m}} = \frac{V_P}{\sqrt{g_P L_P}}$$

where,

g_m = Acceleration due to gravity at a pl[ace] model is tested.

3) Euler's model law:

$$[Eu]_{model} = [Eu]_{Prototype}$$

(or)

$$\frac{V_m}{\sqrt{P_m}} = \frac{V_P}{\sqrt{P_P}}$$

4) Weber model law:

$$[We]_{model} = [We]_{Prototype}$$

(or)

$$\frac{V_m}{\sqrt{\sigma_m / \rho_m \cdot L_m}} = \frac{V_P}{\sqrt{\sigma_P / \rho_P \cdot L_P}}$$

5) Mach model Law:

$$[M]_{model} = [M]_{Prototype}$$

(or)

$$\frac{V_m}{\sqrt{K_m / \rho_m}} = \frac{V_P}{\sqrt{K_P / \rho_P}}$$

Problems:

1) a) State Buckingham's π - theorem.

b) The efficiency η of a fan depends on density ρ, dynamic viscosity μ of the fluid, angular velocity ω, diameter D of the rotor & the discharge Q. Express η in terms of dimensionless parameters.

sol:

$$\eta = f(\rho, \mu, \omega, D, Q)$$

(or)

$$f(\eta, \rho, \mu, \omega, D, Q) = 0 \longrightarrow ① \text{ eqn}$$

The total no. of variables $n = 6$

η = Dimensionless

$\rho = ML^{-3}$

$\mu = ML^{-1}T^{-1}$

$\omega = T^{-1}$

$D = L$

$Q = L^3 T^{-1}$

$\therefore m = 3$ $\quad f(\pi_1, \pi_2, \pi_3) = 0 \rightarrow ②$ eqn

Number of π-terms $= n - m = 6 - 3 = 3$

$$\pi_1 = D^{a_1} \cdot \omega^{b_1} \cdot \rho^{c_1} \cdot \eta$$

$$\pi_2 = D^{a_2} \cdot \omega^{b_2} \cdot \rho^{c_2} \cdot \mu$$

$$\pi_3 = D^{a_3} \cdot \omega^{b_3} \cdot \rho^{c_3} \cdot Q$$

First π-term :

subs. dimensions on both sides of π_1

$$M^0L^0T^0 = L^{a_1} \cdot (T^{-1})^{b_1} \cdot (ML^{-3})^{c_1} \cdot ML^{-1}T^{-1}$$

Equating the powers of M, L, T on both sides,

Power of M, $0 = c_1 + 0$, $\therefore c_1 = 0$

Power of L $0 = a_1 + 0$, $\therefore a_1 = 0$

Power of T $0 = -b_1 + 0$ $\therefore b_1 = 0$

substituting the values of a_1, b_1 and c_1 in π_1,

$$\pi_1 = D^0 \omega^0 \rho^0 \cdot \eta = M$$

second π-term : $\pi_2 = D^{a_2} \cdot \omega^{b_2} \cdot \rho^{c_2} \cdot \mu$

substituting the dimensions on both sides

$$M^0L^0T^0 = L^{a_2} \cdot (T^{-1})^{b_2} \cdot (ML^{-3})^{c_2} \cdot ML^{-1}T^{-1}$$

Equating the powers of M, L, T on both sides,

Power of M, $0 = c_2 + 1$ $\therefore c_2 = -1$

Power of L, $0 = a_2 - 3c_2 - 1$ $\therefore a_2 = 3c_2 + 1$

$$= -3 + 1$$

$$a_2 = -2$$

Power of T, $0 = -b_2 - 1$ $\therefore b_2 = -1$

sub. the values a_2, b_2 and c_2 in π_2

$$\pi_2 = D^{-2} \cdot \omega^{-1} \cdot \rho^{-1} \cdot \mu = \frac{\mu}{D^2 \omega \rho}$$

Third π-term :

$$\pi_3 = D^{a_3} . \omega^{b_3} . \rho^{c_3} . Q$$

substituting the dimensions on both sides,

$$M^0 L^0 T^0 = L^{a_3} . (T^{-1})^{b_3} . (ML^{-3})^{c_3} . L^3 T^{-1}$$

Equating the powers of M, L and T on both sides,

Power of M, $0 = c_3$ $\therefore c_3 = 0$

Power of L, $0 = a_3 - 3c_3 + 3$ $\therefore a_3 = 3c_3 - 3$
$= -3$

Power of T, $0 = -b_3 - 1$ $\therefore b_3 = -1$

sub. the values of a_3, b_3 and c_3 in π_3

$$\pi_3 = D^{-3} \omega^{-1} . \rho^0 . Q = \frac{Q}{D^2 \omega}$$

substituting the values of π_1 and π_2 in ② - eqn

$$f_1\left(\eta , \frac{\mu}{D^2 \omega \rho} , \frac{Q}{D^2 \omega}\right) = 0$$

[or]

$$\eta = \phi\left[\frac{\mu}{D^2 \omega \rho} , \frac{Q}{D^2 \omega}\right]$$

P.NO : 590

2) In 1 in 40 model of a spillway, the velo[city] and discharge are 2 m/s and 2.5 m^3/s. Find the corresponding velocity & discharge in the prototype.

Data:

Scale ratio of length, $L_r = 40$

Velocity of model, $V_m = 2$ m/s

Discharge of model, $Q_m = 2.5\ m^3/s$

Find:

i) velocity of Prototype, $V_p = ?$

ii) Discharge of Prototype, $Q_p = ?$

Sol:

i) velocity of Prototype, V_p

velocity ratio, $\frac{V_p}{V_m} = \sqrt{L_r}$

$$V_p = V_m \times \sqrt{L_r}$$

$$= 2 \times \sqrt{40}$$

$$V_p = 12.64\ m/s$$

ii) Discharge of Prototype, Q_p

Discharge ratio, $\frac{Q_p}{Q_m} = L_r^{2.5}$

$$Q_p = (L_r)^{2.5} \times Q_m$$

$$= (40)^{2.5} \times 2.5$$

$$Q_p = 25298.22\ m^3/s$$

P.No : 604

Classification of models :

1) Undistorted models

2) Distorted models

1) Undistorted models :

"Undistorted models" are those models which are geometrically similar to their prototypes (or) in other words if the scale ratio for the linear dimensions of the model & its prototype is same, the model is called "Undistorted model".

2) Distorted models :

A model is said to be distorted if it is not geometrically similar to its prototype. For a distorted model different scale ratios for the linear dimensions are adopted.

Advantages of Distorted models :

1) The vertical dimensions of the model can be measured accurately.

2) The cost of the model can be reduced.

3) Turbulant flow in the model can be maintained.

P.No : 605

Scale Ratios for Distorted Models :

1. Scale ratio for velocity :

Let,

V_P = velocity in prototype

V_m = velocity in model

Then,

$$\frac{V_P}{V_m} = \frac{\sqrt{2g h_P}}{\sqrt{2g h_m}} = \sqrt{\frac{h_P}{h_m}} = \sqrt{(L_r)_V}$$

$$\therefore \frac{h_P}{P_m} = (L_r)_V$$

2. Scale ratio for area of flow.

Let,

A_P = Area of flow in prototype $= B_P \times h_P$

A_m = Area of flow in model $= B_m \times h_m$

Then,

$$\frac{A_P}{A_m} = \frac{B_P \times h_P}{B_m \times h_m} = \frac{B_P}{B_m} \times \frac{h_P}{h_m} = (L_r)_H \times (L_r)_V$$

3. Scale ratio for discharge :

Let,

Q_P = Discharge through prototype $= A_P \times V_P$

Q_m = Discharge " model $= A_m \times V_m$

Then,

$$\frac{Q_P}{Q_m} = \frac{A_P \times V_P}{A_m \times V_m} = (L_r)_H \times (L_r)_V \times \sqrt{(L_r)_V} = (L_r)_H \times [(L_r)_V]^{3/2}$$

P.No: 608

The discharge through a weir is 1.5 m^3/s. Find the discharge through the model of the weir if the horizontal dimension of the model = $\frac{1}{50}$ the horizontal dimension of the prototype & vertical dimension of the model = $\frac{1}{10}$ the vertical dimension of the prototype.

Sol:

Discharge through weir (prototype) = $Q_p = 1.5\ m^3/s$

Horizontal dimension of model $\Big\} = \frac{1}{50} \times$ Horizontal dimension of prototype

$$\therefore \frac{\text{Horizontal dimension of Prototype}}{\text{Horizontal dimension of model}} = 50 \ [\text{or}]$$

$$(L_r)_H = 50$$

$B_p \times h_p$

$\times h_m$

Vertical dimension of model = $\frac{1}{10} \times$ vertical dimension of prototype

$$\therefore \frac{\text{vertical dimension of Prototype}}{\text{vertical dimension of model}} = 10$$

$$(L_r)_v = 10$$

$\times (L_r)_v$

Using equation,

$$\frac{Q_p}{Q_m} = (L_r)_H \times \left[(L_r)_v\right]^{3/2}$$

$$= 50 \times 10^{3/2}$$

$A_p \times V_p$

$\times V_m$

$\times \left[(L_r)_v\right]^{3/2}$

$$\frac{Q_p}{Q_m} = 1581.14$$

$$Q_m = \frac{Q_p}{1581.14} = \frac{1.50}{1581.4}$$

$$Q_m = 0.00094\ m^3/s$$

$$\Rightarrow \boxed{Q_m = 0.948\ lit/s}$$

UNIT IV

RECIPROCATING PUMP (Single Acting)

▸ 20.2 MAIN PARTS OF A RECIPROCATING PUMP

The following are the main parts of a reciprocating pump as shown in Fig. 20.1 :

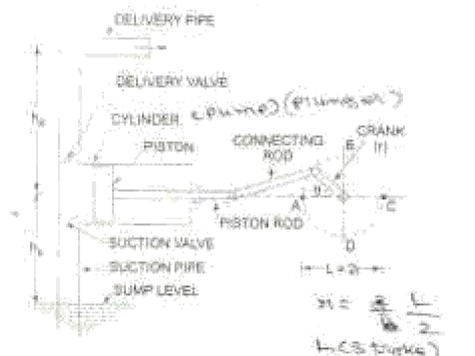


Fig. 20.1 Main parts of a reciprocating pump.

Discharge Through a Reciprocating pump :

D – Diameter of cylinder

A – cross-sec area of piston or cylinder

$= \frac{\pi}{4} D^2$

r – radius of crank.

N – r.p.m. of crank.

h_s – height of the axis of the cylinder from water surface in sump. (suction head) (below pump axis)

h_d – height of delivery outlet above the cylinder axis. (Delivery head)

Volume of water delivered in one revolution or

discharge of water in one revolution

= Area × length of stroke $= A \times L$

Number of revolution per sec, $= \frac{N}{60}$

Therefore:
Discharge pump per second.

$$Q = \text{Discharge in one revolution} \times \text{No. of revolution per second}$$

$$= A \times L \times \frac{N}{60} = \frac{ALN}{60}$$

Weight of water delivered per second.

$$W = \rho \times g \times Q = \frac{\rho g A L N}{60} \quad \therefore g = 9.81$$

Work done by reciprocating pump:

$$\text{Work done per second} = W \times (h_s + h_d)$$

$(h_s + h_d)$ = Total height through which water is lifted.

$$W = \frac{\rho g A L N}{60}$$

$$\text{Work done per second} = \frac{\rho g A L N}{60} \times (h_s + h_d). \text{ KJ}$$

∴ Power required to drive the pump, in KW

$$P = \frac{\text{Work done per second}}{1000} = \frac{\rho g \times A L N \times (h_s + h_d)}{60 \times 1000}$$

$$= \frac{\rho g \times A L N \times (h_s + h_d)}{60{,}000} \text{ KW.}$$

Discharge, Work Done and Power Required to Drive a Double-Acting Pump:

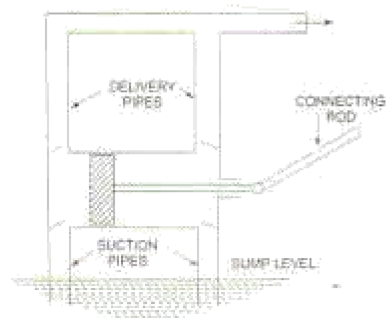


Fig. 20.2

D – diameter of the piston

d – diameter of the piston rod.

Area on one side of piston

$$A = \frac{\pi}{4} D^2$$

Area on the other side of piston, where piston rod is connected to the piston.

$$A_1 = \frac{\pi}{4} D^2 - \frac{\pi}{4} d^2 = \pi/4 (D^2 - d^2)$$

$$Q = \frac{2ALN}{60}$$

Work done by Double - Acting reciprocating pump.

Work done per second $= 2 \rho g \times \frac{ALN}{60} \times (h_s + h_d)$

∴ Power req. to drive the double-acting pump in kW

$$P = \frac{2 \rho g \times ALN \times (h_s + h_d)}{60,000}$$

Slip of Reciprocating pump:

$$\text{Slip} = Q_{th} - Q_{act}$$

$$\% \text{ of slip} = \frac{Q_{th} - Q_{act}}{Q_{th}} \times 100 = \left(1 - \frac{Q_{act}}{Q_{th}}\right) \times 100$$

$$= (1 - C_d) \times 100.$$

$$\left(\because \frac{Q_{act}}{Q_{th}} = C_d\right)$$

where C_d = Co-eff. of discharge

1) **Problem 20.1** *A single-acting reciprocating pump, running at 50 r.p.m., delivers 0.01 m³/s of water. The diameter of the piston is 200 mm and stroke length 400 mm. Determine :*
(i) The theoretical discharge of the pump, (ii) Co-efficient of discharge, and (iii) Slip and the percentage slip of the pump.

G.D:

$N = 50 \text{ r.p.m.}$

$Q_{act} = 0.01 \text{ m}^3/\text{s}$

$D = 200 \text{ mm}$

$= \frac{200}{1000} = 0.2 \text{ m.}$

$L = 400 \text{ mm}$

$= \frac{400}{1000} = 0.4 \text{ m.}$

$\text{Area} = A = \frac{\pi}{4} D^2$

$= \frac{\pi}{4} (0.2)^2$

$= 0.031416 \text{ m}^2$

Soln:

(i) Theoretical discharge of the pump:

$$\boxed{Q_{th} = \frac{ALN}{60}} = \frac{0.031416 \times 0.4 \times 50}{60}$$

$= 0.01047 \text{ m}^3/\text{s.}$

(ii) Co-eff of Discharge:

$$C_d = \frac{Q_{act}}{Q_{th}} = \frac{0.01}{0.01047} = 0.95511$$

(iii) Slip and the % of slip of pump:

$$\text{Slip} = Q_{th} - Q_{act}$$

$$= 0.01047 - 0.01$$

$$= 4.7 \times 10^{-04}\ m^3/s$$

$$\%\text{ of slip} = \frac{Q_{th} - Q_{act}}{Q_{th}} \times 100$$

$$= \frac{0.01047 - 0.01}{0.01047} \times 100$$

$$= 4.489\%$$

RESULT:

$Q_{th} = 0.01047\ m^3/s$

$C_d = 0.95511$

$\text{Slip} = 4.7 \times 10^{-04}\ m^3/s$

$\%\text{ of Slip} = 4.489\%$

3) **Problem 20.2** *A double-acting reciprocating pump, running at 40 r.p.m., is discharging 1.0 m³ of water per minute. The pump has a stroke of 400 mm. The diameter of the piston is 200 mm. The delivery and suction head are 20 m and 5 m respectively. Find the slip of the pump and power required to drive the pump.*

Given Data:

$N = 40$ r.p.m.

$Q_{act} = 1.0 \ m^3/min$

$= \frac{1.0}{60} = 0.01666 \ m^3/s$

$L = 400$ mm

$= \frac{400}{1000} = 0.4$ m

$D = 200$ mm.

$= \frac{200}{1000} = 0.2$ m.

$A = \frac{\pi}{4} D^2$

$= \frac{\pi}{4} (0.2)^2$

$= 0.031416 \ m^2$

$h_d = 20$ m

$h_s = 5$ m

Soln: The discharge for double-acting pump

$\boxed{Q_{th} = \frac{2ALN}{60}} = \frac{2(0.031416)\, 0.4 \times 40}{60}$

$= 0.01675 \ m^3/s.$

$\boxed{\text{slip} = Q_{th} - Q_{act}}$

$= 0.01675 - 0.01666$

$= 9 \times 10^{-05} \ m^3/s.$

Power required the double acting pump

$$P = \frac{2 \rho g ALN \times (h_s + h_d)}{60{,}000}$$

$$= \frac{2 \times 1000 \times 9.81 \times 0.031416 \times 0.4 \times 40 \times (5 + 20)}{60{,}000}$$

$= 4.10921$ KW

RESULT:

$Q_{th} = 0.01675 \; m^3/s$

$Slip = 9 \times 10^{-05} \; m^3/s$

$P = 4.10921$ KW.

Variation of Velocity And Acceleration in the Suction And Delivery Pipes Due to Acceleration of the Piston

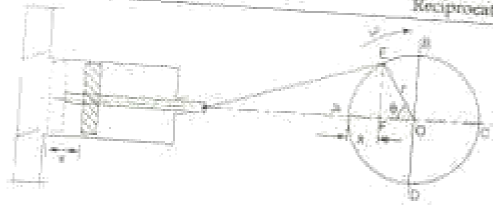


Fig. 20.3 Velocity and acceleration of piston.

ω = Angular speed of the crank in rad/s

A = Area of the cylinder

a = Area of the pipe (suction or delivery)

l = length of the pipe (suction or delivery) l_s l_d

r = Radius of the crank.

$$\omega = \frac{2\pi N}{60}$$

~~ ~~

Pr. head due to acceleration in suction and delivery pipes.

(Pr head due to acceleration = h_a)

ha at suction pipe

$$\boxed{h_{as} = \frac{l_s}{g} \times \frac{A}{a_s} \omega^2 r \cos\theta}$$

ha at deliv. pipe

$$\boxed{h_{ad} = \frac{l_d}{g} \times \frac{A}{a_d} \omega^2 r \cos\theta}$$

Diff value of θ are.

$\theta = 0$ $\cos 0° = 1$

$\theta = 90°$ $\cos 90° = 0$

$\theta = 180$ $\cos 180° = -1$

The

Problem 20.3 *The cylinder bore diameter of a single-acting reciprocating pump is 150 mm and its stroke is 300 mm. The pump runs at 30 r.p.m. and lifts water through a height of 25 m. The delivery pipe is 22 m long and 100 mm in diameter. Find the theoretical discharge and the theoretical power required to run the pump. If the actual discharge is 4.2 litres/s, find the percentage slip. Also determine the acceleration head at the beginning and middle of the delivery stroke.*

Given:

$D = 150\,mm$

$= \frac{150}{1000} = 0.15\,m$ $\qquad A = \frac{\pi}{4} D^2$

$= 0.01767\,m^2$

$L = 300\,mm$

$= \frac{300}{1000} = 0.3\,m.$

$N = 50\,r.p.m.$

$H = 25\,m.$

$l_d = 22\,m.$

$d_d = 100\,mm$

$= \frac{100}{1000} = 0.1\,m$

$Q_{act} = 4.2\,l/s.$

$= \frac{4.2}{1000}$

$= 4.2 \times 10^{-3} = 0.0042\,m^3/s$

Soln:

Theoretical discharge

$$\boxed{Q_{th} = \frac{ALN}{60}} = \frac{0.01767 \times 0.3 \times 50}{60}$$

$Q_{th} = 4.4175 \times 10^{-3}$

Theoretical power

$$\boxed{P_t = \frac{\rho g Q_{th} \times H}{1000}}$$

$$= \frac{1000 \times 9.81 \times 4.4175 \times 10^{-3} \times 25}{1000}$$

$P_t = 1.0833\,kW$

% of slip :

$$\boxed{\% \text{ of slip} = \frac{Q_{th} - Q_{act}}{Q_{th}}}$$

$$= \frac{4.4175 \times 10^{-3} - 4.2 \times 10^{-3}}{4.4175 \times 10^{-3}}$$

% of slip = 4.92 %

Acceleration head at the beginning of delivery pipe :

$$\boxed{h_{ad} = \frac{l_d}{g} \times \frac{A}{a_d} \omega^2 r \times \cos\theta}$$

r — crank radius

g – gravity 9.81

l_d – length of delivery pipe

a_d – diameter , , , — $a_d = \frac{\pi}{4} d^2 = 7.854 \times 10^{-3}$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi 50}{60}$$

$= 5.236$

$$r = \frac{L}{2} = \frac{0.3}{2} = 0.15\,m$$

$$h_{ad} = \frac{22}{9.81} \times \frac{0.01767}{7.854 \times 10^{-3}} \times 5.236^2 \times 0.15 \times \cos\theta$$

$= 20.75 \times \cos\theta$.

At the beginning of the delivery stroke.

$\theta = 0$ and hence $\cos\theta = 1$

$h_{ad} = 20.75 \times 1 = 20.75\,m$

Acceleration head at middle of delivery stroke:

$\theta = 90°$ hence $\cos 90 = 0$

$h_{ad} = 20.75 \times 0 = 0.$

RESULT:

$Q_{th} = 4.4175 \times 10^{-3}$

% of slip = 4.92 %

$h_{ad} = 20.75$ m. (beginning)

$h_{ad} = 0.$ middle

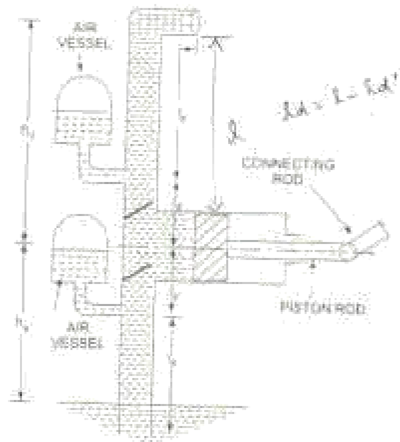


Fig. 20.9. Air vessels fitted to reciprocating pump.

Let A = Cross-sectional area of the cylinder,

a = Cross-sectional area of suction or delivery pipe.

d_d = Diameter of delivery pipe

d_s = " " suction "

l = total length of delivery pipe ([illegible] water reservoir)

l_d = Length of delivery pipe beyond the air vessel,

l_d' = Length of delivery pipe between cylinder and air vessel,

l_s' = Length of suction pipe between cylinder and air vessel,

l_s = Length of suction pipe below air vessel,

h_{ad} = Pressure head due to acceleration in delivery pipe,

h_{as} = Pressure head due to acceleration in suction pipe,

h_{fd} = Loss of head due to friction in delivery pipe beyond the air vessel,

h_{fd}' = Loss of head due to friction in delivery pipe between cylinder and air vessel,

h_{fs} = Loss of head due to friction in suction pipe below the air vessel, and

h_{fs}' = Loss of head due to friction in suction pipe between cylinder and air vessel.

h_s = suction head

h_d = Discharge of water height (delivery head)

$$\omega = \frac{2\pi N}{60} \text{ rad/s}$$

$g = 9.81$

$\rho = 1000$

(*i*) At the beginning of the delivery stroke, $\theta = 0°$, $\sin\theta = 0$ and $\cos\theta = 1$ and hence total pressure head

$$= (h_d + h_{ad} + h_{fd}' + h_{fd}) + \text{velocity head at the outlet of delivery}$$

$$= h_d + h_{ad} + h_{fd}' + h_{fd} + \frac{\bar{V}_d^2}{2g} \qquad (\because \text{ Velocity at outlet is equal to mean velocity})$$

$$= h_d + \frac{l_d'}{g} \times \frac{A}{a_d}\omega^2 r + 0 + \frac{4f \times l_d}{d_d \times 2g} \times \left(\frac{A}{a_d} \times \frac{\omega r}{\pi}\right)^2 + \frac{\left(\frac{A}{a_d} \times \frac{\omega r}{\pi}\right)^2}{2g} \qquad \left(\because \bar{V}_d = \frac{A}{a_d} \times \frac{\omega r}{\pi}\right)$$

$$\boxed{= h_d + \frac{l_d'}{g} \times \frac{A}{a_d}\omega^2 r + \frac{4f l_d}{d_d \times 2g} \times \left(\frac{A}{a_d} \times \frac{\omega r}{\pi}\right)^2 + \frac{1}{2g}\left(\frac{A}{a_d} \times \frac{\omega r}{\pi}\right)^2} \qquad \ldots(20.25)$$

(*ii*) In the middle of the stroke, $\theta = 90°$, $\sin\theta = 1$ and $\cos\theta = 0$ and hence total pressure head

$$= h_d + h_{ad} + h_{fd}' + h_{fd} + \frac{\bar{V}_d^2}{2g} \text{ above atmospheric pressure head}$$

$$= h_d + 0 + \frac{4f \times l_d'}{d_d \times 2g} \times \left(\frac{A}{a_d}\omega r\right)^2 + \frac{4f \times l_d}{d_d \times 2g} \times \left(\frac{A}{a_d} \times \frac{\omega r}{\pi}\right)^2 + \frac{1}{2g}\left(\frac{A}{a_d} \times \frac{\omega r}{\pi}\right)^2$$

$$\boxed{= h_d + \frac{4f \times l_d'}{d_d \times 2g} \times \left(\frac{A}{a_d}\omega r\right)^2 + \frac{4f \times l_d}{d_d \times 2g} \times \left(\frac{A}{a_d} \times \frac{\omega r}{\pi}\right)^2 + \frac{1}{2g}\left(\frac{A}{a_d} \times \frac{\omega r}{\pi}\right)^2} \qquad \ldots(20.26)$$

(*iii*) At the end of the delivery stroke, $\theta = 180°$, $\sin\theta = 0$ and $\cos\theta = -1$ and hence total pressure head

$$= h_d - \frac{l_d'}{g} \times \frac{A}{a_d}\omega^2 r + \frac{4f \times l_d}{d_d \times 2g} \times \left(\frac{A}{a_d} \times \frac{\omega r}{\pi}\right)^2 + \frac{1}{2g}\left(\frac{A}{a_d} \times \frac{\omega r}{\pi}\right)^2 \qquad \ldots(20.27)$$

In equations (20.25), (20.26) and (20.27), the quantities

$$\left(\frac{l_d'}{g} \times \frac{A}{a_d}\omega^2 r\right) \text{ and } \left[\frac{4f \times l_d'}{d_d \times 2g} \times \left(\frac{A}{a_d}\omega r\right)^2\right]$$

are very small and can be neglected.

Problem 20.14 *The cylinder of a single-acting reciprocating pump is 15 cm in diameter and 30 cm in stroke. The pump is running at 30 r.p.m. and discharge water to a height of 12 m. The diameter and length of the delivery pipe are 10 cm and 30 m respectively. If a large air vessel is fitted in the delivery pipe at a distance of 2 m from the centre of the pump, find the pressure head in the cylinder.*

(i) *At the beginning of the delivery stroke, and*

(ii) *In the middle of the delivery stroke. Take f = .01*

G.D:

$D = 15\,cm$

$= \frac{15}{100} = 0.15\,m$

Area $= \pi/_4 D^2$

$= 0.01767\,m^2$

Stroke length $L = 30\,cm$

$= \frac{30}{100} = 0.30\,m$

∴ Crank radius $= r = \frac{L}{2} = \frac{0.30}{2} = 0.15\,m$

$N = 30\,r.p.m \Rightarrow \omega = \frac{2\pi N}{60} = 3.14\,rad/s$

$h_d = 12\,m$

$d_d = 10\,cm$

$= \frac{10}{100} = 0.1\,m \Rightarrow a_d = \pi/_4\, d^2 = 7.854\times10^{-3}$

$l = 30\,m$

$l_d' = 2\,m$

$l_d = l - l_d' = 30 - 2$

$= 28\,m$

Co-efficient of friction $f = 0.01$

(i) At the beginning of delivery stroke:

$$= h_d + \frac{l_d'}{g}\times\frac{A}{a_d}\,\omega^2 r + \frac{4f\,l_d}{d_d\times 2g}\times\left(\frac{A}{a_d}\times\frac{\omega r}{\pi}\right)^2 + \frac{1}{2g}\left(\frac{A}{a_d}\times\frac{\omega r}{\pi}\right)^2$$

$$= 12 + \frac{2}{9.81} \times \frac{0.01767}{7.854 \times 10^{-3}} \times 3.14^2 \times 0.15$$

$$+ \frac{4(0.01)\,28}{0.1 \times 2(9.81)} \times \left[\frac{0.01767}{7.854 \times 10^{-3}} \times \frac{3.14 \times 0.15}{\pi}\right]^2$$

$$+ \frac{1}{2 \times 9.81}\left[\frac{0.01767}{0.1} \times \frac{3.14 \times 0.15}{\pi}\right]^2$$

$$= 12 + 0.6783 + 0.065 + 0.0058$$

$$= \underline{12.75\,m.}$$

(ii) middle of the delivery stroke:

$$= h_d + \frac{4f \times l_d'}{d_d \times 2g} \times \left(\frac{A}{a_d}\,\omega r\right)^2 + \frac{4f \times l_d}{d_d \times 2g} \times$$

$$\left[\frac{A}{a_d} \times \frac{\omega r}{\pi}\right]^2 + \frac{1}{2g}\left[\frac{A}{a_d} \times \frac{\omega r}{\pi}\right]^2$$

$$= 12 + \frac{4(0.01) \times 2}{0.1 \times 2(9.81)} \times \left[\frac{0.01767}{7.854 \times 10^{-3}} \times 3.14 \times 0.15\right]^2$$

$$+ \cancel{40(0.01)58}\ 0.065 + 0.0058$$

$$= 12 + 0.0458 + 0.065 + 0.0058$$

$$= \underline{12.116\,m.}$$

Problem 20.15 *A single-acting reciprocating pump is to raise a liquid of density 1200 kg per cubic metre through a vertical height of 11.5 metres, from 2.5 metres below pump axis to 9 metres above it. The plunger, which moves with S.H.M., has diameter 125 mm and stroke 225 mm. The suction and delivery pipes are 75 mm diameter and 3.5 metres and 13.5 metres long respectively. There is a large air vessel placed on the delivery pipe near the pump axis. But there is no air vessel on the suction pipe. If separation takes place at 8.829 N/cm² below atmospheric pressure, find :*

(i) maximum speed, with which the pump can run without separation taking place, and

(ii) power required to drive the pump, if f = 0.02.

Neglect slip for the pump.

Solution. Given :

Given:

Density $\rho = 1200 \text{ kg/m}^3$

Total vertical height $= 11.5$ m

Suction head $h_s = 2.5$ m

Delivery head $h_d = 9$ m

$D = 125$ mm

$= \frac{125}{1000} = 0.125$

$A = \pi/4 \, D^2$

$= 0.0123 \text{ m}^2$

Stroke length $L = 225$ mm

$= \frac{225}{1000} = 0.225$ m $\Rightarrow r = \frac{L}{2} = \frac{.225}{2} = .1125$ m

Dia of suction and delivery pipe $d = 75$ mm

$= \frac{75}{1000} = 0.075$ m

$a = \pi/4 \, d^2$ Sep $P_a = 8.829 \frac{N}{cm^2}$

$= \cancel{0.0044\ m^2}$ $f = 0.02$

$= 4.4178 \times 10^{-3} = 0.00442 \text{ m}^2$

Length of suction pipe $l_s = 3.5$ m

" " delivery " $l_d = 13.5$ m

Air vessel is placed on the delivery side only. Hence, the velocity in the delivery pipe will be uniform. And there will be no accelerating head on delivery side.

Separation pr = $8.829 \frac{N}{cm^2} = 8.829 \times 10^4 \frac{N}{m^2}$ below atmos pr.

Sep pr head, $\boxed{h_{sep} = \frac{sep\ pr}{\rho \times g}}$

$= \frac{8.829 \times 10^4}{1200 \times 9.81}$

$= 7.5\ m$ below atmosphere.

(ii) max speed with which the pump can run without separation taking place.

Let N = max speed with which pump can run without separation taking place. The separation can take place only at the beginning of suction stroke. As air vessel is not fitted on the suction pipe, there will be accelerating head acting on suction side.

Pr head at beginning of suction stroke.

$= h_s + h_{as}$ below atmosphere.

↑ pr head due to acc in suction pipe

This pr should be equal to h_sep in the limiting case.

$7.5 = h_s + h_{as} = 2.5 + h_{as}$

$h_{as} = 7.5 - 2.5 = 5.0\ m$

But h_{as} at the beginning of suction stroke.

$$\boxed{h_{as} = \frac{l_s}{g} \times \frac{A}{a} \omega^2 r}$$

$$5.0 = \frac{3.5}{9.81} \times \frac{0.0123}{4.4178 \times 10^{-3}} \times \omega^2 \times .1125$$

$$\omega = \sqrt{\frac{5.0 \times 9.81 \times 0.00442}{3.5 \times 0.0123 \times 0.1125}}$$

$$= 6.69 \text{ rad/s.}$$

$$\omega = \frac{2\pi N}{60}$$

$$N = \frac{60 \times \omega}{2\pi} = \frac{60 \times 6.69}{2\pi} = 63.88 \text{ r.p.m.}$$

$\therefore$ max speed with which the pump can run without separation taking place is 63.88 r.p.m.

(ii) Power req to drive the pump:

Now discharge (Q) of the single-acting pump is given by

$$Q = \frac{ALN}{60} = \frac{0.0123 \times 0.225 \times 63.88}{60}$$

$$= 0.00294 \text{ m}^3/\text{s}.$$

Velocity of liquid in delivery pipe will be uniform.

Q = Area of delivery pipe × Velocity

$= a \times V$

$$V = \frac{Q}{a} = \frac{0.00294}{0.00442} = 0.665 \text{ m/s.}$$

∴ Head loss due to friction in delivery pipe.

$$h_{fd} = \frac{4f \times l_d \times V^2}{d \times 2g}$$

$$= \frac{4 \times 0.02 \times 13.5 \times (.665)^2}{0.075 \times 2 \times 9.81}$$

$= 0.324$ m.

During suction stroke, the value of max h_{fs} is given by

$$h_{fs} = \frac{4f\, l_s}{d \times 2g} \times \left[\frac{A}{a}\, \omega r\right]^2$$

$$= \frac{4 \times 0.02 \times 3.5}{0.075 \times 2 \times 9.81} \left[\frac{0.0123}{0.00442} \times 6.69 \times 0.125\right]^2$$

$= 0.834$ m.

Now power req. to drive the pump in kW

$$= \frac{W \cdot Q \cdot H_s}{1000} = \frac{\rho g Q}{1000} \times \left[h_s + h_d + \frac{2}{3} h_{fs} + h_{fd}\right]$$

$$= \frac{1200 \times 9.81 \times 0.00294}{1000} \times \left[2.5 + 9.0 + \frac{2}{3} \times 0.834 + 0.324\right].$$

$= 0.428$ kW

Problem 20.16 *A double-acting reciprocating piston pump is pumping water (diameter of the piston 250 mm, diameter of piston rod, which is on one side of the piston 50 mm, piston stroke 380 mm). The suction and discharge heads are 4.5 m and 18.6 m respectively. Find the work done by the piston during outward stroke. Would the work done change for the inward stroke?*

G.D:

$D = 250\,mm = 0.25\,m$

$A = \frac{\pi}{4} D^2$

$= 0.0491\,m^2$

diameter of Piston rod $d = 50\,mm$

$= \frac{50}{1000} = 0.05\,m$

$a = \pi/4\, d^2 = 1.963 \times 10^{-3}$

$= .001963\,m^2$

Piston Stroke $= 380\,mm$

$L = \frac{380}{1000} = 0.380\,m$

$h_s = 4.5\,m$

$h_d = 18.6\,m$

Hence total work done during outward stroke

$$\boxed{= \rho \times g \times Q_1 \times h_s + \rho \times g \times Q_2 \times h_d}$$

find

$Q_1 = A \times L = 0.0491 \times 0.380$

$= 0.01865\,m^3$

$\frac{ALN}{60}$

$N = 1$ [illegible]

$Q_2 = (A - a) \times L$

$= (0.0491 - 0.001963) \times 0.380$

$= 0.01791\,m^3.$

$\rho \times g = 1000 \times 9.81\,N/m^3$

Total W.D during outward stroke :

$= [1000 \times 9.81 \times 0.01865 \times 4.5 + 1000 \times 9.81 \times 0.01491 \times 18.6]$ N m

$= 4091.27$ N.m.

$= 4.0913$ KJ.

N m convert to kJ.

$\frac{N\,m}{1000} = kJ.$

∴ Total work done during forward stroke :

$\boxed{= \rho \times g \times Q_2 \times h_s + \rho \times g \times Q_2 \times h_d}$

$= 1000 \times 9.81 \times 0.01491 \times 4.5 + 1000 \times 9.81 \times 0.01865 \times 18.6$

$= 4193.637$ N.m.

$= 4.193$ KJ.

RESULT : Total dis

outward stroke $= 4.0913$ KJ

inward stroke $= 4.193$ KJ.

inward stroke will be different.

Problem 20.17 *A single-acting reciprocating pump has a plunger diameter of 250 mm and stroke of 450 mm and it is driven with S.H.M. at 60 r.p.m. The length and diameter of delivery pipe are 60 m and 100 mm respectively. Determine the power saved in overcoming friction in the delivery pipe by fitting an air vessel on the delivery side of the pump. Assume friction factor = 0.01.*

G.D.:

Plunger Diameter = $D = 250\,mm = 0.250\,m$

$L = 450\,mm$

$= \frac{450}{1000} = 0.45\,m$

$r = \frac{L}{2} = 0.225$

$A = \pi/_4\, D^2 = 0.049087$

$N = 60\ r.p.m.$

Angular speed $\omega = \frac{2\pi N}{60} = 2\pi\ rad/s$

Length of Delivery pipe $l = 60\,m$

$d = 100\,mm$

$= \frac{100}{1000} = 0.1\,m$

$a = \pi/_4\, d^2 = 7.853 \times 10^{-3}$

$= .007853\ m^2$

Friction factor $f = 0.01$

Power saved is given by:

Power saved $= \rho \times g \times Q \times \left[\frac{2}{3} (h_f)_{\text{without air vessel}} - (h_f)_{\text{with air vessel}} \right]$

$\rho \times g = 1000 \times 9.81\ N/m^3$

$Q = \frac{ALN}{60} = \frac{0.049 \times 0.45 \times 60}{60}$

$= 0.02205\ m^3/s.$

without air vessel — length of delivery pipe

$$h_f = \frac{f \times L \times v^2}{d \times 2g}$$

$$\left[\because v = \left(\frac{A}{a} \times \omega \times r \right) \right]$$

$$= \frac{0.01 \times 60}{0.1 \times 2 \times 9.81} \times \left[\frac{0.049087}{0.007853} \times 2\pi \times 0.225 \right]^2$$

$$= 23.87\,m.$$

With air vessel:

$$h_f = \frac{f \times L \times \bar{v}^2}{d \times 2g}$$

$$\bar{v} = \frac{A}{a} \times \frac{\omega \times r}{\pi}$$

$$= \frac{f \times L}{d \times 2g} \times \left[\frac{A}{a} \times \frac{\omega r}{\pi} \right]^2$$

$$= \frac{0.01 \times 60}{0.1 \times 2 \times 9.81} \times \left[\frac{0.049087}{0.007853} \times \frac{2\pi \times 0.225}{\pi} \right]^2$$

$$= 2.419\,m.$$

$\therefore$ Power saved $= \rho \times g \times Q \times \left[\frac{2}{3} (h_f)_{\text{without air vessel}} - (h_f)_{\text{with air vessel}} \right] W$

$$= 1000 \times 9.81 \times 0.02209 \left[\tfrac{2}{3} \times 23.87 - 2.419 \right] W.$$

$$= 2924.26\,W$$

$$\frac{W}{1000} = kW,$$

$$= 2.924\,kW.$$

Problem 20.18 *A double-acting reciprocating pump runs at 120 r.p.m. When its suction pipe of 100 mm diameter is fitted with an air vessel on its suction side. The diameter of cylinder and stroke are 150 mm and 450 mm respectively. If piston is to be driven with S.H.M., find the rate of flow from or into the air vessel when the crank makes angles of 30°, 90° and 120° with the inner dead centre. Find also the crank angles at which there is no flow into or from the air vessel.*

G.D:

$N = 120$ r.p.m

$\omega = \frac{2\pi N}{60}$

$= 12.566$ rad/s.

Dia of suction pipe $d = 100$ mm $= \frac{100}{1000} = 0.1$ m

$= 0.1$ m.

∴ Area of suction pipe $a = \pi/4 \, d^2$

$= \pi/4 (0.1)^2 = 7.854 \times 10^{-3}$

$= 0.007854$ m²

Dia of cylinder $D = 150$ mm

$= \frac{150}{1000} = 0.15$ m

$A = \pi/4 \, D^2$

$= 0.01767$ m²

Stroke length $L = 450$ mm

$= \frac{450}{1000} = 0.45$ m

Crank radius $r = \frac{L}{2} = \frac{0.45}{2} = 0.225$ m.

(i) Rate of flow of liquid into air vessel:

$= A \omega r \left(\sin\theta - \frac{2}{\pi} \right)$

$= 0.01767 (12.566)(0.225) \left(\sin\theta - \frac{2}{\pi} \right)$

$= 0.04996 \left(\sin\theta - \frac{2}{\pi} \right)$

$\theta = 30°$

The above rate of flow $= 0.04996\left(\sin 30 - \frac{2}{\pi}\right)$

$= -6.825 \times 10^{-3}$

$= -0.00682\ m^3/s.$

$\theta = 90°$

The rate of flow becomes

$= 0.04996\left(\sin 90 - \frac{2}{\pi}\right)$

$= 0.0181\ m^3/s.$

$\theta = 120°$

The rate of flow becomes

$= 0.04996\left(\sin 120 - \frac{2}{\pi}\right)$

$= 0.01146\ m^3/s.$

(ii) Crank angle at which there is no flow.

But rate of flow.

$= 0.04996\left(\sin\theta - \frac{2}{\pi}\right)$

For no flow from or into air vessel,

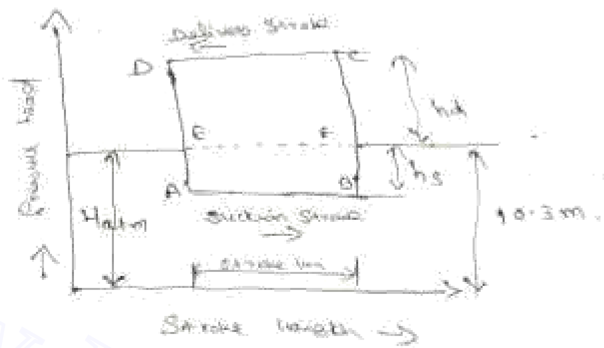
$0.04996\left(\sin\theta - \frac{2}{\pi}\right) = 0$

$\theta = \sin^{-1}(0.6366)$

$= 39° 39'$

INDICATOR DIAGRAM:

Ideal Indicator Diagram:



H_{atm} = atmospheric pressure head

= 10.3 m of water

L = length of stroke.

h_s = suction head

h_d = Delivery head.

We know that the work done by pump per second.

$$= \frac{\rho \times g \times ALN}{60} \times (h_s + h_d)$$

$$= K \times L (h_s + h_d)$$

$$\left[\text{where } K = \frac{\rho g A N}{60} = \text{constant} \right]$$

$$\propto L \times (h_s + h_d)$$

But from Fig 20.4, area of indicator diagram

$$= AB \times BC = AB \times (BF + FC) = L \times (h_s + h_d)$$

Effect of Acceleration in suction and delivery pipes on Indicator Diagram:

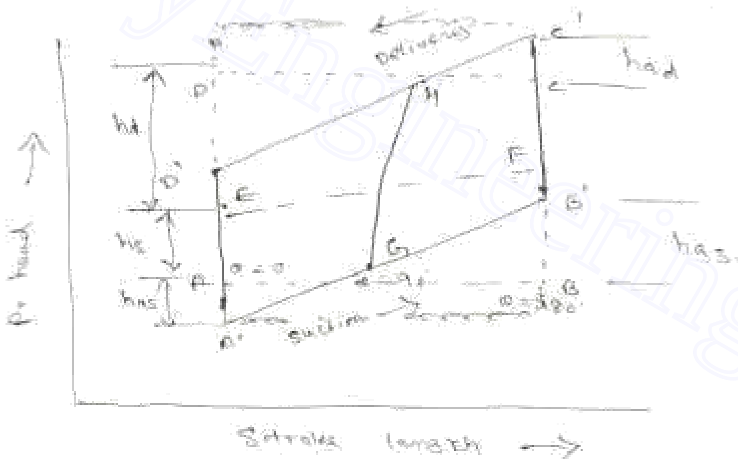
The Pr head due to acceleration in the suction pipe is given by eqn.

$$h_{as} = \frac{l_s}{g} \times \frac{A}{a_s} \omega^2 r\cos\theta$$

when $\theta = 0°$, $\cos\theta = 1$ and $h_{as} = \frac{l_s}{g} \times \frac{A}{a_s} \omega^2 r$

when $\theta = 90°$, $\cos\theta = 0$ and $h_{as} = 0$

when $\theta = 180°$, $\cos\theta = -1$ and $h_{as} = \frac{l_s}{g} \times \frac{A}{a_s} \omega^2 r$.



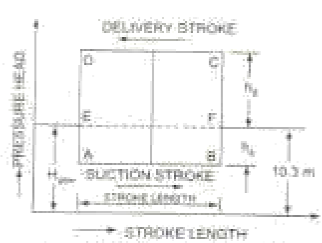


Fig. 20.4 Ideal indicator diagram.

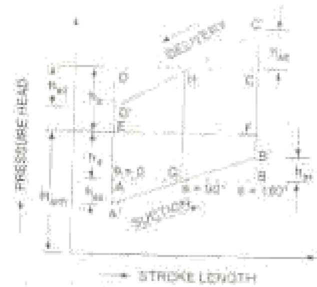


Fig. 20.5 Effect of acceleration on indicator diagram.

Problem 2.4:

The length and diameter of a suction pipe of a single-acting reciprocating pump are 5m and 10cm respectively. The pump has a plunger of diameter 15cm and a stroke length of 35cm. The centre of the pump is 3m above the water surface in the pump. The atm pr head is 10.3 m of water and pump is running at 35 r.p.m. Determine

(i) pr head due to acceleration at the beginning of the suction stroke.

(ii) Max pr head due to acceleration and

(iii) pr head in the cylinder at the beginning and at the end of the stroke.

GiD:

$l_s = 5\,m$

$d = 10\,cm$

$= \frac{10}{100} = 0.1\,m$

$a_s = \pi/4\, d^2 = 7.854 \times 10^{-3}$

$= 0.007854\,m^2$

$D = 15\,cm = \frac{15}{100} = 0.15\,m$

$A = \pi/4\, D^2$

$= 0.01767\,m^2$

$L = 35\,cm$

$= \frac{35}{100} = 0.35\,m$

$r = L/2 = \frac{0.35}{2} = 0.175\,m$

The centre of pump is 3 m above the water surface in the pump } suction head $h_s = 3\,m$

H_{atm} = 10.3 m of water.

N = 35 r.p.m.

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 35}{60}$$

$$= 3.665 \text{ rad/sec}$$

(i) Pr head due to acceleration in the suction pipe.

$$\boxed{h_{as} = \frac{l_s}{g} \times \frac{A}{a_s} \times \omega^2 r \cos\theta}$$

At the beginning stroke $\theta = 0°$

hence $\cos(0°) = 1$

$$\therefore h_{as} = \frac{5}{9.81} \times \frac{0.01767}{0.007854} \times 3.665^2 \, (0.175) \cos 1$$

$$= 2.695 \text{ m.}$$

(ii) Max Pr head due to acceleration in suction pipe.

$$\boxed{(h_{as})_{max} = \frac{l_s}{g} \times \frac{A}{a_s} \times \omega^2 r}$$

$$= 2.695 \text{ m.}$$

(iii) Pr head in the cylinder at the beginning of suction stroke.

$$= h_s + h_{as} = 3.0 + 2.695 = 5.695$$

Pr head in the cylinder is below the atm pr head.

∴ Absolute pr head in the cylinder at the beginning of suction stroke.

$$= \text{Atm pr head} - 5.695$$

$$= 10.3 - 5.695 = 4.605 \text{ m of water (abs)}$$

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$|||^{ly}$, Pr head in the cylinder at the end of suction stroke:

$= h_s - h_{as} = 3.0 - 2.695 = 0.305$ m below atm pr head

$= 10.3 - 0.305$

$= 9.995$ m of water (abs)

Result:

Problem 20.6:

A single acting reciprocating pump has piston diameter 12.5 cm and stroke length 30 cm. The centre of the pump is 4 m above the water level in the sump. The diameter and length of suction pipe are 7.5 cm and 4 m respectively. The separation occurs if the absolute pr head in the cylinder during suction stroke falls below 2.5 m of water. Calculate the max speed at which the pump can run without separation. Take atm pr head = 10.3 m of water.

G.D:

$D = 12.5\,cm = \frac{12.5}{100} = 0.125$ m

$A = \pi/4\, D^2$

$= 0.01227\,m^2$

$L = 30$ cm

$= \frac{30}{2} = 0.3$ m

$r = \frac{L}{2} = \frac{0.3}{2} = 0.15$ m

~~$l_s = 5\,m$~~

$d_s = 7.5\,cm$

$= \frac{7.5}{100} = 0.075\,m$

$a_s = \pi/4\, d^2$

$= 0.004418\,m^2$

$l_s = 7\,m.$

Separation pr head $h_{sep} = 2.5\,m\,(abs)$

Atm pr head $H_{atm} = 10.3\,m.$

(i) Thus the sep can take place at the beginning of stroke only.

(ii) Pr head in cylinder at beginning of suction stroke

$$\boxed{= H_{atm} - (h_s + h_{as})\,m\,(abs)}$$

$= 10.3 - (4.0 + h_{as})$

(iii) In that case pr head in the cylinder at the beginning of stroke becomes $= h_{sep}$

$\therefore h_{sep} = 10.3 - (4.0 + h_{as}).$

$2.5 = 10.3 - 4.0 - h_{as}$

$h_{as} = 10.3 - 4.0 - 2.5 = 3.80\,m, \quad — (1).$

But from eqn, h_{as} at beginning of suction stroke is given by relat

$$\boxed{h_{as} = \frac{l_s}{g} \times \frac{A}{a_s}\,\omega^2 r}$$

$\therefore \theta = 0^\circ \quad \therefore \cos\theta = 1 \quad — (2).$

eqn (1) and (2), we get

$3.80 = \frac{l_s}{g} \times \frac{A}{a_s}\,\omega^2 r$

$3.80 = \frac{7.0}{9.81} \times \frac{0.01227}{0.004418} \times \omega^2 \times 0.15$

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$$3.80 = \frac{7.0}{9.81} \times \frac{0.01227}{0.004418} \times \omega^2 \times 0.15$$

$$3.80 \times 9.81 \times 0.004418 = 7.0 \times 0.01227 \times 0.15 \times \omega^2$$

$$0.16469 = 0.012883\,\omega^2$$

$$\frac{0.16469}{0.012883} = \omega^2$$

$$12.783 = \omega^2$$

$$\omega = \sqrt{12.783} = 3.575 \text{ rad/s}$$

$$\omega = \frac{2\pi N}{60}$$

$$3.575 = \frac{2 \times \pi \times N}{60}$$

$$60 \times 3.575 = 62.83 \times N$$

$$214.5 = 62.83 \times N$$

$$34.13 = N$$

$$N = 34.14 \text{ r.p.m.}$$

Problem [illegible]

The diameter and stroke length of a single-acting reciprocating pump are 100mm and 300mm respectively. The water is lifted to a height of 20m above the centre of the pump. Find the max speed at which the pump may be run so that no separation occurs during the delivery stroke if the diameter and length of delivery pipe are 50mm and 25m respectively. Separation occurs if the absolute pr head in the cylinder during delivery stroke falls below 2.5 m of water. Take atm pr head = 10.3 m of water.

G.D.

Diameter of pump $D = 100\,mm$

$$= \frac{100}{1000} = 0.1\,m$$

$L = 300\,mm$
$= 300/1000 = 0.3\,m$

Crank radius $r = \frac{L}{2} = \frac{0.30}{2} = 0.15\,m$

Delivery head $h_d = 20\,m$

Diameter of delivery pipe $d_d = 50\,mm$

$$= \frac{50}{1000} = 0.5\,m$$

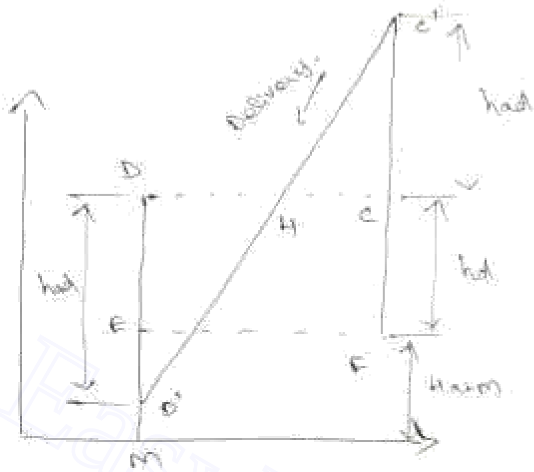
$$a_d = \pi/4\, d^2$$
$$= 0.1963\,m^2$$

Length of delivery pipe $l_d = 25\,m$

Separation pr head $h_{sep} = 2.5\,m$ (ab)

Atm pr head $H_{atm} = 10.3\,m$ of water.

Indicator diagram for delivery stroke only



$D'M = DM - DD'$

$= (DE + EM) - DD'$

$= (hd + H_{atm}) - had$

$\therefore h_{sep} = (hd + H_{atm}) - had$

$2.5 = (20 + 10.3) - had$

$\therefore had = (20 + 10.3) - 2.5 = 27.8\ m$

But acceleration head had at the end of delivery stroke is given by.

$$had = \frac{ld}{g} \times \frac{A}{ad} \omega^2 r$$

$$27.8 = \frac{25}{9.81} \times \frac{\pi/4\, D^2}{\pi/4\, d_d^2} \times \omega^2 \times 0.15$$

$$= \frac{25}{9.81} \times \frac{D^2}{d_d^2} \times \omega^2 \times 0.15$$

$$= \frac{25}{9.81} \times \left(\frac{0.1}{.05}\right)^2 \times \omega^2 \times .15$$

$$= 1.529\, \omega^2$$

$$\omega = \sqrt{27.8/1.529} = 4.264 \text{ rad/s.}$$

We know that

$$\omega = \frac{2\pi N}{60}$$

$$N = \frac{60 \times \omega}{2\pi} = \frac{60 \times 4.264}{2\pi} = 40.72 \text{ r.p.m}$$

UNIT V

TURBINE 1 [redacted]

Velocity triangles and W.D. Pelton wheel:

$$h_f = \frac{4 f L V^2}{D \times 2g}$$

D = Dia of penstock,

V_1 = velocity of jet.

$$\boxed{u = u_1 = u_2 = \frac{\pi D N}{60}}$$

$V_{r1} = V_1 - u_1 = V_1 - u$

$V_{w1} = V_1$

$\alpha = 0,$

$\theta = 0$

$V_{w1} = V_1$ and $V_{w2} = V_{r2} \cos\phi - u_2$.

$$\boxed{= \frac{\rho a V_1 [V_{w1} + V_{w2}]}{\rho a V_1 \times g}}$$

$= \frac{1}{g} [V_{w1} + V_{w2}] \times u.$

Hydraulic η = $\frac{\text{work done per sec}}{\text{K.E. of jet water}}$

A Pelton wheel is to be designed. Pg: 862.

Given:

$S.P = 11,772 \text{ kW}$

$H = 380 \text{ m}$

$N = 750 \text{ r.p.m.}$

$\eta_o = 86\%$ or 0.86

Ratio of jet dia to wheel dia $= \frac{d}{D} = \frac{1}{6}$.

$K_{v_1} = C_v = 0.985$

$K_{u_1} = 0.45$

$\boxed{V_1 = C_v\sqrt{2gh}} = 0.985\sqrt{2 \times 9.81 \times 380}$

$= 85.05 \text{ m/s}$.

$u = u_1 = u_2$.

$=$ speed ratio $\times \sqrt{2gh} = 0.45 \times \sqrt{2 \times 9.81 \times 380}$

$= 38.85 \text{ m/s}$.

$u = \frac{\pi D N}{60}$ $\therefore$

$$\boxed{D = \frac{60 \times 38.85}{\pi \times N}}$$

$\frac{d}{D} = 1/6$

Dia of jet $= d = \frac{1}{6} \times D = \frac{0.869}{6}$

$= 0.165 \, m$

Discharge of one jet $= q =$ Area of jet $\times$ velocity of jet

$= \frac{\pi}{4} d^2 \, V_1 = \frac{\pi}{4} (0.165)^2 \times 85.05 \, m^3/s$

$= 1.818 \, m^3/s$

$$\boxed{\eta_o = \frac{S.P}{W.P} = \frac{11772}{\rho g \times Q \times H}}$$

$$0.86 = \frac{11772 \times 1000}{1000 \times 9.81 \times Q \times 380}$$

$$Q = \frac{11772 \times 1000}{1000 \times 9.81 \times 380 \times 0.86}$$

$= 3.672 \, m^3/s$

$$\boxed{\text{Number of jets} = \frac{Q}{q}} = \frac{3.672}{1.818} = 2 \text{ jets.}$$

2. The penstock supplying water from a reservoir to the Pelton wheel with a gross head of 500 m. One third of gross head is lost

Sol:

$h_g = 500\,m$

$h_f = \frac{h_g}{3} = \frac{500}{3}$

$= 166.7\,m$

$H = h_g - h_f = 500 - 166.7 = 333.30\,m$

$Q = 20\,m^3/s$

$= 165°$

$\phi = 180° - 165° = 15°$

$= 0.45$

$C_v = 1.0$

$\boxed{V_1 = C_v\sqrt{2gH}} = 1.0 \times \sqrt{2 \times 9.81 \times 333.3}$

$= 80.86\,m/s$

$u = \text{speed ratio} \times \sqrt{2gH}$

$\boxed{u = u_1 = u_2} = 0.45 \times \sqrt{2 \times 9.81 \times 333.33}$

$= 36.387\,m/s$

$V_{r1} = V_1 - u_1 = 80.86 - 36.387$

$= 44.473\,m/s$

$V_{w1} = V_1 = 80.86\,m/s$

$V_{r2} = V_{r1} = 44.473$

$\boxed{V_{r2}\cos\phi = u_2 + V_{w2}}$

$V_1 = C_v\sqrt{2gH} = 1 \times \sqrt{2 \times 9.81 \times 90}$

$V_1 = 99.19\ m/s$

$\boxed{W.P = \frac{W \times H}{1000}} = \frac{Q \times \rho \times g \times H}{1000}$

$= \frac{1000 \times 9.81 \times 0.1 \times 700}{1000}$

$= 686.7\ kW$

$\boxed{\eta_h = \frac{2(V_1 - u)(1 + \cos\phi)u}{V_1^2}}$

$= \frac{2(99.19 - 52.76)(1 + \cos 15^\circ) \times 52.76}{99.19 \times 99.19}$

$= 97.18\%$

A Pelton wheel is working under gross head of 400 m. The water is supplied through penstock of diameter 1 m and length 4 km from reservoir.

$C_v = 0.98$

G.D.

$H_g = 400\ m$

$D = 1.0\ m$

$L = 4\ km = 4 \times 1000 = 4000\ m$

$f = 0.008$

$d = 150\ mm = 0.15\ m$

$= 165^\circ$

$\boxed{\rho a V_1 [V_{w1} + V_{w2}] \times u} = \rho Q [V_{w1} + V_{w2}] \times u$.

$= 1000 \times 0.1 [80.86 + 6.571] \times 36.387$

$= 636263.0 \ Nm/s$.

$P = \dfrac{W.D \text{ per sec}}{1000} = \dfrac{636263.0}{1000} = 6362.63 \ kW$.

$\boxed{\eta_H = \dfrac{2[V_{w1} + V_{w2}] \times u}{V_1^2}}$

$= 0.9431 \%$

3. A Pelton wheel is having a mean bucket diameter of 1 m and is running at 1000 r.p.m. The net head on the pelton wheel is 700 m. If the side clearance angle is 15°, ... pg: 544.

Given: $D = 1.0 \ m$

$N = 1000 \ r.p.m$

$\boxed{u = \dfrac{\pi D N}{60}} = \dfrac{\pi \times 1.0 \times 1000}{60}$

$= 52.36 \ m/s$

$H = 700 \ m$

$\phi = 15°$

$Q = 0.1 \ m^3/s$

$V_{r_2} = 0.85\, V_{r_1}$

$u = 0.45 \times$ Jet velocity

$\eta_m = 85\%, = 0.85$

V = Velocity of water penstock

V_1 = Velo of jet of water

Area of penstock $\times V^*$ = Area of jet $\times V_1$

$$\frac{\pi}{4} D^2 \times V^* = \frac{\pi}{4} d^2 \times V_1$$

$$\boxed{V = \frac{d^2}{D^2} \times V_1} = \frac{0.15^2}{1.0^2} \times V_1 = 0.0225 V_1$$

H_g = Head lost due to friction + $\frac{V_1^2}{2g}$.

$$\boxed{400 = \frac{4fLV^2}{D \times 2g} + \frac{V_1^2}{2g}}$$

$$= \frac{4 \times 0.008 \times 4000 \times V^2}{1.0 \times 2 \times 9.81} + \frac{V_1^2}{2g}$$

$$400 = \frac{4 \times 0.008 \times 4000}{2 \times 9.81} \times (0.0225 V_1)^2 + \frac{V_1^2}{2g}$$

$= 0.0033\, V_1^2 + 0.051\, V_1^2$

$$\boxed{V_1 = \sqrt{\frac{400}{0.0543}}} = 85.83 \text{ m/s}$$

$u_1 = 0.45\, V_1$

$V_{r_1} = V_1 - u_1$

$= 85.85$ m/s

Q = Area of jet × velocity of jet $= a \times V_1$

$= \pi/4 \, d^2 \times V_1$

$= \pi/4 \, (0.15)^2 \times 85.83$

$= 1.516 \, m^3/s$.

Work done on wheel per second is given

$$\boxed{= \rho a V_1 [V_{w1} + V_{w2}] \times u = \rho Q [V_{w1} + V_{w2}] \times u}$$

$= 1000 \times 1.516 \, [85.83 + 4.143] \times 38.62$

$= 5023.540 \, Nm/s$.

(i) Power.

$$\boxed{= \frac{W.D}{1000}} = \frac{5023.540}{1000} = 5023.54 \, kW.$$

$$\eta_m = \frac{S.P}{5023.54} = 4268.5 \, kW.$$

$$\boxed{\eta_h = \frac{2 [V_{w1} + V_{w2}] \times u}{V_1^2}}$$

$$= \frac{2 [85.83 + 4.143] \times 38.62}{85.83 \times 85.83}$$

$= 94.34\%$.

Francis turbine:

A Francis turbine with an overall η of 75% is required to produce 148.25 kW power. It is working under a head of 7.62 m. The peripheral velocity

G.D:

$\eta_o = 75\%$

$= 0.75$

S.P = 148.25 kW

H = 7.62 m.

$u_1 = 0.26\sqrt{2gh}$

$= 0.26 \times \sqrt{2 \times 9.81 \times 7.62}$

$= 3.179$ m/s.

$\boxed{V_{f_1} = 0.96 \times \sqrt{2gh}}$ $= 0.96 \times \sqrt{2 \times 9.81 \times 7.62}$

$= 11.738$ m/s.

N = 150 r.p.m.

= 22% of kinetic energy.

$V_{w_2} = 0$ and $V_{f_1} = V_2$

$\eta_h = \dfrac{\text{total head} - \text{loss}}{\text{head at inlet}}$

$\boxed{= \dfrac{H - 0.22H}{H}}$

$= \dfrac{0.78H}{H} = 0.78$

$\boxed{\eta_h = \dfrac{V_{w_1} u_1}{gH}}$

$= \dfrac{0.78 \times 9.81 \times 7.62}{3.179} = 18.34$ m/s.

Flow ratio, Ratio $= \dfrac{V_{f_1}}{V_{w_1}} = \dfrac{11.738}{18.34} = 0.64$

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The inlet angle

$$\tan \alpha = \frac{V_{f1}}{V_{w1} - u_1} = \frac{11.738}{18.34 - 3.179}$$

$$= 0.774$$

$$\alpha = \tan^{-1} 0.774 = 37.74^\circ$$

Diameter of wheel:

$$u_1 = \frac{\pi D_1 N}{60}$$

$$\boxed{D_1 = \frac{60 \times u_1}{\pi \times N}} = \frac{60 \times 3.179}{\pi \times 150}$$

$$= 0.4049 \text{ m}$$

Width of wheel at inlet

$$\eta_o = \frac{SP}{WP} = \frac{148.25}{WP}$$

$$\boxed{WP = \frac{WH}{1000}} = \frac{\rho \times g \times Q \times H}{1000} =$$

$$= \frac{1000 \times 9.81 \times Q \times 7.62}{1000}$$

$$\eta_o = \frac{148.25}{\frac{1000 \times 9.81 \times Q \times 7.62}{1000}}$$

$$= \frac{148.25 \times 1000}{1000 \times 9.81 \times Q \times 7.62}$$

$$Q = \frac{148.25 \times 1000}{1000 \times 9.81 \times 7.62 \times \eta_o}$$

$= 2.664\ m^3/s$

$Q = \pi D_1 \times B_1 \times V_{f_1}$

$\boxed{2.664 = \pi \times 0.4447 \times B_1 \times 11.738}$

$B_1 = \dfrac{2.664}{\pi \times 0.4447 \times 11.738}$

$= 0.1[illegible]\ m$

24. The following data is given for a F.T. [illegible] head

$H = 60m$, speed $N = 700$ r.p.m, shaft power 294.3 kW

$\eta_0 = 84\%$, $\eta_h = 93\%$, flow ratio $= 0.20$, [illegible]

Soln:

$H = 60m$

$N = 700$ r.p.m

$= 294.3$ kW

$\eta_0 = 84\% = 0.84$

$\eta_h = 93\% = 0.93$

$\boxed{\dfrac{V_{f_1}}{\sqrt{2gh}} = 0.20}$

$V_{f_1} = 0.20 \times \sqrt{2gh}$

$= 0.20 \times \sqrt{2 \times 9.81 \times 60}$

$= 6.862\ m/s$

Breadth ratio

$\dfrac{B_1}{D_1} = 0.1$

$D_1 = 2 \times$ inner diameter $= 2 \times D_2$

Actual area flow $= 0.95\,\pi D_1 \times B_1$

Discharge at outlet = Radial.

$V_{w_2} = 0$ and $V_{f_2} = V_2$

$$\boxed{\eta_o = \frac{S.P.}{W.P.}}$$

$$0.84 = \frac{294.3}{W.P.}$$

$$W.P. = \frac{294.3}{0.84} = 350.357\ \text{kW}.$$

$$\frac{1000 \times 9.81 \times Q \times 60}{1000} = 350.357$$

$$Q = \frac{350.357 \times 1000}{60 \times 1000 \times 9.81} = 0.5952\ m^3/s.$$

Q = Actual area of flow $\times$ velocity of flow.

$$\boxed{= 0.95\,\pi D_1 \times B_1 \times V_{f_1}}$$

$= 0.95 \times \pi \times D_1 \times 0.1 D_1 \times V_{f_1}$

$0.5952 = 0.95 \times \pi \times D_1 \times 0.1 \times D_1 \times 6.262 = 2.648 D_1^2$

$$\boxed{D_1 = \sqrt{\frac{0.5952}{2.648}}} = 0.54\ m.$$

$$\frac{B_1}{D_1} = 0.1$$

Speed runner at inlet

$$\boxed{u_1 = \frac{\pi D_1 N}{60}} = \frac{\pi \times 0.54 \times 700}{60}$$

$= 19.79\ m/s$

$$\eta_H = \frac{V_{w1} u_1}{gH}$$

$$V_{w1} = \frac{0.73 \times 9.81 \times 60}{17.77} = 24.66 \text{ m/s.}$$

Guide blade angle.

$$\tan\alpha = \frac{V_{f1}}{V_{w1}} = \frac{6.862}{24.66} = 0.278.$$

$$\boxed{\alpha = \tan^{-1}(0.278)}.$$

$$= 13.928^\circ$$

Runner vane angle at inlet and outlet.

$$\boxed{\tan\theta = \frac{V_{f1}}{V_{w1} - u_1}} = \frac{6.862}{24.66 - 17.77} = 0.892.$$

$$\theta = \tan^{-1} 0.892.$$

$$\tan\phi = \frac{V_{f2}}{u_2} = \frac{V_{f1}}{u_2} = \frac{6.862}{u_2}$$

$$\boxed{u_2 = \frac{\pi D_2 N}{60}} = \frac{\pi D_1 N}{2 \times 60}$$

$$= 9.896 \text{ m/s.}$$

$$\tan\phi = \frac{6.862}{9.896} = 0.6934.$$

$$\boxed{\phi = \tan^{-1} 0.6934^\circ}.$$

Centrifugal pump.

When 1.2 m diameter [illegible] at [illegible] will pump 1880 l/s. the [illegible] which the vanes make

Pg: 9.47.

Sol:

$D_2 = 1.2\,m.$

$N = 200\,r.p.m.$

$Q = 1.880\,m^3/s.$

$H_m = 6.1\,m.$

$\phi = 26^\circ$

$V_{f_2} = 2.5\,m/s.$

$D_1 = 0.6\,m.$

(i) $\eta_{man} = \dfrac{g\,H_m}{V_{w_2}\,u_2}.$

$\boxed{u_2 = \dfrac{\pi D_2 N}{60}} = \dfrac{\pi \times 1.2 \times 200}{60}$

$= 12.56\,m/s.$

$\boxed{\tan\phi = \dfrac{V_{f_2}}{u_2 - V_{w_2}}}$

$= \dfrac{2.5}{\tan 26^\circ} = 5.13.$

$V_{w_2} = u_2 - 5.13 = 12.56 - 5.13 = 7.43\,m/s.$

$\eta_{man} = \dfrac{9.81 \times 6.0}{7.43 \times 12.56} = 0.63 = 63\%$

$\boxed{\dfrac{u_2^2}{2g} - \dfrac{u_1^2}{2g} = H_m}$

$u_2 = \omega \times r_2$ and $u_1 = \omega \times r_1$

$\dfrac{(\omega \times r_2)^2}{2g} - \dfrac{(\omega \times r_1)^2}{2g} = H_m.$